Living in the Real-World-System: technology and mathematics as Systemic Thinking mediators

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Abstract: In this paper *Systemic Thinking* is considered as a general philosophy that, by suggesting a "thinking globally, but acting locally" approach, can represent a paradigm shift in how we view the world and a shift away from the view of reductionism or thinking about isolated parts that fit a mechanistic model. After dwelling on the characteristics of Systemic Thinking, particularly on its cross-disciplinary nature, we analyse why and how some mathematical application software implemented as general problem solving tools involving low-medium level mathematical knowledge, as well as some mathematical competences themselves, could play the role of mediators in supporting and creating the conditions for systemic thinking development and in deepening external world knowledge.

Introduction

Systemic Thinking (ST) is described by Licon Khisty as a general philosophy that suggests "thinking globally, but acting locally". She writes that "Systemic thinking represents a major paradigm shift in how we view the world; it is a shift away from the view of reductionism or thinking about isolated parts that fit a mechanistic model." (Licon Khisty, 1997). The term "systemic thinking" refers to the concept of "system". In this sense, the word "system" stands for "a (possibly complex) set of elements that may be of different nature and which interact towards the accomplishment of certain objectives". As this definition implies, a system's components are not only the individual elements composing it but also the network of interactions among them and their purpose.

Systemic thinking is applied in various areas of real-world-life as a tool for analysing and designing systems: economic systems, health systems, educational systems, electoral systems, measurements systems, etc.; it is a cross-disciplinary capability. Systemic thinking presupposes that one builds within his head a global scenario that represents the reference system to be worked on locally. Systemic thinking is therefore a complex thinking process that develops through a sequence of individual thinking acts, even of different nature.

Systemic thinking requires to be supported in each phase by one or more representations systems (particularly mathematical representations) and operating modalities concerning the elements involved, both actual and virtual, mental or external (Bruner, 1966; Chesa & Tarrago, 1988; Tall, 1994).

In this paper we analyse why and how some mathematical application software implemented as general problem solving tools involving low-medium level mathematical knowledge and some mathematical competences (modelling above all) could play the role of mediators (Vigotsky, 1987) in supporting and creating the conditions for systemic thinking development and in deepening external world knowledge.

Technology and Mathematics as Systemic Thinking mediators

We consider a situation (leasing contracts) that may be seen as an open problem; it is a very common everyday situation that contains a strong mathematical component (Lemut & Greco, 1999).

A brief account of the situation is provided, followed by an analysis of the reasoning performed by an adult, Enrico, who is fairly confident with mathematical tools and is used to handling computing devices (calculator, spreadsheet and software programs for symbolic computation).

Signing a lease is a fairly common operation performed, for instance, when running a shop, business or public organisation. Leasing involves two subjects, the leaser and the leaseholder. A prospective leaseholder may be given a choice between various types of contract as the number of instalments, the frequency of repayments (monthly, six-monthly, yearly), the amount of each instalment, the deposit to be paid on signing the contract, the final price for redemption. Both parties will seek to enter the type of contract which, from their individual point of view, is the most convenient.

Problem*: Suppose you are a prospective leaseholder who wants to figure out how leasing plans work and evaluate which of various options are the most convenient.*

Phase 1

Enrico's reasoning: Enrico examines the first leasing plan: eight six-monthly instalments of 2,4 million lire each, for a commodity costing 15 million lire (for simplicity's sake, both the deposit and final redemption are disregarded). The lessor informs him that under this scheme the applicable annual interest rate is approximately 12 percent. Using a pocket calculator, Enrico reckons the overall expense straight away as the product of 2,400,000*8=19,200,000. Then, by calculating 19,200,000-15,000,000)/15,000,000*100, he infers that the interest payable in four years is 28%, so the annual interest is 7% (28:4). But, in this way, the result does not match the lessor's claim.

Analysis of Enrico's reasoning: Enrico instinctively performs a few simple (albeit incorrect) calculations suggested by the data at hand and by his competences, both in mathematics (percentages) and economics. At the end of this phase he applies a first stage of systemic thinking *(ST.1)* when he checks whether the amount of annual interest he worked out is consistent with the information received from the vendor. He then realises that his way of calculating the interest rate is too simplistic, hence he is stimulated towards the acquisition of new knowledge.

