Promoting mathematical investigation in class: reflections on the rôle of the teacher on the basis of experiences conducted in the secondary school

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Summary In this work is presented an analysis of the different functions carried out by the teacher during class discussion, a fundamental phase of the educational process which, if well carried out, can assume the characteristics of a real research activity for the entire class. The analysis described are the results of a collaborative endeavour with secondary school teachers and is based on several didactic experiences, that can be considered as an interpretation of the current ideas of social constructivism.
After the description of examples which illustrate some of the functions analysed, some open problems are highlighted, particularly connected with the professional preparation of the teachers.

1. Introduction

Beginning in the 80's, theories of teaching-learning which say that both individual processes and interaction with others are central in the educational development of students, i.e. theories which stress both the individual and social dimensions, have become continuously more diffuse. Today, the idea that the student should not be considered as an object but rather as a subject who continuously acquires more importance within his relationships with others is widely shared. The theories that have contributed to the dissemination of such convictions have all developed from the constructivist studies of Piaget and have then assumed unique connotations, differentiating themselves one from the other, but highlighting the common matrix which is the 'Piagetian' one. Such are, for example, radical constructivism (Von Glasersfeld, 1995), social constructivism (P. Ernest, 1995 and H. Bauersfeld, 1995), theory of didactic situations (G. Brousseau, 1997) and the Inquiry model (R. Borasi, 1996).
According to the theories cited, teachers and students should carry out in class research activities in which they can work together, posing and solving questions beginning from opportune problematic situations. The work of the teacher is no longer that of preparing the pupils by the repetition and memorization of formulas and concepts, but that of promoting, in the pupils, the qualities of a researcher, thus knowing how to confront complex and mutable situations, even outside of the scholastic environment. To arrive at such a goal, it is essential that teachers learn to construct opportune didactic situations, thus creating conditions that encourage the maximum participation of the class and then know how to manage such situations to the fullest; thus succeeding in carrying out an effective cognitive progression in each pupil.
After having carried out several didactic experiences based on the ideas expressed, (E. Castagnola, C. Joo, A. Pesci, 1996, A. Pesci, 1998, E. Giuliani, A. Pesci, M. Reggiani, F. Tomassini, 1998), in collaboration with a group of teachers who we have coordinated for some years (A. Pesci, M. Reggiani, 1994) we felt the need to give full details about the rôle of the teacher during class discussion, an important fundamental phase which if well accomplished, can truly take on the characteristics of a research activity.
The objective of this presentation is that of describing the various typologies of activity which the teacher is called on to perform during a class discussion and also to supply some particularly significant examples: both the description of the various activities and the examples surfaced during several encounters with the teachers who carried out the didactic experiences cited and shared with us considerable debate and reflection

2. The rôle of the teacher during class discussion

During the discussion, the job of the teacher is not that of supplying ideas and methods useful to the students for solving the questions posed, but that of guiding the spontaneous search for resolution strategies on the part of the students. For this reason, the functions which the educator fills within the class are several and often difficult to carry out. The list that follows does not pretend to be exhaustive, but establishes however, some fundamental functions

which the class teacher is required to know how to do: The poor management of some crucial moments during a discussion might give rise to significant obstacles in the cognitive development of the class and therefore construct negative experiences for students. The importance of an analysis of this kind proves therefore to be essential so that the teacher is aware of the multiplicity of rôles which he is expected to play.

