# A CONSISTENCY QUALITATIVE ANALYSIS OF COMPLEX REALLIFE MODELS <br> Meznik I, Dohnal M. <br> Brno University of Technology, E-mail: dohnal@fbm.vutbr.cz; meznik@fbm.vutbr.cz 


#### Abstract

A qualitative quantification is based on four values only - plus, minus, zero, unknown. Therefore to develop a qualitative model requires very limited knowledge. This is the most important advantage of the qualitative models. To develop a qualitative model of a complex process is an iterative procedure. It could be very time consuming, which, however, requires highly qualified experts. Unfortunately especially at the early stages of the development the models can contain mutually inconsistent relations. Therefore the inconsistent models has no solution. The paper analysis some aspects of the inconsistent model and presents a simple procedure how to eliminate the inconsistencies. A case study is based exclusively on equationless knowledge items. The following variables are taken into consideration: Borrowing Requirements, Collective Burden, Deficit (Government), Economic Activity, Government Expenditure, Interest Payments, Long-term Interest Rate, Social Security Payments, Taxes, Unemployment. Complete lists of all possible qualitative scenarios are generated.


## INTRODUCTION

An applicable model of a sophisticated problem represents an extremely complex, multidimensional, absolutely unique and vaguely described system. A conventional quantitative model is prohibitively inaccurate and its results and consequently any conclusions based on them could be misleading.
At present, most of the techniques employed for the analysis of a broad spectrum of problems possess analytical and/or statistical natures. Unfortunately these precise mathematical tools do not always contribute as much as is expected towards a full understanding of tasks under study (Davis, 1990).

It is no paradox that less information intensive methods of analysis often achieve more realistic results in cases where the system which is being modelled is very complex, and/or ill known (Gallo, Gill, 1990). Modern computers are extremely powerful tools in terms of number manipulation. However their contribution to solving vague problems from finance and investment using common sense has been practically very small.
Therefore a qualitative trend analysis is used to generate a set of all possible time scenarios (Parson et al 1992). Even very uncertain knowledge is valuable. It is the effectiveness with which uncertain knowledge is used which is very often the main distinction between good and bad models.
Equationless knowledge is such knowledge that cannot be formalised by equations because of (Cohn, 1989)

- vagueness
- complexity
- transparency of the final results i.e. inability of the final users (top managers) to understand conclusions based on sophisticated mathematical / logical theories
There are two basic types of knowledge items, namely deep and shallow. Deep knowledge is knowledge that represents the basic laws of nature. Roughly speaking a deep knowledge item is such item, which is accepted without any questions by the corresponding professional community. The key deep knowledge item in engineering is the law of mass and energy conservation. Unfortunately there are no deep knowledge items in economics and finance. Any knowledge item has its exceptions and simplifications. Moreover there are different specific interpretations by different experts.
The shallow knowledge item is not related to any deep knowledge item. All sorts of statistical analysis are used to generate shallow models. However, the mathematical forms of these knowledge items, usually mathematical models (e.g. exponential, polynomial) are dictated not by
reasoning or by the very nature of the problem under study but by the statistical theories and, quite often, by tradition and rigid applications of statistical packages.
A semi-deep knowledge item represents such information, which is generally known and rather frequently used but not generally accepted. An example is the Interest Rate Parity model. The model is represented by an equation and this equation may be used for forecasting purposes (Dohnal, 1998).


## QUALITATIVE MODELS - TUTORIAL INTRODUCTION

A qualitative model is the best calculus that can be used as a common sense theoretical background. Relatively large and realistic problems can be tackled using existing software (Dohnal 1991), (Meznik et al, 1999).
Qualitative modelling is a combinatorial problem. It is not the goal of this paper to analyse algorithms suitable for the qualitative analysis. A simple and efficient algorithm is the brute force approach. It means that all possible combinations are tested against the qualitative model. An extensive description of qualitative calculi is in (Davis, 1990). A general methodology how to simplify/degrade quantitative models into qualitative models is given in (Dohnal, 1991).
Suppose there are only three qualitative values:
Positive (+), negative (-), zero (0), unknown or not relevant (*)
A qualitative solution of a qualitative model is specified if all its n qualitative variables
$\mathrm{X}_{1}, \mathrm{X}_{2}, \ldots \ldots . \mathrm{X}_{\mathrm{n}}$
are described by the qualitative triplets
(X, DX, DDX)
where DX and DDX are the first qualitative and second qualitative derivatives with respect to time.