Phase 2

Enrico's reasoning: He therefore requests further information and obtains the following explanation: a) each instalment contains a capital share and an interest share; b) the capital and interest shares of each instalment depend on the interest rate related to the number of instalments per year; c) the capital shares that are to be paid back in succession must be calculated using the compound interest formula after *n* periods:

$$\text{instalment} = \text{capital}*(1+\text{interest})^{n},$$

hence $$\text{capital} = \text{instalment}*(1+\text{interest})^{-n}.$$

Analysis of Enrico's reasoning: Enrico seeks out additional information in order to find further relationships within his data that might tell him what he wants to know. In other words, he realises that he must discover other elements of the system underlying lease contracts *(ST.2)*. The intricate situation emerging from the new data acquired tells Enrico that he needs a suitable aid for representing, quantifying and processing the relationships he is trying to express. He is supported by an algebraic model he can manipulate.

Phase 3

Enrico's reasoning: Enrico decides to give up the calculator and resort to a spreadsheet (Fig. 1). At the top of the sheet, he describes and represents the elements relevant to the situation: value of the commodity (A1-B1), deposit (A2-B2), number of instalments (A3-B3), frequency of repayments (A4-B4), instalment amount (A5-B5), redemption price (A6-B6) and total cost (A7-B7). Further down he applies the suggested formulas and represents: a) the progressive number of instalments from 1 to 8 (from A 10 to A17); b) the capital share paid back within each instalment (from B10 to B17); c) the related interest share (from C10 to C17). Finally, he describes the present value in A19-B19.

Analysis of Enrico's reasoning: Enrico sets out a systemic table *(ST.3)* representing the data he has, the results he wants to achieve and the relationships between them. This organisational effort is to a large extent aided by the software tool he employs, the spreadsheet, and by his mathematical competences (formulas' writing and manipulation), either previously acquired or deepened just while using the spreadsheet.

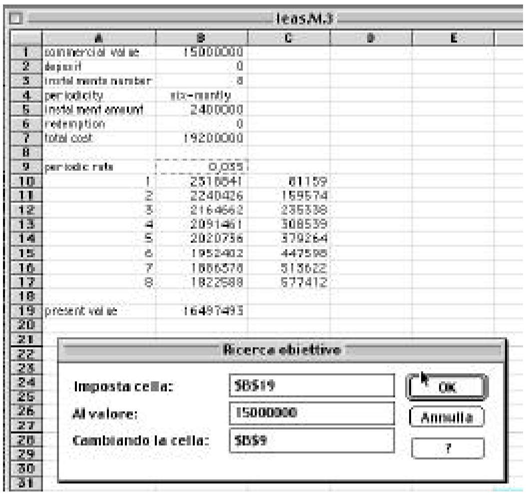
Phase 4

Enrico's reasoning: To use the sheet in an effective way, Enrico needs to approximate roughly a six-monthly rate (SR) he considers reasonable (0,035=0,07:2) and enter it in cell B9 (fig.2a). Having done so, he automatically obtains a first lease plan, but the sum of the capital shares therein (B19 in fig.2a) is out by 15 million lire (commercial value of the commodity). He subsequently realises that, to be consistent with the plan at the top of the sheet, he needs to come up with a six-monthly rate that produces a sum equal to 15 million lire. He is told that a "search objective" command fulfils this purpose and so applies it (fig.2a). In this way, he obtains in cell B9 (fig.2b) the six-monthly interest rate that has been actually applied.

