# FROM MATHEMATICS FOR LIVING TO LIVING FOR MATHEMATICS

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*"Life is good for only two things, discovering mathematics and teaching mathematics".*
Siméon Poisson (1781-1840)

Mathematics for living and living for mathematics are related to the goals of mathematics education. On the one hand everybody is in need to learn mathematics to be able to deal with everyday life situations. On the other hand we need to teach mathematics to get also mathematicians. The question here is related to mathematical curricula and teaching strategies in particular. In teaching mathematics we have to teach how to solve everyday life problems. We as well can use everyday life problems to learn about mathematical structure and develop mathematical thinking. Developing mathematical thinking is in need of developing mathematical writing. Mathematical thinking and mathematical writing can not wait to start in secondary school, it has to start from childhood. The procedures of solving everyday life situation can help, but teacher's communication skills are of special meaning. This we can see from discussing in details one example, where as well using of real coins, visualisation and understanding of mathematical signs are parts of our teaching strategies.

## 1. The Paper's Philosophy

It is difficult to think that living was possible without mathematics. From the first beginning primitive mathematics was needed. From the other hand, even in primitive mathematics, the potential power of man's abstraction was there. The invention of the concepts of just numbers one and two reflects this power. Since that time somebody was behind this invention. Those people we can call prehistoric mathematicians. The industrial revolution, especially the so-called second one, started at the late 19th century has grown rapidly with the help of mathematics. Such mathematics invented by mathematicians, who devoted a special part of their life to mathematics. An extreme feeling we can find in Siméon Poisson's words (1781-1840); "Life is good for only two things, discovering mathematics and teaching mathematics". When we come to study our modern life of modern inventions we find that mathematics' role is a special one. In this concern, it is remarkable to notice that some mathematics, discovered before for its own sake, has become a decisive tool for modern technology and as well as modern living.

In our modern societies, education has become a right for all. Therefore education is neither only of mathematics, nor is offered to only children of high mathematical abilities. In one class we normally find children of different abilities. The question is how to reach different goals for different children in one classroom. Indeed everyday life mathematics is an object to reflect cultural and pragmatic goals of mathematics education. What about developing children thinking? What about getting some children interested in mathematics as a field of future profession? First question is related to one of the most important general goals of mathematics education and the second one is related to the outcome of specific goals of mathematics education as a professional goal. We need to search for strategies, which can be of different goals to different children.

### 2.1. Finding the Difference of Two Numbers in Everyday life Situations

When you ask children of early school years to find out the difference between 80 and 28 you have a chance to use everyday life real situation, to deal with the problem for different levels. Here we start with the Decimal System understanding and everyday life situation.

In this case the teacher can start the discussion by writing on the board the open statement **8**0 - **2**8 = $\boxed{\phantom{0}}\boxed{\phantom{0}}$. (Here the teacher can use special colour for writing the digits of tens)

After that, the teacher can translate the problem into everyday life situation. For this task he can design a strategy to produce the next dialog or something similar.

Teacher: *Look carefully at the board. Who can tell me how many ten-mark-coins I had in my pocket at the beginning?*

Child: 8 coins.

Teacher: *Right. Look now again at the board. Can you see that I bought a book?*

Child: yes.

Teacher: *What was the price of this book?*

Child: 28 marks.

Teacher: *How many coins of ten marks I had to give to the seller?*

Child: 3.

Teacher: *How many ten-mark-coins I had to keep in my pocket?*

Child: 5.

Teacher: *Did the seller give me back any money?*

Child: Yes.

Teacher: *How much he gave me back?*

Child: 2.

Teacher: *Are these 2 tens?*

Child: No.

Teacher: *Yes. These 2 are not 2 tens. Who can tell me, which kind of 2 then?*

Child: 2 ones.

Teacher: *So, we know how many tens I kept in my pocket. How many?*

Child: 5

Teacher: *Yes. Where I have to write digit 5?*

$$\mathbf{8}0 - \mathbf{2}8 = \overset{\text{T}\ \ \text{O}}{\boxed{\phantom{0}}\boxed{\phantom{0}}}$$

Child: Under the "T".

