Theorizing for Non-theoretical Approaches to Mathematics Education

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Introduction

The theme of this conference supports the paradigm shift in mathematics from seeing mathematics as the study of formal systems to seeing mathematics as a living body (See: 13 and 15:611). This shift has been reflected in primary school mathematics programmes "from seeing mathematics as a large collection of concepts and skills to be mastered in some strict partial order to seeing mathematics as something people do" (16:3655), and in secondary school mathematics programmes from the "formal" teaching of mathematics to introducing mathematics as human activity in order to provide a basic preparation of learners for full participation as functional members of society (17:3661). Both the paradigm shifts of mathematics and their reflections on school mathematics programmes call for non-theoretical approaches to mathematics education, ie approaches based on practice and personal experience, not on theories (7: 678). The present paper is an attempt to justify this proposition and shed light on it, as well as to raise dialogue about the whole issue and the details of the ideas included.

To deal with the issue, the coming sections contain the following:

Some basic assumptions.
Developing a framework for a new view of "mathematical truth".
Concluding remarks.

Some Basic Assumptions:

The present attempt to theorize for non-theoretical approaches to mathematics education is based on some assumptions, which also constitute an essential part of the process of "theorization". The most important of these assumptions, while keeping in mind that they are interactive, are as follows:

Mathematics is one of the existing disciplines, which cannot be seen apart from the contemporary and future trends of knowledge. These trends are characterized basically by "complexity", which implies at least transdisciplinarity in viewing knowledge (through the general systems theory, cybernetics, non-linearity … and so on). (See: 12), in addition to the accelerated development of knowledge and its cohesion with its applications. Such a "postmodernism" science goes beyond Newtonian view, which is almost called the Cartesian Synthesis by Ormell (See: 14:9-15), and refuses the reductionist approach.

Education ought to cope with such developments in knowledge, especially with the diminishing of differences between practices of the ordinary citizen - or at least the "educated citizen" - and the researcher's behaviour and activities (12). Some of the pathways to that are to deal with knowledge in an integrated way, concentrating on problem solving and avoiding the thought of "simplification" and "linearity". To mange that, some "non-traditional" methods of teaching are to be used, eg self-learning (with the use of different media of different levels of sophistication), collective work, dialogue, brain storming….etc. Needless to say, such changes must be associated with introducing radical changes in means and tools of evaluation. Within such a framework, two points should be emphasized; the first is that application of knowledge will constitute a major concern of curricula, of which "modelling" will be an essential part at all educational stages; the second point is that criticizing the existing knowledge will be a continuous process, leading to creativity.

There is a need to develop further "new mathematics" in order to represent a "behaviour of systems", where the chaos theory and the catastrophe theory can be referred to as examples.

The following quotation from Woodcock and Davis about catastrophe theory would explain better what we mean (18:9):

"Catastrophe theory is a controversial new way of thinking about change – change in a course of events, change in an object's shape, change in a system's behaviour, change in ideas themselves… The theory is controversial because it proposes that the mathematics underlying three hundred years of science, though powerful and successful have encouraged a one- sided view of change. These mathematical principles are ideally suited to analyse - because they were created to analyse - smooth, continuos, quantitative change: the smoothly curving paths of planets around the sun, the continuously varying pressure of a gas as it is heated and cooled, the quantitative increase of hormone level in the bloodstream. But there is another kind of change too, change that is less suited to mathematical analysis : the abrupt bursting of a bubble, the discontiuous transition from ice at its melting point to water at its freezing point, the qualitative shift in our minds when we "get" a pun or a play on words."

If it is rather difficult for students at a certain educational level to deal with knowledge at a high level of sophistication, they must at least be aware of the embodied assumptions and their limitations(1).