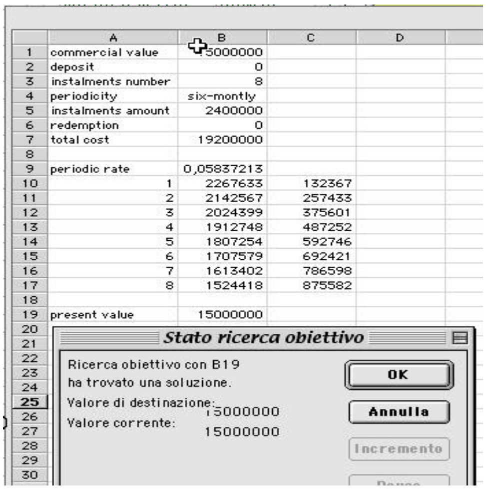
Analysis of Enrico's reasoning: This phase is clearly guided by the characteristics of the spreadsheet itself. Until Enrico comes up with a preliminary hypothesis regarding the six-monthly rate, the sheet calculates evidently incoherent results in the cells from B10 to B17 and from C10 to C17. When Enrico applies the "search objective" command correctly, he is prompted to clarify the relationship between the entities in question, i.e. the amount of the individual instalments and the interest rate applied, the sum of all capital shares and the present value of the commodity. Enrico uses this command to calculate at what interest rate (B9 in fig.2b) the sum of the capital shares is actualised as 15 million lire, given the instalment rate set in the leasing plan (B5) (*ST.4)*. The same command also reckons the instalment amount the vendor should fix if there was a rise or fall in the applicable interest rate (the concepts of variable and parameter are crucial). This facility offered by the spreadsheet allows Enrico to contemplate the situation from two points of view, his own as a buyer and the vendor's, thus expanding his systemic view of the situation *(ST.5)*.

	A	B	C
1	commercial value	15000000	
2	deposit	0	
3	instalments number	8	
4	periodicity	six-montly	
5	instalment amount	2400000	
6	redemption	0	
7	total cost	=B2+B3*B5+B6	
8			
9	periodic rate		
10	1	=$B$5*(1+$B$9)^(-A10)	=$B$5-B10
11	=A10+1	=$B$5*(1+$B$9)^(-A11)	=$B$5-B11
12	=A11+1	=$B$5*(1+$B$9)^(-A12)	=$B$5-B12
13	=A12+1	=$B$5*(1+$B$9)^(-A13)	=$B$5-B13
14	=A13+1	=$B$5*(1+$B$9)^(-A14)	=$B$5-B14
15	=A14+1	=$B$5*(1+$B$9)^(-A15)	=$B$5-B15
16	=A15+1	=$B$5*(1+$B$9)^(-A16)	=$B$5-B16
17	=A16+1	=$B$5*(1+$B$9)^(-A17)	=$B$5-B17
18			
19	present value	=SOMMA(B10:B17)	
20			

Fig.1 – Formulae used



	A	B	C
1	commercial value	15000000	
2	deposit	0	
3	instalments number	8	
4	periodicity	six-montly	
5	instalment amount	2400000	
6	redemption	0	
7	total cost	19200000	
8			
9	periodic rate	0,035	
10	1	2318841	81159
11	2	2240426	159574
12	3	2164662	235338
13	4	2091461	308539
14	5	2020736	379264
15	6	1952402	447598
16	7	1886378	513622
17	8	1822588	577412
18			
19	present value	16497493	



	A	B	C
1	commercial value	15000000	
2	deposit	0	
3	instalments number	8	
4	periodicity	six-montly	
5	instalments amount	2400000	
6	redemption	0	
7	total cost	19200000	
8			
9	periodic rate	0,05837213	
10	1	2267633	132367
11	2	2142567	257433
12	3	2024399	375601
13	4	1912748	487252
14	5	1807254	592746
15	6	1707579	692421
16	7	1613402	786598
17	8	1524418	875582
18			
19	present value	15000000	

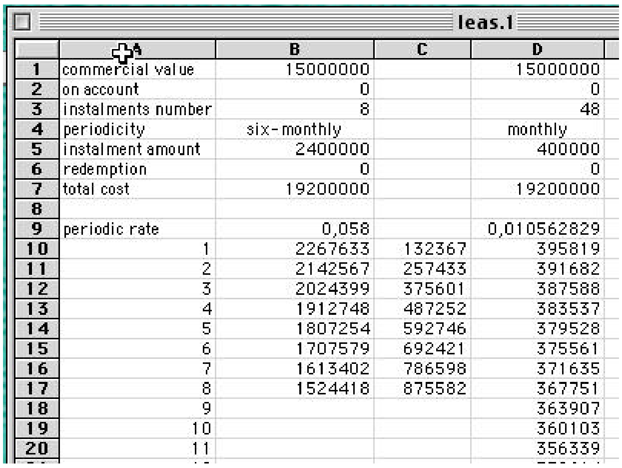
Fig. 2a – Present value (in B19) in case the six-monthly rate (B9) was 3,5% and "search objective" command.
Fig. 2b – The actual value of the six-monthly rate (in B9) has been automatically calculated

Phase 5

Enrico's reasoning: To compare the six-monthly rate (SR) he obtained with the annual lessor's rate, Enrico tries to find out the annual rate (AR) on the basis of the six-monthly rate produced. He notices that, if an initial capital H is expected to yield the same amount of interest no matter whether this is calculated on an annual or six-monthly basis, the equivalence $H(1+SR)^2=H(1+AR)$ has to be posed and manipulated. From this, the relationship between the six-monthly rate and the annual rate can be worked out.

