A Classroom Use of the Geometer's Sketchpad in a Mathematics Pre-Service Teacher Education program

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ABSTRACT Over the last two decades or so mathematics teaching, geometry teaching in particular, has gone through continuous trials incorporating the use of technology. Since the early 80s several instructional software were developed for the purpose of helping teachers and students to use computers as teaching and learning tools through exploration rather than just as drill-machines. The main purpose for these technological developments is to guide students through a discovery process that encourages students first visualize and analyze a problem, and then making conjectures before attempting a proof. This approach is based on van Hieles levels of geometric thinking: Visualization, Analysis, Informal Deduction, Formal Deduction, and Rigor. In this paper, The Geometer's Sketchpad software with exemplary applications will be introduced.

Background

Mathematics teaching, geometry teaching in particular, has been reshaped due to certain innovative developments in recent years. For centuries geometry has been taught through deductive reasoning approach. Although the deductive reasoning approach is perfectly acceptable it does fail reaching the majority of students. The 1982 report of the National Assessment of Educational Progress (NAEP) stated that deductive reasoning in proving was the least liked mathematics topic of the 17-year-olds. When these students were asked about the importance of the topic, less than fifty percent of them rated the topic as important.

In 1989, the National Council of Teachers of Mathematics (NCTM) called for substantial changes in the way mathematics is taught (the *Standards*, 1989). In teaching geometry, the *Standards* called for an increase in exploration, conjecture making, and use of geometric transformations. The *Standards* recognized the impact of technology on the teaching and learning mathematics through freeing students from lengthy time-consuming tasks and taking them in the world of observation and exploration of mathematical ideas.

The 2000 NCTM *Principles and Standards for School mathematics* indicate that instructional programs from prekindergarten through grade 12 should enable all students to-
   (1)   analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
   (2)   specify locations and describe spatial relationships using coordinate geometry and other representational systems;
   (3)   apply transformations and use symmetry to analyze mathematical situations;
   (4)   use visualization, spatial reasoning, and geometric modeling to solve problems. (p. 41-43).

Making a case for what have been missing in school mathematics is a noble task. The point however is how these gaps can be filled; it is not an easy task at all.

In recent years, in particular since early 80s, a few technological developments have been introduced and made available for teachers and students. This marked the beginning of a major change in the way we teach geometry. An instructional software known as the Geometric

Supposers for Apple II computers has been developed in 1985 that enabled teachers and students to use computers as teaching and learning tools. The software helped creating an environment in which students explore geometric figures and make conjectures about their properties. Learning geometry would then be turned into a sequence of part-to-part, part-to-whole, and whole-to-whole interrelationships discovery of geometric figures. This has been viewed as a process that would open the door wide for proofs. This approach reflects the research done by the Dutch mathematics scholars Pierre van Hiele and Dina van Hiele-Geldof. Based on their research finding in the classrooms, the van Hieles observed that students pass through a sequence of thought development levels in geometry: Visualization, Analysis, Informal Deduction, Formal Deduction, and Rigor. Most of the texts in the market assume that students are able to employ formal deduction right away. Little if any has been done to enable students to visualize, to analyze, and to make conjectures about a geometric shape. The three levels have been skipped at once.

The main goal therefore is to bring students through the first three levels: Visualization, Analysis, and Informal Reasoning. The Geometr's Sketchpad has been created to bring students through these three levels. It fosters a process that encourages observation, discovery, and making conjectures; a process that closely reflects how mathematics is normally created.

Geometer's Sketchpad

Briefly, upon activating the Sketchpad program, a drawing window will be opened with the following:

       Title bar
       Menu bar
       Tool Box (or Drawing Tools)
       Tool Status Box

Then what we can do with Sketchpad?
We can do seven major tasks:

    (1)   Euclidean constructions
    (2)   Transformations
    (3)   Analytic Geometry – working with rectangular or polar coordinates
    (4)   Mix Graphs and Text
    (5)   Change the Visual Properties of the displayed Figures
    (6)   Create Animations
    (7)   Creating Scripts (with which a complex construction can be encapsulated in a single step).