Thought might not be controlled by logic. It is rather, the contrary, that logic is controlled by thought. Both Gödel's theory of the impossibility to prove the validity of a formal system without using another – external - system (having isomorphism between them) and the collapse of positivism have contributed much in such directions (See, for example; 2, 10 and 12). As a result, induction and deduction are seen as an integrated means of reasoning, there is no single description of a system – as it depends on the observer (2) (See: 5:5-10), "objectivity" is questionable and the components of a system can be re-built (synthesized) in many different ways. (3)

Forcasting becomes an essential part of knowledge, whether in its making or findings. So, scenario building, "conditional" prediction as well as "simulation and modelling"(4) should be a part of any educational programme to be introduced at any level.

Future education is supposed to be based on the existence of developing "multiple intellegences" (See : 6). Some of the relevant implications are a wide range of choices of the content of study, flexibility (in almost all respects) as well as viewing different "disciplines" as of "equal importance".

Changing curricula of education depends on many interactive factors, whether educational, societal, regional or human(5). Although societal factors are decisive in this respect, it seems that there is an increasing role of factors related to the human culture with regard to the current growing process of "globalization"(6).

Developing a Framework for a New View of "Mathematical Truth":

The issue of the definition of "mathematical truth" seems to be the central problem in the philosophy of mathematics. Curry pointed out that (4:3).

"If mathematics is to be a science, then it must consist of propositions concerning a subject matter, which propositions are true in so for as they correspond with facts".

In dealing with mathematical truth, we are concerned with the nature of this subject matter and these "facts". Curry claimed that there are three main "types of opinion" as to the nature of this subject matter (may be seen as theories of mathematical truth). He specified them as (Ibid):

"1) Realism, or the view that mathematical propositions are true insofar as they correspond with our physical environment; 2) idealism, which relates mathematics to mental objects of one sort or another; and (3) formalism".(7)

Some major criticisms have been directed to the first two types which led to excluding them from consideration in the context of mathematical truth, keeping the priority of "formalism". Curry explained that as following (4: 3-5):

"The realist point of view is now not taken seriously by most mathematicians… Today, however, the view is untenable; for one reason because there is nothing corresponding to infinity in the external environment… All forms of idealism are subject to the same fundamental criticism: viz., that the resulting criterion of truth is vague at best, and depends on metaphysical assumptions…".

Coming to formalism, which is adopted by most mathematicians, it means that mathematics is the study of formal systems. A formal system refers to the ordered pair (set, structure) and their interrelations, ie the rules of formation (of new terms), the rules of procedure (for deriving further theorems), and the elementary propositions or predicates [8] (See: 4:56). Gödel's theory, referred to above, makes the formal view of mathematical truth questionable, a matter which may point to a "crisis" of mathematical thought .[9] (See; 10: 16-17).

Other implications of the formal view are that mathematics has superior position among other disciplines being freed from the physical world [10] and that the whole of mathematics is characterized by "certainty", even when dealing with probability, as being the study of formal systems and deducing its statements by "formal logic". We can easily note that these implications contradict the above mentioned assumptions.

It might be helpful if we reconsider the different views of mathematical truth assuming that:

Reality is to be extended to include "virtual reality" as well as the content of "conditional propositions", and not to be confined to "physical reality".

Human behaviour can be more easily explained assuming that the mind constructs mental models of reality, rather than by assuming the existence of a "mental logic"[11] (5 : 7).

Mathematical systems are open systems, which have been influenced by supra systems, introducing changes on all of their components.

It seems that the three views of mathematical truth are integrated in such a sense and that mathematics is much more equipped, in the same sense, to be developed in order to cope with and provide much more contributions to the emerging paradigm of science, ie complexity.

Concluding Remarks

The most important of these remarks are as follows :

It will be wrong to assume that this paper has presented the "happy solution" for the issue of mathematical truth [12]. It might provide an "acceptable" way of thinking, which may pave the way to put both the paradigm shifts in mathematics and mathematics education in practice. It has at least revealed the need to introduce basic changes to the concepts of "reality", "mental objects" and "mathematical systems" in order to cope with the developments in science, and to look at both the views of mathematical truth and to "science" in an integrated way, consequently, it is much more likely to highlight the use of non-theoretical approaches to the study of mathematics, based on the assumptions mentioned above.