A) Coordinating the discussion

Among the principal rôles of the teacher is that of coordinating the discussion and deciding, therefore, the order of the interventions according to the following situations:

- To initiate the discussion: for example, it can be opportune to choose neither a particularly good pupil nor one whose ability is very limited, but to opt for a child who is average for the class. It is more likely, in this way, that uncertain or incorrect solutions will emerge; for example, the presentation of a correct strategy especially if by one of the best pupils in the class, could block the presentation of other strategies by his classmates.
- In the course of the discussion the choice of which pupil to let speak depends on different motivations. Occasionally, it can be opportune to question a child because he is momentarily distracted or because he does not usually participate spontaneously. Therefore it is always indispensable to let the pupils speak who want to participate and who raise their hand preferring the one who proposes arguments which in that moment can be important for the discussion. For example, because it is assumed that he will expound a strategy or an argument different from those which have already come up.
- Generally, before ending the discussion, the teacher questions those pupils who have not been particularly participatory in such a way as to assure himself that the arguments presented have been understood by everyone

B) 'Making the point' of the situation

So that everyone is aware of what has been said, it is important, during the discussion, to give a synthesis of the resolution strategies which have emerged or of the more important examples. This task can be carried out directly by the teacher or by one or more of the pupils. In this latter case, it is a good idea to choose students who are quite able to express themselves, thus having the best possible effectiveness.

Example 1

In the course of a discussion, at the conclusion of work on a card which asked who was the best of four tennis players, where each player was assigned the number of matches played and the number won (the objective being that of initiating the construction of proportional reasoning) since no new ideas emerged and everyone agreed to the fact that Carlo was the best player (which was correct), the teacher decides to summarize the resolution strategies which emerged.

Teacher: *"OK, listen to this, to know who is the best among the four players, in this situation, did we all respond more or less in the same way? Did we use the same method to arrive at saying that Carlo is the best?"*

Class: *"No!!!"*

Teacher: *"No. What are the ways in which we have tried to give a solution? Let's hear what the ways are."*

To Tomaso, a pupil with a good ability for expressing himself, was given the task of beginning to present some of the strategies which had emerged, then other pupils intervened and shortly all of the methods used, (among which some were correct and others not) were recalled by the class, therefore opening the debate phase on the correctness of the resolution strategies used.

C) Asking to write strategies or examples

The teacher can decide to have written down, either on the board or in a notebook, some

syntheses or particular examples which both encourage discussion and give a way for everyone to reflect on what has emerged. Often the different results obtained from the various procedures are written on the board in such a way that everyone can have them available during the entire discussion. On the other hand, some particularly important examples or quite complex ones can be written on the board to give everyone time to reflect on them or so that the children can refer to them after they have been proposed.

Example 2

With reference to the situation described in Example 1, Marco maintains the strategy where if two players have won fewer games than they lost, then the better player is the one who has a smaller difference between the games lost and won. Tomaso wants to oppose the correctness of this strategy:

Tomaso: *"I want to quickly write a table on the board. One player played 3 games and of them won one, another played 5 games and won 2. (He writes on the board*)

G	*3*	*5*
V	*1*	*2*
P	*2*	*3*
	0,3	*0,4*

Now, I've done the difference and I see that this one lost 2 and the this other one lost 3. Therefore, according to Marco, this one (indicating the first one) *should be the better player. However, I used Edoardo's and my method and in the first case I did 1 divided by 3 and got 0.3 recurring and for the other I did 2 divided by 5 and got 0.4. Therefore, in my opinion this one is the better player* (indicating the second).

Marco: *"But I said to do the games lost less those won. Therefore, in the first you get 1 while in the second… what do you get?….1."*

Tomaso: *"And so….they're both equally good!"*

Teacher: *"Marco, what do you say?"*

Marco: *"...It's a special case!"*

Teacher: *"Agreed, so let's think. Let's go on, the example is on the board so it's always present. Then Marco will have to say something."*

It is interesting to note that in the course of the discussion reference was made several times to the same situation which was written on the board and obviously within view of everyone. From this, it clearly arose that Marco's strategy and Tomaso's could not be equivalent and this was a help for some classmates.

D) Refraining from judgement based on knowledge

It is well known that it is fundamental to ensure good development of the discussion and that the teacher should not express any judgement on the merits of the correctness of the resolution methods or on the arguments proposed by the pupils. It frequently happens therefore that, even when an incorrect methodology is expressed, the teacher simply accepts the proposed solution with a simple, apparently out of place affirmation such as "OK" or "Right", leaving any refuting to the classmates.

