MULTIMEDIA TECHNOLOGIES: A NEW WAY TO ANALYZE THE CONNECTIONS BETWEEN THE MATHEMATICS, ARCHITECTURE, AND ARTS.

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ABSTRACT The academic mathematics course has become, for many students, a highly structured sequence of definitions, theorems, and proofs that lead only to additional definitions, theorems, and proofs. Modern university education is becoming more and more dependent on the new media. This work describes an educational approach, in a undergraduate course (first year), where some mathematical concepts are presented with their artistic connections, using also the new media (e.g., hypertext, hypermedia, virtual reality, the World Wide Web, and so forth). I refer of my experience at the Academy of Architecture of Mendrisio, University of Italian Switzerland, where I have researched the interconnections between Mathematics, Arts and Architecture (e.g., the polyhedra, the golden section, the fractal geometry) using multimedia solutions [9, 10]. I have observed that the new media, in support to the traditional educational method, can help to reach some good cognitive goals, to increase the students' attention, to do more interactive the learning process.

1. INTRODUCTION

At the Academy of Architecture of Mendrisio (University of Italian Switzerland) there is a course of Mathematics (its name is Mathematical thought) where the students learn some different and interesting aspects of this subject [1]. The course, conceived by Prof. Sergio Albeverio (chief of the chairs of Mathematics) and by me, intends to provide an introduction to basic facets of mathematical thought (logic, algebra, geometry, topology, analysis and stochastic). Also, it aims to introduce a host of applications.

The themes of the course include: the concept of symmetry and broken symmetry; numbers and algebra: natural, real, complex numbers. The relationships between structures and algebraic constructions; the infinity in mathematics: potential (infinite process) and real infinity. The infinite numbers and infinitesimal. The concept of limit and the fundamentals of analysis; the concept of proximity-deformations, transformations, and elementary topology; logical structures, how they relate to the theory of foundations ("Cantor's Paradise" or what is left of it). Chance: tools to analyze it. The basic concepts of probability theory and stochastic processes.

Fractal forms: geometry, measures, architecture. Dynamic systems and their attractors, strange attractors as fractal sets. Complex systems: measuring complexity, algorithmic complexity, entropy, dissipative systems, biological and ecological systems. In this course I have used some multimedia technologies (e.g. CD-ROM, computer-aided design (CAD) tools, scientific documentaries, educational hypertext and hypermedia) in support to the traditional educational tools (overhead projector and blackboard) [9]. There are many mathematical subjects that can find more applications in arts and architecture, for example: the platonic solids, the polyhedra, the golden section, the fractal geometry. It is possible to present them using the new media inside of a traditional lecture or in the laboratory activities. The computer plays a central role in this environment and it coordinates the use of various symbol systems [7]. For this reason, in the lecture hall I have a personal computer connected with a LCD projection device.

2. THE PLATONIC SOLIDS AND THE POLYHEDRA

Through history, polyhedra have been closely associated with the world of art. For example, Plato found an association in the *Timaeus* between the Platonic solids and the elements of fire, earth, air, and water (and the universe). The peak of this relationship was certainly in the Renaissance. For some Renaissance artists, polyhedra simply provided challenging models to demonstrate their mastery of perspective. For others, polyhedra were symbolic of deep religious or philosophical truths of great import in the Renaissance. This was tied to the mastery of geometry necessary for perspective, and suggested a mathematical foundation for rationalizing artistry and understanding sight, just as renaissance science explored mathematical and visual foundations for understanding the physical world, astronomy, and anatomy. For other artists, polyhedra simply provide inspiration and a storehouse of forms with various symmetries from which to draw on. To explain the platonic solids and the polyhedra I have used some hypertexts and hypermedia (on CD-ROM and online) and I have also researched in the Internet some interesting animations using Applet Java.

At the Internet address: http: / home.a-city.de / walter.fendt /mathengl/ platonengl.htm there is an Applet Java which contains an animation where we can choose and rotate the platonic solids (changing the rotation angle).