## QUALITATIVE INCONSISTENCIES

Macroeconomics, finance, environment are examples of such areas of research which generates prohibitively complex models. The models are based on semi subjective, semi inconsistent and predominantly shallow knowledge. Therefore the first versions of the complex models are always inconsistent. The following sequence of steps summarises the basic features of qualitative methodology:

- Modification of heterogeneous knowledge structure by using unified terminology and notation
- Transfer of quantitative knowledge items to quantitative information (qualitative degradation)
- Integration of all qualitative knowledge items into a single model
- Consistency test
- Elimination of inconsistencies

The last to steps cannot be described by generally applicable rules and each model must be treated on ad hoc basis. A simple problem is used to demonstrate the basic idea.
The following qualitative relations are taken into consideration (for details see (Berndsen, 1995) where the important aspects of macroeconomics are presented):
1 T D Negative
2 T CB Positive
3 D BR Positive
4 BR IP Positive
5 BR R1 Positive
6 R1 IP Positive
7 R1 EA Negative
8 EA U Negative

| 9 | G | EA | Positive |  |
| :--- | :--- | :--- | :--- | :--- |
| 10 | G | D | Positive |  |
| 11 | EA | T | Positive |  |
| 12 | CB | EA | Negative |  |
| 13 | SSP | CB | Positive |  |
| 14 | U | SSP | Positive |  |
| 15 | IP | D | Positive | where |
| BR | Borrowing Requirements |  |  |  |
| CB | Collective Burden |  |  |  |
| D | Deficit (Government) |  |  |  |
| EA | Economic Activity |  |  |  |
| G | Government Expenditure |  |  |  |
| IP | Interest Payments |  |  |  |
| R1 | Long-term Interest Rate |  |  |  |
| SSP | Social Security Payments |  |  |  |
| T | Taxes |  |  |  |
| U | Unemployment |  |  |  |

Represents a simple qualitative model using the first derivatives only. For example, the first qualitative relation
1 T D Negative (5) can be described by two qualitative statements:
If T goes up then D goes down
If T goes down then D goes up

## ELIMINATION OF INCONSISTENCIES

A qualitative analysis discovered that the qualitative model represented by the set of relations (4) has no solution because certain subset of relations are not consistent. There are several possible subsets of relations, which can be removed from the model to achieve solvability.
Roughly speaking the problem of identification of inconsistencies is a combinatorial problem. Several potential approaches were tested (brutal force, different heuristics). However, a dialog between the group of experts and a computer seems to be the best variant. The main reason why the dialog like approach gives the best solution is a simple fact that there are nearly always many different ways hot inconsistencies can be eliminated.
Unfortunately, there is no a generally applicable rule how to identify the best subset of relations causing the inconsistency of the model as a whole. There are different points of view:

- Minimise the modification of the model under study, i.e. minimise the number of removed relations
- Remove the least accurate / reliable relation and test the consistency
- Identify such variable which is involved in many different relations and reconsider its application The fact that the model is not consistent is a reason for a careful study of all elements of the model (4). Let us suppose that the conclusion of a study is that for example, the relations 2 and 10 are potential troublemakers. If the relations 2 and 10 are removed then the modified model has the following three solutions (see triplets $(3,1)$ ) which

|  | BR | CB | D | EA | G | IP | R1 | SSP | T | U |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ |  |
| 2 | $+0^{*}$ | $+0^{*}$ | $+0^{*}$ | $+0^{*}$ | $+0^{*}$ | $+0^{*}$ | $+0^{*}$ | $+0^{*}$ | $+0^{*}$ | $+0^{*}$ | $(6)$ |
| 3 | $+^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ | $++^{*}$ |  |

indicates that the second derivatives are ignored. The reason is very simple, the model is based on the first derivatives only and therefore no second derivatives can be evaluated. Moreover, it is usually very useful to tune the model using the first derivatives only and include the second derivatives if the model based on the first ones is considered as perfect. The conclusions which can be reached using the set of three scenarios is simple
The third scenario (see 6)) is the only one, which increases the goal function EA.
A group of experts introduced the second derivatives into the modified first derivatives model (4)

| 1 | 24 | T | D |
| :--- | :--- | :--- | :--- |
| 3 | 21 | D | BR |
| 4 | 23 | BR | IP |
| 5 | 23 | BR | R1 |
| 6 | 21 | R1 | IP |
| 7 | 26 | R1 | EA |
| 8 | 24 | EA | U |
| 9 | 22 | G | EA |
| 11 | 23 | EA | T |
| 12 | 26 | CB | EA |
| 13 | 21 | SSP | CB |
| 14 | 21 | U | SSP |
| 15 | 23 | IP | D |