It can happen that some pupils will explicitly ask for the teacher's opinion with regard to the resolution method or an argument expressed. Even in these cases the teacher should not express himself on the correctness of the method or even less on what develops from it but it is his task to bring the interests of the class back to the judgements expressed by the children themselves.

E) Asking for further explanations

Another job of the teacher is that of asking for new clarification or explanation of the motivations of a strategy expressed. The motivations for such a request can be various:

- The teacher has not completely understood a pupil's statements. It often happens that the

pupils, when reading their protocol, do not realize that they have proceeded too quickly without allowing the one who listens to follow their reasoning; the teacher thus invites them to reread it more slowly. Other times the pupils do not explain clearly or they are too concise and also in this case it is important to ask them to reformulate what they want to express.

- The one who explains needs support to be able to reflect better and evolve his thought process. In certain cases, it is necessary to have some reasoning repeated to give the pupil a way to be able to reflect. Sometimes that is sufficient because the pupil realizes the error in what he is saying or because he manages to develop better a thought upon which even he has not yet focused.
- The classmates have not understood what has been said. It often happens that a pupil does not explain his thought with sufficient clarity. Sometimes his classmates ask for clarification while in other cases it is the teacher who, from the expressions on their faces, realizes that it is necessary to repeat an argument.

F) Cutting or evaluating statements

Furthermore, it is within the authority of the teacher to cut possible repetitive statements and to evaluate instead those which, correct or incorrect, could construct a positive turning point opening a new path not already travelled. For example, if the discussion becomes sterile because strategies or arguments already expressed are repeated, it is a good idea for the teacher to redirect the discussion to the heart of the problem which is being debated, possibly resuming traces of the discussion which still need to be developed.

Although it is not simple but it is very constructive to succeed in intervening with the proper judgement of the classmate's arguments, the teacher often invites the pupils to intervene on what had been said by the others and when that happens, that is, that a pupil intervenes to respond to a classmate, the teacher explicitly underlines the importance of such an intervention.

Example 3

During the same discussion referred to in the preceding examples, the teacher stops to underline Pierpaolo's intervention who wants to oppose Allesandra's theory, according to which two players are equally as good if they lose the same number of matches.

Pierpaolo: *"If one plays 100 matches and loses 50 of them and another plays 50 and loses 50, since the difference is equal they are equally good?"*

Teacher: *"I don't understand. Alessandra have you understood the example that Pierpaolo gave? Try to repeat it."*

Pierpaolo: *"Ok. One plays 100 matches and loses 50, the other plays 50 and loses 50 should they be equally good?"*

Thanks to the request for repetition on the part of the teacher, the entire class focused its attention on Pierpaolo's example, particularly useful for refuting Alessandra's strategy.

G) Being flexible

One quality which the teacher must possess is that of being flexible according to the humour of the class, as a matter of fact while sometimes it is necessary to select the interventions because everyone would like to speak, other times it has to be the teacher's task to solicit a class which is not very participatory. It should be remembered, moreover, that the level of concentration of the class is not constant in that it is influenced by various factors such as the excitement about an upcoming school trip, or the tiredness typical of the last hours of lessons. The work programmed prior to going into the classroom therefore, could undergo notable changes following ascertainment of a class atmosphere which is not completely positive. The impression could be of a "waste of time", nevertheless it has been established on many occasions that class work proves to be most useful only if one manages to sense the positive disposition of the class.

