Mathematical Modeling, Technology, and the Environment

Beverly J. Ferrucci, Keene State College, Keene, New Hampshire, USA
Jack A. Carter, California State University, Hayward, California, USA

Abstract Several environmental problems that can be modeled using the graphing, curve-fitting, and programming technology of graphing calculators are presented in this paper. These problems reflect real world environmental concerns and social issues that help motivate student interest in mathematical modeling. Models for the controlled harvesting of animals, waste disposal, and the world's consumption of natural resources are illustrated using technology available in the current versions of today's mathematically powerful calculators.

Introduction

Mathematical modeling can be viewed as the process of describing phenomena in terms of mathematical equations. This paper accepts this notion and presents problems that can be applied as models and simulations of the environment. The goal here is to promote applications of calculator-based models and simulations as a tool for planning, implementing, and monitoring a sustainable future for environmental resources.

Resources are discussed from the perspective of mathematical modeling and data analysis by using the basic statistical regression features found on a graphing calculator such as the TI-83 or TI-89. The discussion involves three basic parts: (1) introduction of an environmental problem; (2) development of a mathematical representation for the problem; and (3) application of the mathematical model using the graphing calculator.
Environmental problems such as the ones included in this paper are now used in classes and workshops for both prospective and practising teachers. Since these problems present real-world environmental concerns and social issues, student interest and motivation in mathematical modeling is apt to be heightened.

The first problem explores trends in the world-wide consumption of natural gas. It investigates various modeling techniques by analyzing data from seven different regions of the world.
A major concern of many municipal agencies, the disposal of garbage into landfills, is the focus of the second problem. It utilizes the capabilities of the graphing calculator to investigate the best models for predicting the amount of garbage produced in the future.

The third problem looks at an environmental situation dealing with the North American bison. Federal wildlife officials periodically consider the implementation of a controlled harvesting program for bison, and this problem models the population growth and impact of various harvesting rates on a herd's size. It is another good illustration of a successful mathematical modeling effort.

Problem 1

One of the most critical environmental problems facing humanity today is the depletion of natural resources. The trends in natural gas consumption present interesting insights into the characteristics of exponential growth and raise questions about the future availability of these resources.

An analysis of resource depletion data provides an opportunity to find a function which best fits the data. This analysis allows for the creation of symbolic, tabular, and graphical models that clarify past activities and justify making conjectures about the future. The consumption of natural gas in various regions of the world for the years 1989-1998 is detailed in Table 1. These data allow students to explore relationships about regional trends in the consumption of natural gas.

Table 1

World Natural Gas Consumption 1989 - 1998

(10^9 Cubic Feet)

Region	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
North America	22.77	22.73	23.10	23.83	24.69	25.35	26.18	26.87	26.92	26.29
Central and South America	2.31	2.21	2.35	2.37	2.55	2.70	2.86	3.06	3.24	3.41
Western Europe	9.86	10.12	11.06	11.10	11.46	11.69	12.57	13.92	13.75	14.24
Eastern Europe and Former USSR	28.09	28.42	27.74	26.30	26.23	24.11	23.21	23.60	22.34	22.21
Middle East	3.75	3.77	3.77	4.20	4.47	4.75	4.96	5.52	6.13	6.54
Africa	1.50	1.52	1.69	1.66	1.71	1.79	1.88	1.95	1.95	2.02
Far East and Oceania	5.64	6.02	6.31	6.77	7.29	7.95	8.36	9.09	9.45	9.67
World Total	**73.43**	**74.78**	**76.02**	**76.23**	**78.40**	**78.34**	**80.01**	**84.01**	**83.77**	**84.40**

This tabular model spotlights the relatively consistent decline in consumption in Eastern Europe and the former USSR, while the total world consumption, after a temporary slowdown, continues to grow.

For example, an analysis of the consumption data for the Middle East using the regression models on a graphing calculator shows that the linear ($Y = a + bX$) and exponential ($Y = a\, e^{bx}$) models produce correlation coefficients of 0.97. The logarithmic ($Y = a + b\ln X$) and power regression ($Y = a\, X^b$) models each result in correlations of 0.98. In addition, the linear, logarithmic, exponential, and power regression models for the data analysis of the World Total of natural gas consumption result in correlations of 0.98. Environmental resource data such as these provide many worthwhile modeling explorations for students, and may motivate students to investigate the use of natural resources in their own countries.

Table 2 displays the magnitudes of natural gas consumption in the United States from 1890 to 1990.