The application of the integrated curricula and non-theoretical approaches to mathematics education are not easy processes, though necessary. They need not only to introduce curricular changes, but rather to change the mentality of teachers, educators, parents, and the whole "public opinion", not to mention a "great struggle". In case of deciding to apply them, many models can be used. Nevertheless, it is most likely to attempt applying them first in developed countries and some "elite" schools in developing countries.

The writer has suggested a model called "functional encyclopedism" to deal with the studied "transdisciplinary" units, where data collecting for dealing with any problem or issue will take an "encyclopedism" form by using high technology, so as to allow students to analyse, study and discover by themselves the great scientific points of departure in different fields. Nevertheless, such a model is applicable only in certain countries and among particular classes within these countries and across some other countries. We must be aware of the danger of "polarization" of people having enough access to computers and the internet and those who do not. There might appear a form of "knowledge stratification", which could be much more worse than the classical "social stratification", although they are interrelated.

Finally, when emphasizing non-theoretical approaches to mathematics education and calling to apply them, we must examine the whole curriculum and other educational conditions, the whole societal conditions as well as the whole global changes. The issue goes beyond technical aspects, being subjected to many other considerations. However, we have to call for plan, work hard and struggle to introduce such improvements in curricula and mathematics education in the widest possible range.

Notes

1. Eg in dealing with “regular” velocity, students should be aware that it cannot exist in reality, and that the straight line indicating the relationship between distance and time in that case is due to the lack of our knowledge, and is based on an “unrealistic” assumption. Considerable attention should be paid to developing students’ habits of criticising knowledge stating explicity the embodied assumptions.
2. Note that Fioretti suggests that complexity is a property of the relation between a system and its observer, a matter which has introduced radical changes into science, the relation between “logic”, thought and research methodology.
3. A matter which implies the need to get many different relevant data and information from many different resources, by the use of some different “methodologies”.
4. Predication as such, if not conditional, is meaningless under the new way of thinking (particularly, from the perspective of complexity).
5. This is due to the view of education as an open system, which is a sub-system of some other wider systems, including: the national, regional and human cultures (See: 8: 20-24). The system of education includes - basically - the following sub-systems: Aims, structure, administration, finance, curricula, teacher education and educational research (8: 21-22).
6. By globalization we mean “the increasing obvious interactions of economic, sociological, political, cultural and behavioural matters without any consideration of the political borders (of “dependent” states)” (1:7).
7. In this concern, Mina (9:25) pointed out that: "From the point of view of historical development, realism could be considered the basic thoughts of Egyptians, Phoenicians, and all primitive mathematicians in general; idealism as the main thought of Greeks-especially Plato- and “intuitionists”; and formalism as the mode of thinking in the last few centuries. But, it seems that such a historical perspective might not be comprehensive, because the idealism point of view is still- in some way or another - influencing some mathematicians".
8. Some of the consequences of the formalistic conception of mathematical truth suggested by Mina in 1978 (See: 9: 26-30) are: Independence of mathematical structure from the "physical world". Also, deduction is the methodology of mathematics and "pure" mathematics is independent from the physical universe. Obviously, these consequences contradict the assumptions stated in the previous section. Note that the author is the same person, but his thought has changed considerably rather recently.
9. Nevertheless, there might not be a direct connection between such crisis and producing new mathematics through research in the subject (See: 10:16).
10. Eg Bell's famous statement "Mathematics is the queen and servant of sciences". Furthermore, some -if not many, mathematicians feel superiority among some other scientists.
11. This conclusion is quoted from Johnson-Laird. Fioretti added, referring to the basic ideas of connectionism, that “We can think, about mental categories as implemented by paths in which information flows in closed loops, and mental models as connections between these categories” (5:7).
12. Simply because there might not exist such a solution in the framework of complexity. In all cases, the issue needs much more discussion and dialouging as well as “negotiation”.

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