H) Refraining, sometimes, from 'mathematical' clarifications

Very often the teacher must stop himself from making corrections and clarifications of a mathematical nature so as not to block the flow of the discussion, particularly if it is complex. This deals with a behaviour which is not easy to get used to because traditionally the mathematics teacher is usually expected to always have the task of clarifying terms and definitions without letting even one such occasion slip by. However, in some situations it can be better to refrain from such a task so as not to derail the attention of the class, as appears evident in the following segment of a discussion

Example 4

Francesco: *"I did this: so, I took these results as if they were fractions, that is Alberto 15/30."*

Edoardo: *"Me too!"*

Francesco: *"Bruno 20/90, Carlo 28/52 and Dario 48/100. So, all the fractions, so first I took the first fraction which is 20/90 and I, what's it called, the second figure, the denominator is 90 and I changed, I took the highest common denominator, the common denominator which is 90 in all of the fractions, I transformed them all into 90 for the denominator. After that……."*

Edoardo: *"Teacher, excuse me, it's impossible!"*

Francesco: *"And I did it! So, Alberto has 15/30, since 90 is 30x3, I also multiplied 15x3 and I got 45/90 for Alberto"*

Teacher: *"That is what your tried to do? What was your goal?"*

Francesco: *"To find the best. For Bruno I left him as is, 20/90, then for Carlo who is 52, to arrive at 90 I multiplied by a number that I don't remember and I got 48, and then……well, ok, for Dario on the other hand I have a denominator of 100, to arrive at 90 I divided by a certain number5 and I got 43."*

Teacher : *"And at the end?"*

Francesco: *"At the end I had…..well……45/90, Bruno 20/90, Carlo 48/90 e Dario 43/90.*

Teacher : *"So who is the best?"*

Francesco: *"The best is Carlo, because pf 90 matches he won 48, then the second is Dario"*

Teacher: *"OK…..has someone else used the same method or has someone used a different method?"*

What is under discussion is the correctness of the resort to the comparison of fractions rather than the comparison of the differences (between matches played and won by one player or between matches won by different players)therefore, the teacher suitably did not have to remind that class how to obtain the lowest common denominator correctly. It is interessting to observe that the teacher did not correct the wrong terms used by Francesco who said "the second figure" instead of "the second number" and "the highest common denominator" instead of "the lowest common denominator". What the teacher held to be important, in this phase, was that the resolution strategies which emerged were discussed and therefore did not offer any mathematical clarification at this time which would certainly be an obstacle to the pupils discussions.

3. Concluding observations

To conclude what has been shown, it can be interesting to highlight some positive aspects of the didactic experiences which have been referred to, which were conducted with particular attention to instances of investigation and collective discussion. Also what follows, like that which was shown in section 2, emerged during encounters with the class teachers who carried out these experiences.

First of all, it can be noted that through the didactic modality described, greater class participation in the work proposed is obtained and the children very quickly acquire the awareness of their direct involvement in the construction of knowledge.

Leaving open a problematic situation even for quite a long time, namely the fact that the teacher does not express himself on the correctness of the strategies which the students come up with, allows everyone appropriate time for maturation and offers many opportunities for expression and debate. Moreover, if the pupils are left free for quite a while to express themselves, they make the path of their learning and the obstacles and misunderstandings which they encounter very evident, therefore facilitating the construction of more profitable successive didactic moments.

Encouraging the interaction between equals also develops the ability to argue their own point of view and to interpret it to others, educating them to the respect of and comparison with the ideas of others.

It also seems important to indicate several open problems relative to the carrying out of the didactic modality illustrated and that clearly constitute an obstacle for anyone who wants to carry them out in their own classes.

First of all the appropriate didactic material for such a project is still all to be done, at least in our country. The traditional textbook, by its nature, cannot propose open situations and overall it is difficult for these to be adapted to the evolution of mathematical discourse of a specific class.

It is necessary, moreover, to be adequately prepared to know how to carry out this new rôle for teachers, to know how to interpret this new way of being in class. It is not simple to manage a discussion positively and neither is it easy to choose the problematic situations best adapted to initiate mathematical investigation in class. Even before this, to prepare oneself to carry out a different rôle as teacher, it is necessary to agree with the necessity of such a change, that is the urgency of placing the pupils at the centre of the didactic process and constructing around them the most favourable environment possible for their effective educational growth. This deals with ideas which are certainly not original in the literature, but which are still a long way from being realized in practice in class.

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