Table 2

United States Natural Gas Consumption

(10^9 Cubic Feet)

1890	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990
0.3	0.3	0.5	0.8	1.9	2.7	6.3	12.8	21.9	20.2	17.2

An observation of the sequence of terms does not result in a set of differences that are constant. Thus, it may be concluded that the relationship between natural gas consumption and time in years can not be given by a polynomial function. However, since there is an approximately constant multiple between the terms, these data can best be described by a geometric sequence. The geometric mean of these ratios is approximately 1.84 and the resulting associated equation that generates these data can be given as $C = 0.3\ (1.84)^t$.

Notice that up to 1970, except for the depression years of 1930 to 1940, there is an approximate doubling of annual natural gas consumption every 10 years. As these two observations suggest, the annual growth of consumption up to 1970 can best be described by the exponential function $C = C_0(1 + r)^t$ where r, the average annual growth rate, can be calculated to be 0.063 with $C_0 = 0.3$ and t being the number of years since 1890.

From the graph of this exponential function, $C = 0.3(1.063)^t$, it can be shown that the consumption increases slightly at first and then displays a sharp rise once the magnitude becomes sufficiently large. By entering the data from Table 2 into a graphing calculator, the exponential model of $C = 0.142e^{.0628t}$ can be confirmed as the best fit for these data resulting in a correlation coefficient of r = 0.997. However, students can easily see that these two exponential models are not equivalent, and this becomes readily apparent when the graphs of the two models are examined.

Another interesting aspect of this problem is its interdisciplinary nature. A remarkable mathematical, as well as environmental phenomenon of exponential growth is its approximate annual increments of consumption, with an average fixed rate of growth that leads to a doubling of consumption in fixed periods of time. Thus, if the United States had continued the same consumption rate as it did from 1900 to 1970, annual consumption would have reached approximately 40 units by 1980 and 80 units by 1990.

The ultimate quantity of natural gas was estimated to be 1290 units by Hubbert (1969). Hubbert also pioneered the application of modeling to environmental problems and predicted a decline in both the US production and consumption of natural gas as a result of the usage and availability of fossil fuels. Further, he described the complete cycle of production as a bell-shaped curve with a peak production of 25 units per year.

Prominent mathematician, Maria Agnesi, is often associated with the bell-shaped curve. This curve is named the versiera of Agnesi and its equation is $x^2y = a^2(a - y)$. The interesting mathematical properties of this curve present many intriguing applications for students.

An interesting graph by which to investigate Hubbert's vision of the complete cycle of natural gas consumption can be created by a translation of the versiera of Agnesi along the independent axis. By starting at a = 21.9 (the maximum consumption level from Table 2) and by translating the versiera of Agnesi 90 units to the right of origin, students have a wonderful opportunity to explore this phenomenon.

Students also may wish to investigate the prediction by Hubbert in 1969 to one made 30 years later. Hakes (1999) predicted that over the next twenty years the United States' natural gas market will increase by more than two and one half times. By this model, natural gas consumption is expected to account for more than 28% of the total US energy consumption, compared to 24% in 1997. Particularly, gas consumption is expected to increase at the rate of 1.7% annually from 1997 to 2020 due mainly to the growth of gas-fired electricity generation.

Problem 2

The United States is the world's largest producer of garbage. Each year Americans throw out ten times their weight in garbage which is the equivalent of about one half ton per person. However, less than one quarter of it is recycled. and the rest is either incinerated or buried in landfills. The Environmental Protection Agency (www.learner.org) estimates that half of the landfills that were operating in 1990 are now closed for one of two reasons: They are filled to capacity or contaminating nearby groundwater. In addition, about 86% of landfills are leaking toxic materials into lakes, rivers, and streams. Once groundwater is contaminated, it is extremely expensive, difficult, and sometimes even impossible to clean.

With some forethought, people could recycle landfilled waste which includes materials such as metal, paper, and glass. This would help reduce the demand on these items and help to eliminate potentially severe environmental, economic, and public health problems.

This classroom problem involves a municipality that currently deposits over 95% of its trash into a landfill. Since the landfill is expected to close in five years, the city's planners need an alternative means of waste disposal. As part of their planning, they need to determine the amount of garbage that the city will produce by the year 2005. Table 3 lists the data on the municipality's past garbage production.