The second column identifies the shape (see Fig. 1) and indirectly the second derivatives as well. A computer program solved the model (7) and therefore there are no inconsistencies. The following set of 27 time scenarios exist if the model (9) is used to generate them:

|  | CB | D | EA | G | IP | R1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| +++ | +++ | +++ |  |  |  |  |  |  |  |
| 2 +++ | +++ | +++ |  |  |  | + 0 | +++ |  |  |
| 3 +++ | ++ | +++ |  |  |  |  | +++ |  |  |
| +++ | ++ | + |  | +-+ | +++ |  |  |  |  |
| +++ | ++ | + |  | +-+ | ++ |  |  |  |  |
| +++ | ++ | + |  |  |  |  |  |  |  |
| ++ |  | ++ |  |  |  |  |  |  |  |
| 8 +++ |  | ++ |  |  |  |  |  |  |  |
| +++ |  |  |  |  | ++0 |  |  |  |  |
| 10 +++ | ++ | ++ |  | + |  |  |  |  |  |
| $1++0$ | ++- | ++- | +-+ | +-+ | +- | ++- | ++- |  |  |
| 2 ++- | ++- | ++- | +-+ | -+ | + | ++ | ++- |  |  |
|  | +0+ | + |  |  |  | +0+ | +0+ | +0- |  |
|  | + | +0 | +00 | + |  |  | +00 | +00 |  |
|  | +0- | +0- | +0+ |  |  | +0- | +0- |  |  |
| -+ | +-+ | +-+ |  |  |  | +-+ | +-+ |  |  |
| -+ | +-+ | -+ | ++- |  |  | +-0 | +-+ |  |  |
| + | +-+ | +-+ |  |  |  | +- | -+ |  |  |
| -+ | +-- | +-+ | ++ | +++ | -+ | +- | +-- |  |  |
| 20 +-+ | +-- | +-+ | +++ |  | +-+ |  |  |  |  |
| 21 +-+ | +-- | +-+ | +++ | +++ | +-+ | +- |  |  |  |
| 22 +-+ | +-- | +-0 | ++ |  | +-+ | --- | +-- |  |  |
|  |  |  |  |  |  |  |  |  |  |

```
24 +-+ +-- +-- +++ +++ +-0 +-- +-- +++ +--
25 +-+ +-- +-- +++ +++ +-- +-- +-- +++ +--
26 +-0 +-- +-- +++ +++ +-- +-- +-- +++ +--
27 +-- +-- +-- +++ +++ +-- +-- +-- +++ +--
```

The scenarios 16-27 have positive first derivative of EA and therefore correspond to the scenario 3 (4). It is rather difficult to comment the results and consequently choose a subset of scenarios 16 27 as the goal scenario. We believe that a multicriterial approach is needed. However, for example unemployment is always decreasing, see the last column (8). In this specific case no compromise is needed.

## CONCLUSIONS

The consistency represents a very important obstacle, which must be eliminated to achieve a meaningful qualitative model of a complex problem. The proposed concept has been successfully applied to several finance, managerial and risk problems. This is, however, not enough to believe that the methodology presented above can be used to analyse all types of complex problems.
A possible explanation of the success, which the qualitative method has enjoined, is that it has generated good questions to the decision-makers. Field experts are forced to re evaluate their attitudes. The scenario generator is a useful generator of "provocative" questions.
The qualitative modelling itself is very flexible. It is possible to perform any union or intersection of different models. This is a very useful feature for verification of models and their simplifications. This aspect is important for development of classical quantitative models.

## REFERENCES

Berndsen R. (1995) Causal ordering in economic models, Decision Support Systems, 15, 157 165
Cohn A. G. (1989) Approaches to qualitative reasoning, Artificial Intelligence Review, 3, 177-232 Davis E., (1990) Representations of Common-sense Knowledge (Morgan Kaufmann, San Mateo)
Dohnal, M., (1991), A Methodology for Common-sense Model development, Computers in Industry, 16, 141 - 158
Dohnal M. (1998): A Qualitative Approach to Pattern Identification for Financial Data Mining, Journal of Computational Intelligence in Finance, 5, No. 3, 1997, 27-36
Gallo, G.M., Gill, S.M. (1990) How to Strip a Model to Its Essential Elements, Computers in Economics and Management, 3, 199-214
Dohnal M., Meznik I. (1999), Multidimensional Screening of Qualitative optimisation Scenarios, In: Proceedings of The Second International Conference - MATHTOOLS'99, Tools for Mathematical Modelling, Saint Peter (To appear)
Parsons S. ,Dohnal M., (1992) Qualitative, Semiqualitative and Interval Algebras and their Application to Engineering Problems, Eng. Applic. Artif. Intelligence, 5, 553-560.

Authors' address:
Brno University of Technology, Faculty of Business and Management, Technicka 2, 61669 Brno, Czech Republic