Enrico describes the annual rate in A60-B60 as ((1+B9)^2-1) (fig.3).

Analysis of Enrico's reasoning: Enrico needs to formalise in mathematical terms the new relationship he has found in the system *(ST.6)*. In this phase, Excel cannot provide any significant help and could easily be replaced by the calculator Enrico used earlier on.



leas.1

	A	B	C	D
1	commercial value	15000000		15000000
2	on account	0		0
3	instalments number	8		48
4	periodicity	six-monthly		monthly
5	instalment amount	2400000		400000
6	redemption	0		0
7	total cost	19200000		19200000
8				
9	periodic rate	0,058		0,010562829
10	1	2267633	132367	395819
11	2	2142567	257433	391682
12	3	2024399	375601	387588
13	4	1912748	487252	383537
14	5	1807254	592746	379528
15	6	1707579	692421	375561
16	7	1613402	786598	371635
17	8	1524418	875582	367751
18	9			363907
19	10			360103
20	11			356339

Fig. 3 – Comparing B60-D60 suggests the leasing plan more convenient for the buyer.

Phase 6

Enrico's reasoning: At this point, Enrico wonders whether he may be better off paying back in monthly instalments, which are one sixth the amount of the six-monthly ones. He then inserts the data concerning this hypothesis on the sheet, so as to compare the two situations. After the calculation (Fig.3**),** he finds the monthly interest rate that would be applicable and calculates the related annual rate. He then realises that this second hypothesis would be to his disadvantage since the annual rate (D60) is higher than before (B60).

Analysis of Enrico's reasoning: Enrico is guided by his curiosity and supported by the easy of representating a situation and getting information in a spreadsheet environment, using mathematical competences (formulas' generalisation). Making small changes to the sheet layout, he can compare the two hypotheses *(ST.7)* he himself formulated (Fig.3) and weigh up their financial impact both from his point of view and from the lessor's. In this phase the spreadsheet is no longer just a mere support, but (as a strong representation system) guarantees the conditions that make systemic thinking (and new knowledge acquisition) possible and allow it to develop *(ST.8)*.

Phase 7

Enrico's reasoning: Enrico, who had never before wanted anything to do with matters of this kind (even when he might have needed to), starts wondering what is actually behind leasing plans. He especially wonders what relationships there are between the capital and interest shares of each instalment. After representing the interest shares Cn in the column C (as (B5-Bn) in fig.1) and operating on the sheet in a free way, he notices that C10=B9*B10.

Enrico wonders whether C11=B9*B11, C12=B9*B12 and so on. As a result, he can see that this relationship does not hold true because the capital share diminishes while the interest share grows (first free exploration).
Adding temporary columns or using empty cells in sheet areas that do not interfere with the calculations, Enrico enters formulas to explore the sheet and make conjectures (second and third explorations, fig.4). For instance, he notices that C11-C10 is equal to B9*B10, so he calculates C12-C11 and notices that the result is equal to B9*B10+B9*B11, and so on. He thus assumes that the interest payable for the nth instalment is the result of

B9*B10+B9*B11+B9*B12+...+B9*Bn, i.e.
Interest rate*(first.instalment.capital+second.instalment.capital+.........+nth.instalment.capital).