Teacher: *How many tens I gave to the seller?*

Child: 3.

Teachers: *How many marks I got back?*

Child: 2.

Teacher: *Where I have to write this digit 2?*

Child: Under the letter "O".

Teacher: *Why?*

Child: Because they are ones.

Such a dialog, in a longer or shorter form, the teacher can plan to get. After such a discussion, the teacher has to get confirmation of children's understanding. In a case of finding that some children are not sure of their understanding, the teacher can discuss the problem again using iconic visualisations. He can draw in an iconic way the coins and as well the book and its tag. After that, the teacher can discuss similar problem such as:

$$70 - 28 = \overset{\text{T}\ \ \text{O}}{\boxed{\phantom{0}}\boxed{\phantom{0}}}\ .$$

In this time the teacher can ask directly about the amount of tens he had to keep in his pocket and where he has to write it now. Then, he can ask about the number of tens he had to give to the seller,

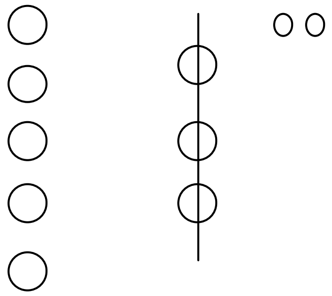
the amount of marks he got back in change and where he has to write this amount. The teacher can give a third example like "90 - 56 = ". In this time he has to ask the children to provide by themselves a similar story related to the new problem. As well, they have to write the appropriate digits of the remainder (difference) in the appropriate places. This work of children can be organised in different ways. After that the teacher can give as exercises similar problems where both the minuend and subtrahend are different from one exercise to another. Through exercises the teacher has to give in a kind of revelation some problems like
"100 - 76 = $\boxed{\phantom{0}}\boxed{\phantom{0}}$" and "120 -83 = $\boxed{\phantom{0}}\boxed{\phantom{0}}$
The teacher has to take care in the discussion of the last examples to bring understanding to students of having 10 tens in a hundred and 12 tens in 120".

## 2.2. Playing with Numbers and Building Structures

In the above mentioned examples, mathematics was used as a tool to solve an everyday life situation problem. Here we shall discuss how to use the above mentioned everyday life situation to learn about the associative property of addition of integers in an intuitive base.
This can be done in the next lesson. However there is a necessary condition for going ahead to the new lesson. The condition is that the teacher has already taught his children the meaning of equality, among others through the case of number decomposition. So, the children understand that the sign of equality includes two parallel segments having the same measure (length), one represents the left-hand side expression and the other represents the right-hand side expression. On the other hand, this means that both expressions are of the same number, they are equal names of a number. Balance as well is a good metaphor for this understanding of equality. My aim here from mentioning the using of the physical view of the sign of equality is to represent some of my own principles of teaching and learning mathematics. One of these principles is the teaching for understanding of each mathematical object, as much as we can. This includes the understanding of symbols. Nevertheless this issue, and as well the teaching of equality itself, is beyond our discussion here.
In the new lesson the teacher has to first bring to children's mind the meaning of equality, this can be done through some exercises. Then he has to bring back the first example of the last lesson and its solution. Here the teacher can put a box on a table, inside the box he has real coins of ten marks. The teacher has to write on the board "80 - 28 =". After that he can discuss the problem through a similar dialog to the next one.
Teacher: *How many ten-mark-coins I had in the beginning?*
Child: 8.
Teacher: *Who would like to play my role? Who can come to take from the box the needed coins?*
Child: May I.
Teacher: *Okay. Which coins you will keep in your pocket?*
Child: These 5.
Teacher: *Right. So, which coins you have to give to the seller?*
Child: These 3.
Teacher: *Let us write what you have said:*
80 - 28 = 50 + 30 (The teacher can colour the numerals 80, 50 and 30 in one colour, 28 in another colour and "-" in a third colour.)
Teacher: *Have I written a correct statement?*
Child: No.
Teacher: *Who can tell me, what I actually wrote?*
Child: This is writing of only 80 marks.

