Geometric Thought Within School Mathematics Textbooks in Jordan

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Introduction

Geometry is one of the core school mathematics content and one of the major content standards that faced changes in regard of its structure and its teaching and learning in school. The informal structure of geometry started by measurement and experimentation before 3000 B.C, while its logical structure started before 300 B.C.

In the last three decades, the changes in geometry were represented by two points of view; the first is epistimological which consider geometry as a deductive knowledge and a socio-constructivist knowledge that depends on doing mathematics by discovering patterns , modeling ,and visualizing . The second is a psychological view which describes the development of geometric thinking levels by the Van Heile model of reasoning (Nott, 1996 ;Romberg , 1992).

Students always respond to geometry: we could never do proofs, many of them do not understand it . These responses might be based on the nature of geometry curriculum materials that use the abstraction approach of geometrical structure.

This analytical study is conducted in regard of four issues related to teaching and learning geometry . These issues are : the role of mathematics textbooks in school mathematics education reform ; geometry as a main mathematics content standards ; the structure of geometric thought in school mathematics textbooks with regard to Van Hiele levels and his teaching and learning model of geometric thinking ; and what related research had revealed .

The role of math Textbooks in school math education reform

School mathematics textbooks are the major source of mathematics content in Jordan's schools, and the best to represent the national math curriculum. Also, it is a major source of instructional methods for math teachers and thinking processes for their students. In Jordan, although the development of math curriculum and textbooks has been taken place for the last decade, more work still needed for the 21st century. In the meanwhile neither technology of calculator nor computer instructional software are used to promote teaching and learning of geometric thought, except very few research conducted by the faculties in the universities .

In the new math education reform, many questions are raised . One of these questions deals with the role of textbooks in math education reform. Nationally, there is stress that mathematics textbooks should be one of the many resources available to students. In addition to other curriculum materials, good math textbooks should help the teacher to emphasize important mathematical ideas and change his routine instructional methods to a different approach that depend on the role of students to apply the mathematical ideas (conceptual or procedural) and use them to solve nonroutine problems (NCTM ,1997-1998 Handbook).

Comparing this international demand with the role of textbooks in Jordan, there is a national mathematics curriculum which is integrated with textbooks as essential element and unique resource in implementing that curriculum . So, it is important to take care of the math textbooks to be built carefully.

Geometry as A Major School Math content Standard

Teaching and learning geometry is an essential task for both math teachers and students in both school stages (basic (1-10) and secondary (11- 12) grades).In the NCTM 1998 discussion draft of principles and standards for school mathematics, geometry and spatial sense is the third content standard for grades k-12 under four major standards (see page 61). these four standards emphasize that teaching and learning geometry is integrated with the geometric thought and the model of teaching and learning geometry suggested by Van Hiele. Also, it is clear that the standards of geometry can be achieved by using different models; physical, pictorial, words, symbols, or software tools.

In Jordan, the outlines of school mathematics curriculum for the basic stage stated several goals for teaching geometry not as one standard but as different modules. These goals can be summarized in studying geometric shapes of two and three dimensions through visualization and analyzing their properties to grasp the logical geometric structure in mathematics
secondary education , and using formal reasoning and proof to solve mathematics problems. But the major point is that these goals are not stated as clear standards, where this enforce the teachers to depend on the specific objectives to treat the subject matter as parts and not as a whole .In the meanwhile, this weakens the ability of students to learn .The previous goals are achieved through a content represented by plane, coordinate , and three dimensional geometries

Comparing the NCTM standards and the national mathematics curriculum goals, there is a common view for teaching geometry in schools. But the national curriculum does not have a clear view about what is called standards, also mathematics teachers are not oriented to state their own standards that fit with the international standards .

***The Nature of Van Hiele Geometric Thought*:**

Reviewing the theoretical and research background of student cognition in geometry (Van hiele,1994 ;Teppo ,1991 ;Swafford Janes and Therton ,1997) ,Van Hiele learning and teaching model describes the different levels of thinking that students pass through as they move from the perception level to the formal deduction level . Five geometrical thought levels have been substantiated by research: recognition, analytical, relational or ordering, informal deduction, and formal deduction.

In regard of the role of mathematics textbooks, geometry as a major content standard , and the student cognition in geometry , mathematics textbooks should be built stressing this cognition represented the previously stated levels. And if we assume that teaching and learning mathematics are proceeding in the right way , van Hiele geometric thought should be taken into account ; either while constructing mathematics textbooks , or through classroom instruction .

***Research Based Knowledge Regarding geometric Thought* :**

Most of the researchers were concerned about classifying student's geometric thinking levels through certain content; i.e assessing the geometric thought in regard of Van Hiele levels . Few researchers used the experimental approach by teaching geometrical concepts to one group using Van Hiele model, and rarely they stress the content analysis approach of geometrical content in mathematics textbooks.