What makes Sketchpad interesting is that it is a dynamic geometry environment where *dragging* plays a central role – a set of demonstrations will be made highlighting this feature of Sketchpad

Classroom Uses of Sketchpad in a Pre-Service Program

In 1999 the Ontario Ministry of Education, Canada, has purchased the right for Sketchpad package to be distributed to all high schools, colleges and universities across the province. The Ministry has strongly recommended the use of Sketchpad in high schools mathematics curriculum (grades 9-12). At the Faculty of Education, Lakehead University, the Intermediate-Senior (I/S) Mathematics Education class of 14 future mathematics teachers has been set for the experience. Eighty-minutes-session per week for 12 weeks were reserved for the experiment. The materials used were handouts prepared to guide the students in a step-by-step fashion. There were initial hands-on guided instructions at the computer lab at the beginning of each session through a data projector. Then the students follow the handout materials on their own while the

instructor was available for help whenever needed. Each student has a computer on his/her own. The sessions were progressing smoothly. For me, the instructor, I had the sense that the class has been deeply involved in the program and well inspired for the experience. The version for the Sketchpad used was for Windows® 95 and 98. At the end of each session, each student has to prepare a reflection-journal on what was happening throughout the session. At the end of all sessions, the students were instructed, for an overnight duration, to respond to the following:

> *Explain how you would use the Sketchpad in your future teaching. Identify themes that you would consider introducing to your classes through this software.*
> *You may pick one of your themes and be more specific in your detail.*

In the Students Own Words

The following is a collection of students' responses to the above take-home report:

Response 1(M)

"Themes within Geometer's Sketchpad:

• Graphing

• Plotting functions

• Graphical Representation of Mathematical Operations (derivative as slope)

Over these past few months I have come to earn a respect for this software. Initially I felt restricted and limited in its application to any high school math class. I felt as though the activities presented were geared towards the intermediate elementary level and I could not see incorporating GSP into the Ontario Curriculum for grade ten through 12. Ironically I now feel that GSP is an effective tool to be incorporated in any high school math class. GSP has the ability to demonstrate complex graphing and equations, record sketches and make animations. It is not the software that would pose any restrictions but the time available to incorporate it into the classroom. The use of GSP in the classroom reaffirms the belief that school is an institution of learning and not job specific training program. GSP may not be on the desktop of this country's top engineers, mathematicians or physicists but it does offer a visual aid to somewhat abstract and complex mathematical concepts. I feel by incorporating GSP into a classroom you are recognizing the varied learning styles of your students and making an attempt help them develop a confidence in mathematics."

Chris included with his response an example lesson titled "Coordinate Hide & Seek" as an example of how to incorporate GSP into the classroom.

Response 2(M)

"Geometric sketch pad is a good program that we have used since coming back from our Christmas vocation. I have found the sketchpad to be informative, easy to use, and generally a useful program for a teacher. On the other hand, I have a hard time seeing myself, or any other teacher using this program as a learning tool in their classrooms.

I think the best thing about sketchpad is it is so easy to use. I would definitely use this program to draw diagrams for tests and quizzes. I would be able to make accurate diagrams and draw pictures that the students would be able to understand easily. I also like the segment and shape drawing capabilities of this program.

If I were forced to teach something using this program, I would hope that it would be the measuring of circles, angles and arcs unit. I think that sketch pad makes it easy to create large enough diagrams for all the principles to be demonstrated and observed by all the students. I also think that measuring circumferences and radii would be made easy by using the sketchpad. Students would be able to see the different arc angles areas change as the points and diagrams were made larger or smaller.

My personal thoughts on this program is not as positive as everyone else in the classes are. And I am sure that everyone wrote a glowing review of this program. I admit that if I became literate enough in this program I would use it to draw diagrams for tests and quizzes, but other than that I see this program as another venture that will be forgotten about just as soon as a better version or program comes along. That is my major concern about this program – it is not user friendly. It took weeks to figure out which keys did which, and even then I would forget the next time I went to use the program.

I think that students would find this a waste of time because there is little practical use of this program. I can't even imagine the scene that would occur if I pulled my math class into the computer lab to work with the sketchpad. The students would be questioning me the whole time why they were doing this instead of class work and I would have no good answer.

In closing, I think that geometric sketch pad is no good for what is going on in the education system right now. Cut backs in education have lead to less time to fool around with software that is not 100% curriculum based. I am not going to jump on the geometric sketch pad bandwagon because I am not convinced it is the best program that we could be using in the classroom. I think I will wait until I see an effective math program that will help students understand more about mathematics."

Response 3(M)
"Education today is in a constant state of change. This is greatly due to the advancements in electronics, technology, and media. Geometer's Sketchpad is one computer program that will change the way that mathematics is taught.

Geometer's Sketchpad can be used to teach many units in mathematics. I have personally witnessed this program being used in a classroom setting, and I have seen some amazing results. Students are actively engaged in their learning while using this program, and thus they learn, understand, and retain much more information. They feel a part of the learning process and thus respond remarkably well to this style of learning.