Table 3
Municipal Garbage Production, 1970-1995

Year	Garbage Produced per day (in tons)
1970	398
1975	451
1980	510
1985	603
1990	749
1995	814

These data can be entered into a graphing calculator to investigate the efficacy of various models. As with the data on natural gas consumption, several regression models produce correlation coefficients that are quite close to 1. The logarithmic model produces the best correlation, and is possible to compare the year 2005 prediction with this model to those of the other models. Table 4 displays various models, their associated correlation coefficients, and their predictions for the amount of garbage produced per day in 2005.

Table 4
Predictive Models for 2005 Municipal Garbage Production

Model	Correlation Coefficient	Predicted Tons Per Day in 2005
Linear	.986	981.83
Logarithmic	.980	939.11
Exponential	.994	1118.40
Power	.992	1041.90

Table 4 shows that the predictions produced by the four models provide a good demonstration of the considerable differences that can result from using models with similar correlation coefficients in predicting future environmental situations.

Problem 3
An estimated 40 million bison were roaming the western United States in 1830. Sixty years later only about 200 bison remained due to poor policies for managing this resource. Today, on lands under auspices of government agencies, there are about 26,000 bison distributed as follows:

Adult males	10,400	Male calves	3,380
Adult females	9,100	Female calves	3,120

The agencies are considering the implementation of a controlled harvesting program for these animals. Particularly, they are considering allowing the harvesting of 1,000 adult males annually. What will be the effect of this program on herd size over the next ten years?

To answer this question, we can first develop a model of population growth for the herd. Recent studies provide the following information:

1. Calves become sexually active adults at two years of age.

2. For every 100 adult female at the beginning of a year, 90 calves are born, of which 48 are males and 42 are females.

3. Only 50 percent of the calves reach one year of age, and of these, 60 percent reach maturity. The survival rate for adults is 0.90.

If we let M and F denote the number of male and female adults, then initially M = 10,400 and F = 9,100. Without additional information about the age distribution of the calves, we could assume that two-thirds of the males and two-thirds of the females are newborns and denote these numbers by N and P, respectively. So, N = (2/3) (3,380) and P = (2/3) (3,120). We can then assume the remaining calves are one year of age and denote the number of these males and females by B and G. At the beginning of the period one year from now, the bison population can be described as follows:

1.	New male calves (N)	.48F
2.	New female calves (P)	.42F
3.	One-year-old male calves (B)	.5N
4.	One-year-old female calves (G)	.5P
5.	Adult males (M)	.90M + .6B - 1000
6.	Adult females (F)	.90F + .6G

This yields a herd of M + F + N + P + B + G.

Using the information we have developed, we can complete the following chart to solve the problem.

Bison Population Distribution					
Time	Herd Size	Adult Males	Adult Females	Male Calves	Female Calves
0	26000	10400	9100	3380	3120
1					
2					
.					
.					
.					
10					

The following TI-83 program can be used to explore and extend this model.

```
PROGRAM:BISON
:Disp "ADULT MALES?"
:Input M
:Disp "ADULT FEMALES?"
:Input F
:Disp "MALE CALVES?"
:Input C
:Disp "FEMALE CALVES?"
:Input D
:(2/3)*C→N
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:C-N→B
:(2/3)*D→P
:D-P→G
:0→1
:Lbl   1
:Disp I
:I+1→I
:M+F+N+P+B+G→S
:Disp S
:Disp M
:Disp F
:N+B→K
:P+G→L
:Disp K
:Disp L
:F→U
:N→V
:P→W
:.9M+.6B-1000→M
:.9F+.6G→F
:.48U→N
:.42U→P
:.5V→B
:.5W→G
:Disp
:Pause
:If I<11
:Then
:Goto 1
:End
:Stop
```

One motivation in utilizing these modeling problems is to provide students with realistic opportunities to connect mathematics to significant environmental and social problems while incorporating recent advances in technology. Problems such as these serve to demonstrate that mathematical models can be developed in a variety of forms: graphs, tables, charts, equations, and programs.

We have found that students' motivation is increased when they participate in pertinent mathematical applications that aim to alleviate the degradation of the environment. Also, the students' responses to these learning activities appear to have changed some of their attitudes. While they understand the limits of using these functions to model data, and that models may not always fit the data well, they also learn that just observing data patterns may not give all the information needed to mathematically model and solve environmental problems.

The depletion, recycling, and controlled management of resources are critical environmental issues facing the world today. By integrating data analysis and mathematical modeling techniques with the capabilities of graphing calculators, it is hoped that a deeper understanding of environmental problems and their impact on society will result.

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