Locally in Jordan, two studies were conducted regarding Van Hiele levels of geometric thought. One examined the levels among the student teachers of elementary education at Yarmouk university (khasawneh, 1994) . The other study examined the effect of LOGO environment for learning and teaching geometry on the eighth graders development of Van Hiele geometric thought (Khasawneh and Algamedi, 1997) .The results revealed that the students teachers were ranked at low levels of van Hiele geometric thought , and there was positive effect of LOGO environment on the development of geometric thought among eighth graders who were ranked at high levels.

Based on the previous revision of the role of mathematics textbooks , geometry as a major standard of mathematics content , the structure of geometric thought , and research based knowledge , this analytical study is conducted to answer the following questions :

1. What is the structure of geometric thought in mathematics textbooks for grades six to nine ? Does that structure develop according to Van Hiele model ? .
2. What is the structure of geometric skills in mathematics textbooks for grades six to nine ? Does that structure develop according to Hoffer
model ? .
3. Does the geometric thought develop through mathematics textbooks according to the students' class level?

❖ ***METHODOLOGY***

content analysis approach was used to answer the questions of this study . Mathematics, copyright 1998, textbooks for grades six, seven, eight, and nine were the sample of analysis .These textbooks were assigned to be taught to the previous grades by the ministry of education in Jordan by the decision of the council of education, number 4\92 on January ,8,1992 . The first step was to analyze the geometric modules included in each textbook in order to assign the units of analysis and to facilitate the analysis in light of the instrument . Appendices 1,2,3 and 4 summarize the geometric content according to the conceptual , procedural , and applications and problem solving knowledge .

❖ ***Units of analysis*:**

Different units of analysis were used : activities, examples, definitions, generalizations, and questions(exercises, routine and nonroutine problems).These units are included in the different lessons within each geometric module, and defined as the followings:

- ***Activity: any task presented on the explanation pages that help student to learn different ideas by himself (individually).***
- ***Example: any solved routine or nonroutine practice or problem which is integrated with the presentation of the different ideas on the explanation pages, that help student to get the different attributes of either the conceptual or procedural knowledge.***
- ***Definition: statement that included all the critical attributes of geometrical concept, and appears on the explanation pages.***
- ***Generalization: a mathematical relation that connects two or more geometrical concepts such as axioms, principles, and theories.Where it appears on the explanation pages.***
- ***Questions: exercises for the purpose of drill and practice, and routine and nonroutine problems for the purpose of applications and problem solving. All of these questions appear on the question pages at the end of each lesson or module.***

Each unit of analysis was analyzed according to the highest level of geometric thought that it represents.

- ***Instrumentation* :**

Hoffer matrix (4*5) was used, as instrument of content analysis. The first dimension of this matrix is the Van Hiele geometric thinking level (recognition, analysis, ordering, and deduction), while the second is the geometric skills(visualization, description, drawing, logical, application). (See Hoffer,1981:table 1,p.15 ;Kasawneh,1994:table 1 , p.449)

***Reliability of the content analysis*:**

The researchers conducted the content analysis, where each one analyzed the four modules for grades six to nine. After one month, two geometric modules were randomly selected by each researcher and reanalyzed. Intrarater and interrater reliability was considered by finding coefficient of alignment between the first and the second analysis for each researcher, and between every two researchers on the first analysis. The results recorded high coefficients (the lowest 90.6, the highest 93.2) .(See Odeh,1993,p.362)

RESULTS

In order to answer the different questions of the study, the different geometrical modules were analyzed in light of the 4*5 matrix(geometric thinking levels by the geometric skills) using the different units of analysis. (See appendices 5, 6, 7, 8).

Depending on the appendices 5,6,7, and 8, tables 2, 3, 4, and 5 summarize the percentages of the units that represent the geometric thinking levels and the geometric skills in each geometric module for each grade level.

Table 2
Percentages of the geometric thinking levels by the geometric skills\ sixth grade math textbook

Thinking level / Skills	cognition	analysis	ordering	Deductive	Sum	%
Visual	3	8	5	5	21	26
Descriptive	12	9	12	1	34	41
Drawing	6	4	1	1	12	16
Logical	3	4	3	-----	10	12
Application	3	1	1	-----	5	5
Sum	27	26	22	7	82	----
%	33	31	27	9	-----	100

It is clear from table 2 that the geometric content - within the sixth grade textbook - develops according to the geometric thinking levels, where the percentages were 33% for the recognition level, 31% for the analytic, 27% for the ordering, and 9% for the deductive level . While the geometric skills recorded 26%, 41%, 16%, 12%, and 5% for the visual, the descriptive, the drawing, the logical, and the application skills respectively. Regarding the previous percents, the tasks within the sixth grade textbook that need recognition, analysis, and ordering levels are distributed approximately equally , while the deduction has low percent and the descriptive skill has the highest percent among the other skills.

Table 3
Percentages of the geometric thinking levels by the geometric skills/seventh grade math textbook

Thinking level / Skills	cognition	Analysis	Ordering	Deductive	Sum	%
Visual	9	7	6	2	24	31
Descriptive	9	6	6	11	32	41
Drawing	3	2	-	-	5	6
Logical	2	1	2	4	9	13
Application	-	3	4	-	7	9
Sum	23	19	18	17	77	-
%	30	25	23	22	-	100

It is clear from table 3 that the percentages of the tasks integrated with the geometric thinking levels decreases as the level increases, where it begins with 30% for the recognition and ends with 22% for the deductive level .In the mean while, tasks integrated with the descriptive skill recorded the highest percent(41%), followed by the visual skill (31%) ,and the logical (13%) .But the application and the drawing skills recorded 9% and 6% .It is clear that the ranges of the percentages of the tasks integrated with either the geometric levels or the geometric skills for the seventh grade are small compared with that for the sixth grade.