The unit that I witnessed, which was taught using Geometer's Sketchpad, was the unit on slope. This program is perfect to teach slope, especially to a Grade 9 class. Students appreciated slope much more because they were able to physically see examples in which they were able to maneuver lines, thus changing their slopes. This change in slope really put into perspective the relationship between slope and the "steepness" of the line.

Students drew a line by either drawing two points then connecting them with the "construct line segment" command, or they used the icon for constructing a line segment. They would then click on the "calculate" command, and then click "slope". The slope of the line segment was then displayed on the screen. Students could then use the grid to discover where the points were, and thus discover the relationship *rise/run*. Students could then plot points on the grid and connect them with a line segment, and figure out the slope themselves before the computer did it for them.

The best part of using Geometer's Sketchpad for this unit on slope is that students could highlight a point on a line segment, and move the point, thus changing the slope of the line. Students have a visual representation of how slope changes relative to the "steepness" of the line.

Geometer's Sketchpad is limited in the sense that one cannot enter equations of lines and have them drawn on the screen. Thus, the TI-83 (he refers to graphic calculator) is an excellent compliment to this program for teaching slope. Together, students have two powerful visual tools to help them understand the challenging concept of slope.

I feel that there are many topics that can be taught with the aid of Geometer's Sketchpad. Transformations are difficult topics for students to grasp because it is difficult to teach visual

topic in a non-visual way. Thus, Geometer's Sketchpad can be an effective tool to teach these topics.

Teachers can also use this program to teach the equation for the length of a line segment. Geometer's Sketchpad is equipped with a function, which measures the length of any line segment in inches or centimeters. Area, perimeter, and circumference can also be taught using this program. Geometer's Sketchpad is also equipped to measure circumference of circles, as well as area of closed figures. By using the length of a line segment feature, students can also calculate perimeters.

Trig function are difficult for some to understand. Geometer's Sketchpad can help in the comprehension of these ratios by using the calculator feature. Students can also learn about the "special triangles" ($30°$, $60°$, $90°$, and $45°$, $45°$, $90°$) and the trig functions relating to each angle of these triangles by using the "calculate angle" command.

Geometer's Sketchpad is a versatile program. I know I will use it in my future career as a teacher of mathematics."

Response 4(F)
As an example of the use of Geometer's Sketchpad in a lesson, she wrote:

"Measuring Circles, Angles, and Arcs, is an example of an activity that models the specific expectations in the Ontario Curriculum documents. This investigation concentrates on: (1) Measuring the circumference and radius of a circle and calculating their ratio, (2) constructing an arc on a circle and measuring its arc angle and arc length, (3) measuring and comparing central and inscribed angle, (4) constructing a section and comparing its area to the area of the circle.

When designing lessons using this medium, three strategies should be included: (1) guided investigations (i.e. tours), (2) open-ended exploration (i.e. where students are given a problem and they are to explain what they did and discovered) and (3) demonstrations (the use of pre-made sketches and diagrams by the teacher)."

Response 5M)
"With such a capable program as Geometer's Sketchpad, there are a large number of potential areas for the incorporation of this program into the curriculum. Sketchpad could be introduced into the curriculum in grade nine and utilized through out grade twelve. Such areas of mathematical instruction which could be greatly enhanced with Sketchpad, include parallel lines, angles, properties of triangles (and other polygons), geometric proofs, properties of circles, slope, equations of lines, rectangular and polar coordinates, and the list goes on.

For this exercise, I will focus on the use of Sketchpad in assisting students with the geometric proofs, which take place around grade ten. Before using the program, the students would already be expected to be knowledgeable in performing proofs by hand, and familiar with the various theorems (opposite angle, transverse parallel, and congruence).

For an introduction into the geometric proofs, students would first be asked to investigate the congruence postulates of triangles.

Construct triangle ABC and then create a copy of this triangle onto another portion of the sketch (label the vertices of the new triangle A', B' and C'.

In order to demonstrate side-side-side (SSS) congruence, have the students measure corresponding line segments.

By seeing the corresponding line segments are equal, the students then have visual proof of the SSS congruence theorem."

This student made an error here. He could use the dragging feature to drag one of the triangles and place it on the other.

Response 6(M)

The geometer's sketchpad is a valuable tool that can be used in the classroom to meet the technological requirements of the curriculum. In my future classes I would definitely incorporate geometer's sketchpad into my lessons. Specifically, I would address the Measurement and Geometry strand of the grade 9-mathematics curriculum. The overall expectation states: Students will:

-formulate conjectures and generalizations about geometric relationships involving two-dimensional figures, through investigations facilitated by geometry software where appropriate."

