Statistics and Measuring: an experimental educational research in Italian High Schools

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Abstract. Measuring is a very common activity, belonging to pupils' everyday experience. We consider that an informal point of view is really important, and in our opinion it can be effective, in order to introduce some basic concepts of Statistics. In this paper we describe an experimental research activity: we presented to students aged 17-18 years a short test based upon a common measuring activity. The greater part of the pupils showed difficulties in the interpretation of the role of standard deviation.

INTRODUCTION

The history of researches related to the Laws of Chance is ancient and several great mathematicians, in very many periods, contributed in exploring and analyzing foundations of Theory of Probability and of Mathematical Statistics (let us indicate, for instance, as historical references: Daston, 1980; Lakoma, 1998; Todhunter, 1965; Maistrov, 1974).

We shall not give now a full presentation of researches upon the didactic introduction of Probability and Statistics (let us quote the interesting summary in: Gagatsis, Anastasiadou & Bora-Senta, 1998): in this paper we shall propose the beginning of a research activity on an approach to basic concepts of Statistics from an informal point of view: we shall consider some experimental results in order to point out reactions and obstacles and to evaluate them.

In particular, we wanted to investigate by a test if High School pupils, traditionally taught in basic statistical concepts (e.g. the standard deviation), really understand the full meaning of them.

As we shall see, our test is based upon a well-known paradox (as regard paradoxes, let us remember some words by G.J. Székely: "Just like any other branch of science, mathematics also describes the contrasts of the world we live in. It is natural therefore that the history of mathematics has revealed many interesting paradoxes some of which have served as starting-points for great changes": Székely, 1986, p. XI; see moreover: Pflug, 1981; Dall'Aglio, 1991, p. 62, Lolli, 1998).

By the test we wanted to point out:

(a) pupils' understanding of the meaning of the standard deviation, particularly as regards the standard deviation of a result obtained by two different measurements (of two different lengths) made by an instrument whose standard deviation is given;

(b) pupils' understanding of the effective possibility to improve their results (from standard deviations' point of view) by using a different and unusual procedure. So we wanted to point out if pupils choose a complex procedure in order to obtain a lower standard deviation of the result, or if they choose the common procedure although the standard deviation of the result is higher.

As we shall see in the next paragraph, our test will be given by two cards:

(card A) referred to the point (a) above stated;

(card B) referred to the point (b) above stated.

METHOD OF OUR RESEARCH

We considered 49 High School pupils that knew main statistical concepts (4th class of Italian *Liceo Scientifico*, pupils aged 17-18 years, in Treviso, Italy), in particular the standard deviation. We proposed to them the following test (the employed paradox is quoted in: Székely, 1986, pp. 125-126; see moreover: Hotelling, 1944):

Card A

You are going to measure the length of two (different) rods by two measurements; you use an instrument, which measures length with random error whose standard deviation is σ.

1) How do you measure the length of your rods? Describe your procedure.

2) What is the standard deviation of your result?

Time allowed: 5 minutes (we wanted that students examine the problem 'at a glance'). No textbooks or electronic calculators allowed.).

Then we gave to the pupils the following card:

Card B

Once again, you are going to measure the length of two (different) rods by two measurements; you use the same instrument above described, which measures length with random error whose standard deviation is σ.

A friend of yours suggests to you the following procedure (in his opinion it is better than measuring the rods one by one!): you will measure the total length T of the two rods (by putting them end to end) and then their difference D (by putting your rods side to side). So the measured lengths of the rods will be: (T+D)/2 and (T–D)/2 respectively.

1) Do you agree with your friend?

2) Why?

Time allowed: 8 minutes.

Of course, the correct answer (card B, question 1) is: "yes". In fact, the standard deviation of the lengths measured according the method described in the second card (card B) is $\frac{\sigma}{\sqrt{2}}$, which is clearly lower than the standard deviation σ previously obtained (see card A).

Results of the test (as previously noticed, with reference to 49 High School pupils) are given in the following tables:

Answers to question A-1	*Pupils*	*Percentage*
Measuring the rods one by one	47	96%
No answer	2	4%

Answers to question A-2	*Pupils*	*Percentage*
The standard deviation is σ	46	94%
No answer	3	6%

Answers to question B-1	*Pupils*	*Percentage*
I agree with my friend	10	20%
I disagree with my friend	27	55%
No answer	12	25%

As regard pupils' justifications (i.e. answers to question B-2), the greater part of the pupils that preferred measuring of the rods one by one stated, in several ways, that this method is simple, direct and quick. It is remarkable that only 3 pupils (out of 10 that preferred the second method) calculated the correct standard deviation.

CONCLUSIONS

While the first part of our test (card A) showed a satisfactory knowledge of the standard deviation, as regards the card B, why the calculation of the standard deviation in considered and correctly made only by 3 pupils out of 49 (i.e. only by 6% of the whole-considered sample)?

Of course we must underline that our sample is rather small and further deeper researches would consider moreover detailed characteristics of teaching and the possible presence of pre-course intuitions; however, in our opinion, data here obtained reveal that many pupils have hardly understood correctly the practical meaning of the standard deviation.

Situation previously described shows that "simple" procedures (often seen as natural and reassuring ones) are frequently preferred and extended to a lot of cases, sometimes without controls: this behaviour can cause mistakes. So many pupils, although they knew the concept of standard deviation, in a practical situation preferred a common, simple procedure without calculating the standard deviation of obtained results: this behaviour reveals the presence of a real obstacle.

We do not think that such obstacle can be considered as properly an epistemological one or (only) as an educational one (we now refer to the fundamental classification of obstacles in: Brousseau, 1983; see moreover: Vergnaud, 1989). If we consider it as educational obstacle, for instance caused by any weakness in teaching, we must underline that the influence of affective aspect is surely remarkable. Then, in our opinion, it would be regarded as an *affective obstacle*, too; and it would be difficult to overcome it completely just by educational means (like for example showing of counterexamples; see interesting situations described in: Kaldrimidou, 1987).

Of course the possibility to overcome obstacles like the obstacle examined in this paper would be clarify by further researches.

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