Analysis of Enrico's reasoning: As well as in the phase 6, Enrico interacts continuously with the sheet in order to make conjectures about further relationships between the system's elements *(ST.9)* and to formalise them. His capabilities in formulas' manipulation are crucial to look at the spreadsheet globally but operate on it locally *(ST.9)*.

leas.M.7

	A	B	C	D	E	F
1	commercial value	15000000				
2	depos[illegible]	0				
3	instalments number	8				
4	periodicity	six-montly				
5	instalment amount	2400000		free exploration	hypotheses :	
6	redemption	0		*Cn - C(n - 1) =*	*B9*Bn ?*	YES
7	total cost	19200000		*Cn = B9*(B10+B11+.....Bn) ?*		YES
8						
9	periodic rate	0,058372132				
10	1	2267633	132367			132367
11	2	2142567	257433	125066	125066	257433
12	3	2024399	375601	118168	118168	375601
13	4	1912748	487252	111651	111651	487252
14	5	1807254	592746	105493	105493	592746
15	6	1707579	692421	99675	99675	692421
16	7	1613402	786598	94178	94178	786598
17	8	1524418	875582	88984	88984	875582
18						
19	present value	15000000				
20	annual rate	0,120151569				
21						

Fig. 4 – The second and third free explorations

Phase 8

Enrico's reasoning: He now verifies this conjecture in formal terms. In doing so, he needs to draw upon his knowledge of algebra with the aid of a software program for symbolic computing. This check is based on the assumption that the lessor, at the end of each instalment term, has the same capital s/he would have had if s/he had invested the leased amount at the same interest rate s/he charged the lease-holder.

Analysis of Enrico's reasoning: The verification of his conjecture in this phase leads Enrico to concentrate on the economic significance of the objects involved *(ST.11)* in order to find a key for formalising his conjecture. Once again he is encouraged to put himself in the vendor's shoes and get a deeper insight into the system *(ST.12)*. At this level, the new software employed (a tool for symbolic computation) and the mathematical competences necessary for using it in an effective way, are once again able to support the process.

Conclusions

The analysis of Enrico's thinking processes while approaching the problem discussed in the previous section, can clarify how systemic thinking progresses during the study of an open situation and as both programs such as Excel (involving strong mathematical representation systems) and suitable mathematical capabilities may play the role of mediators in supporting and creating the conditions for the development of systemic thinking, that has to be considered as a very important and cross-disciplinary way of reasoning.

Specifically, our analysis points out that: a) open problem solving activities require students to perceive and gradually untangle the "system" that lies at the core of the problem situation; b) achieving a global view of a situation (e.g. Phase 4) and being able to understand and use mathematical models (algebraic models and the "search objective" model embedded in Excel) may help discover and formalise local actions that describe particular relationships between certain elements of the system; c) reasoning on local relationships (e.g. Phases 6 and 7) that are to some extent formalised may improve the skill of re-thinking a system globally and reveal aspects that might otherwise have remained hidden.

We believe that among the reasons why certain software programs and suitable mathematical knowledge may play the role of mediators in supporting and creating the conditions for systemic thinking development, the following can be included: a) they offer a great variety of representation techniques and inherent functions; b) students are forced to identify a "system" on which to operate; c) students' attention is concentrated on interpreting and finding out relationships within the system; d) the focus is on how to operate and not on executing operations; e) it is possible to satisfy student curiosity as well as to formulate, validate and verify conjectures, getting a better understanding of a given system; f) students are encouraged to view matters from various points of view and to analyse the meaning of specific software output.

From our analysis, it appears also that certain software environments can play a crucial role in systemic thinking development when the following conditions occur: users are asked to solve open situations; tutors are very attentive in grasping what suggestion the user needs, and when; users have a strong motivation or curiosity to acquire new knowledge; users are inclined to gamble on the potential of both mathematics and the software in use.

Finally, we want to address the mutual influence between systemic thinking and mathematical capabilities. In this paper, indeed, we discuss the influence of some mathematical capabilities to systemic thinking development, while in other previous papers (Lemut & Greco, 1998, 1999), we discussed the opposite way of influence, namely the central role of systemic thinking in the acquisition of algebraic and geometrical problem solving capabilities (Arzarello, 1998; Gallo, 1994) .

Concluding, we feel that follow-up studies should focus on whether conscious activation of systemic thinking in significant problem situations under tutor guidance (that is crucial in several reasoning phases (e.g. Phases 2, 3, 5, 7, 8 in our example) could enable students to apply this intellectual "tool" in other cases. Preliminary experience gained with adults and 14-15 year-old students seems to provide a positive answer to this, and appears to be in line with the statement that systemic thinking is a "...way of thinking that, once adopted, permeates all thinking regardless of situations or context." (Licon Khisty, 1997).

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