He is referring to the recent Ontario Ministry of Education Curriculum Guide.

He attached to his report a lesson focusing on the "Specific Expectations" of the Ministry stating that "the lesson that is included is one theme, which I would address. Additionally, it meets the specific expectations: Students will

-illustrate and explain the properties of the interior and exterior angles of triangles and quadrilaterals, and of angles related to parallel lines.

-pose questions about geometric relationships, test them, and communicate the findings, using appropriate language and mathematical forms.

-confirm a statement about the relationships between geometric properties by illustrating the statement with examples or deny the statement on the basis of a counter example."

Response 7(F)

"I would use Geometer's Sketchpad in all my high school math classes. However for grade nine and ten I would most likely use it as an introductory tool to basic algebra, and trigonometry. I would use it as an exploration or introduction into a new topic. This would give the students an idea, especially visually of what this new topic is about and also a chance to play with it before studying the topic.

In the higher level classes, grade eleven and twelve, I would use this more as a teaching and assessment tool. I think the greatest use would be in the areas of trigonometry, geometry.

Another area for application of the Geometer's Sketchpad is with perspective drawings and fractals. I would most likely use these two areas for an enrichment tool."

She attached a detailed lesson titled "Polygons-measurement and calculations".

Response 8(M)

"Geometer's Sketchpad is a fantastic tool for helping students understand relationships and visualize concepts in many areas of Geometry. Many students have difficulties with Geometry, and the principal reason for this is their inability to get beyond the abstract ideas. Geometer's Sketchpad helps in this area by showing relationships, and allowing students to construct shapes and lines for themselves. For most concepts this is much more effective than having a teacher simply explain. It addresses more learning styles (tactile and visual for example), and makes mathematics fun for those students who might find it otherwise in a normal class.

Geometer's Sketchpad can be used in Grade 9, 10, 11, and even 12 math. I will focus on the unit dealing with the following areas: opposite angle theorem; complementary angle theorem; supplementary angle theorem; parallel line theorem; angle sum of a triangle theorem. I taught the unit on Geometry in both Grade 9 and 10 math both of which on the above theorems. I found it difficult to get students conceptualize these ideas, and I know that Geometer's Sketchpad will help the next time I teach this subject.

To use Geometer's Sketchpad effectively in a mathematics course, you need a plan for how you will use the software and how you will assess you students (based on both their work in class and in the computer lab).

The Plan: Most of the first day would be spent getting organized in the computer lab - making sure everybody can log in correctly, teaching people how to operate in a Windows environment, and getting oriented in Sketchpad. One of the problems inherent to working with a class on computers is the vast differences in abilities among students – some are quite comfortable on computers, while others have little or no experience with computers. Subsequent days in the computer lab would focus on the above mentioned theorems.

Assessment: Assessment would be based on the following: • ability to work effectively in the computer lab (a subjective mark); • ability to apply knowledge within Sketchpad (a practical mark); • ability to extend knowledge beyond Sketchpad (a theoretical mark)."

Response 9(M)

"I think that this is an excellent program and could definitely be effectively utilized in schools between grades 8 and 12.

I am not 100 percent certain of when each aspect of geometry is taught so to place a specific application of the program to particular grade will be difficult. However, regardless of grade level I believe that one of the most important things teachers can do is familiarize the students with the program so that future lessons will allow more time for learning and less time for orientation." He offered a detailed description of a plan that he would implement in his future teaching using the program.

Response 10(M)

Sketchpad is very useful for enabling students to explore relationships dynamically so that they can see changes in geometric figures as they manipulate them. The simplest example of this, would be an activity in which students construct a circle. I would use this circle, and use Sketchpad to calculate **circumference/diameter**. Then, I would ask them what familiar number this ratio gives! The next instruction I will give my students will integrate Sketchpad's most useful aspect: the ability to give students a way to explore relationships dynamically so that they can see changes in geometric figures as they manipulate them. I would allow students to do this by picking a point on the circle and manipulating the circumference of the circle (making it bigger and smaller). In this manner, students will see that Sketchpad automatically changes the diameter as well! The most important thing for them to notice, is that the ratio **circumference : diameter** DOES NOT CHANGE!!!"

Final Comment

These reports seem to show a swing in the students' reaction to the 12 weak experience with Geometer's Sketchpad ranging from uncertainty to overconfidence about the potential use of the software in the classroom in their future teaching of mathematics. They all agreed that geometry, algebra, and trigonometry would be the areas where they may use the software. There were good pieces of lesson plans attached by most of them. However, there is no room to include them here.

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