Table 4
Percentages of the content units regarding geometric thinking levels by geometric skills/eighth grade textbook

Thinking level / Skills	cognition	analysis	ordering	Deductive	Sum	%
Visual	1	1	3	5	10	15
Descriptive	2	5	5	10	22	32
Drawing	-	-	-	-	-	-
Logical	2	6	7	10	25	37
Application	1	4	5	1	11	16
Sum	6	16	20	26	68	-
%	9	24	29	38	-	100

Table 4 revealed that the percentages of the tasks integrated with the higher geometric levels in the eighth grade textbook increases as the grade increases. This means that the deductive tasks recorded the highest percent (38%) ,followed by the ordering level(29%)and the analysis one(24%).While the recognition tasks has the lowest percent(9%). In addition ,table 4 showed that the logical skill has the highest percent among the other skills.

Table 5
Percentages of the content units regarding geometric thinking levels by geometric skills/ninth grade textbook

Thinking level / Skills	cognition	Analysis	ordering	Deductive	Sum	%
Visual	-	-	2	9	11	16
Descriptive	3	2	3	8	16	23
Drawing	3	1	2	-	6	9
Logical	2	6	7	12	27	38
Application	1	4	5	-	10	-
Sum	9	13	19	29	70	-
%	13	19	27	41	-	100

Table 5 shows that the deductive tasks has the highest percent(41%) within the ninth grade textbook ,followed by the ordering level(27%). While the analysis (19%) and the recognition(13%) levels recorded lower percents. In the meanwhile, the logical skill has the highest percent(38%) ,followed by the descriptive(23%), the visual(16%), the application(14%), and the drawing(9%).

To answer the third question tables 6 and 7 are formed by using tables 2,3 4, and 5.

Table 6

Percentages of the geometric thinking levels by the instructional levels

Instructional Level / Thinking Level	Sixth Grade	Seventh Grade	Eighth Grade	Ninth Grade
Recognition	33%	30%	9%	13%
Analytic	31%	25%	24%	19%
Ordering	27%	23%	29%	27%
Deductive	9%	22%	38%	41%

Table 7

Percentages of the geometric skills by the instructional levels

Instruction Level / Geomet Skills	Sixth grade	Seventh grade	Eighth grade	Ninth grade
Visual	26%	31%	15%	16%
Descriptive	41%	41%	32%	23%
Drawing	16%	6%	----	9%
Logical	12%	13%	37%	38%
Application	5%	9%	16%	14%

It is clear from table 6 that the geometrical tasks that represent the visual and the descriptive skills decreases with the increase of the instructional level , while the ordering level is approximately the same(27, 23, 29, and 27) through the four grade levels .

On the opposite, the percent of the deductive content increases with the increase of the instructional level (9%, 22% , 38%, and 41%) respectively , and it is clear that the increase in the higher level is great in the eighth and ninth grades . This means that the textbooks concentrate on using the logical reasoning and using proof to solve problems at the higher student's levels.

At the same time table 7 emphasizes that the visual, the descriptive, and the drawing skills decrease with the increase of the instructional level , while the logical and the application skills increase with the growth of the student level. It is also clear that the logical skills recorded high percentages at eighth and ninth grades, while percentages of the content that represent the application skill still low at all grade levels .

Discussion and conclusions :

Geometry and geometric thinking is a vital component of classroom mathematics programs, and it is a focus issue for the 21st century.Regarding this issue, school mathematics textbooks play an important role in developing student's geometric cognition .The results of this analytical study emphasize that role and stress that the structure of geometrical content is shaped to fit with Van Hiele thinking levels. The results revealed that the geometrical modules in the textbooks of the different grades are structured in a way that fit students' experience through their instructional levels .The distribution of the tasks' percents is accomodated with the Van Hiele geometric thought .These percents are affected by the instructional level , the type of geometrical content , and the geometrical experiences presented in the curriculum for grads 1-5 .

Another issue is that the tasks integrated with the geometric thought levels develop through the textbooks of six to nine .This development is concentrated on the ordering and deductive levels .The major question raised : is there any acceptable criterion for the distribution of the percents of the geometrical tasks within the different textbooks ?.

It is difficult to give one answer to this question. If school math textbooks are the major resource for teachers and their students, math textbooks should be built carefully with an acceptable structure of geometric standard in regard of Van Hiele geometric levels.Otherwise ,the teacher's role should be changed, and he should be aware of the geometric thinking levels. In addition, instructional software should be designed to enhance this field.

Another question can be raised : do we need all students to be problem solvers in geometry? .If we propose to achieve equity among students, they should be able to do logical arguments and construct geometrical proofs.

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