Teacher: *Correct. So, let us try to make a correction:*
80 - 28 = (50 + 30).
Teacher: *Is it now correct?*
Child: No.
Teacher: *Why?*
Child: This is only the minuend, 80.
Teacher: *Right. So, what I need to write more.*
Child: Minus 28.
Teacher: *Absolutely correct.*
80 - 28 = (50 + 30) - 28
Teacher: *Is this statement now true?*
Child: Yes.
Teacher: *These coins are of 50 marks and those are of 30 marks. From which of both parts we have to pay 28 marks, and which part we have to keep in pocket?*
(The teacher has to demonstrate real coins and at the same time he has to point to the numerals related in the written statement.)
After getting the answers to both questions, the teacher adds a new line as follows.
80 - 28 = (50 + 30) - 28
= + ( - )
Teacher: *Please, write in your notes these two lines and try to fill the spaces in the second line correctly*.
This is as well some of my strategies, through which each child gets a chance to think and to rediscover by himself. When you ask students, only in oral way, you can not be sure that each child is really an active thinker. In addition, some students need to have peaceful time to think alone. Of course oral discussion has as well its own positive sides, but when you ask the children to answer in a written form, you give every child a chance to develop his thinking. Also questions can be given in written form.
The teacher has to discuss children's solution to get the second line correct. The most important here is that the teacher has to ask children to explain how we modified the original expression to get the other equal expressions. The teacher has to ask the children to give an explanation based on the everyday life situation story. Then he has to move to get explanation of pure number and operations understanding.
After that the teacher has to add a new line as follows.
80 - 28 = (50 + 30) - 28
= 50 + (30-28)
= +
The teacher has to ask students about, what he should write in the empty spaces. Finally he has to get the last line as follows.
80 - 28 = (50 + 30) - 28
= 50 + (30 - 28)
= 50 + 2
= 52
The teacher has to ask the children to give an explanation of what is written on the board in the final form. Coins used can help to bring understanding of each step of the solution writing, especially to children of modest abilities.

Also the use of visualisation in a dynamic way can be of help to these students and it is a way as well to develop children's spatial abilities. So, the steps of solving the problem can lead at the end to the next figure.



The most important thing to get students to learn such mathematics writing is to give to them to solve similar problems and to ask them to write logically all the steps needed in each problem. Also the teacher has to discuss students' writing later with the whole class. To some students of low abilities the teacher has to offer special individual instruction. In this case, the child has to show in a concrete way, how he behaves to buy the book, where each step has to be translated in formal writing and in time. The teacher has to give such students to demonstrate to each other what they have got. This is a long way to make students interested in playing with numbers, modifying expressions and solving problems.

## 3.**Conclusion**

In this paper we discussed in details only one example of the way of teaching mathematics, from one side for living, and from the other side a way for some to live for mathematics.
Everybody is in need to learn mathematics for practical everyday life situations. But as well, everybody is in need to learn about mathematics as a way of thinking. This way discovered by man and every man is in need to rediscover this way. This includes beside cultural aim, a personal aspect for developing the thinking of every child. On the other hand we are in need to get mathematicians for developing mathematics itself and as well its applications in science and technology. To solve human problems related to health and environment we are in need of mathematics and mathematicians. The problem here is that mathematical thinking has a growing aspect. It has to start from childhood. Therefore we have to take care of mathematics thinking from early years of education. Mathematics thinking is related to mathematics writing. Today mathematics writing is a problem, which we face in secondary school. The solution of this problem has to start from early years.
Using examples of everyday life can be enough for some to understand mathematics as a subject for everyday living, for very few children these examples can open the way to live for mathematics. For the third part mathematics is in a place between these two points, where this place differs from one person to another. So, we need to develop such strategies, which can be a way to gain different goals of mathematics education, general and specific ones.
The above mentioned examples and strategies are a part of the work done for mathematical clubs established in Finland by the author.

## References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