At the Internet address: http: // www. li.net/~george/virtual-polyhedra/vp.html there is a fine hypermedia, by George W. Hart [6] which contains a collection of over 1000 virtual reality polyhedra . This site is a self-contained easy-to-explore tutorial, reference work, and object library for people interested in polyhedra. We may choose to simply view the virtual objects for their timeless, serene aesthetics, or to read the related mathematical background material at various levels of depth. Of course, as an academic type, there are few exercises. Using this hypermedia online (at the address: http: // www. li.net/~george/ virtual-polyhedra/art.html), the students can analyse the presence of the polyhedra and the platonic solids in the Arts (e.g., Pre-Renaissance, German and Italian Renaissance, Post Renaissance and Twentieth Century). This hypermedia also introduces the interactivity in the learning process, and this is not possible using the traditional textbooks.

The use of Virtual Reality for teaching offers a series of advantages [2]: learning efficacy, and safety in interaction (because the knowledge objects are virtual). In this case the students have used some virtual objects, created using VRML (Virtual Reality Modelling Language) language, to know and to manipulate the polyhedra, to rotate and to observe the platonic solids from the different points of view, to analyze the connection between the shape of the fullereni molecules (C_{60}) (Figure 1) and the geodetic cupolas (designed by the engineer Buckminster Fuller).

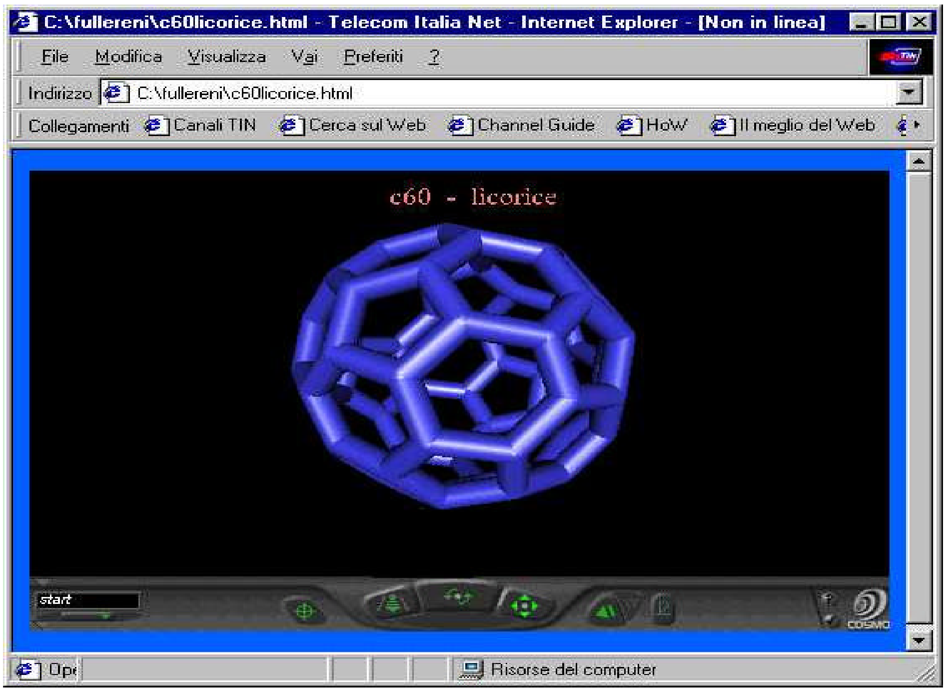

Figure 1. Virtual fullereni.

To introduce the polyhedra in the Escher's work I have used the CD-ROM Escher Interactive™ which contains an audio/ visual life of M. C. Escher and videos of the artist at work [11]

This is not an instructive tools but I have built an educational path to present some subjects of my students. I have explained the sections dedicated to:

- the impossible shapes;
- the tessellations;
- the morphings;

- convex and concave (this is a game which describes a space that could be convex or concave depending upon the perspective of the objects placed within it);
- the spheres (which describe the distortion produced when a spherical lens is placed over a two dimensional picture space) (Figure 2) [9, 10].

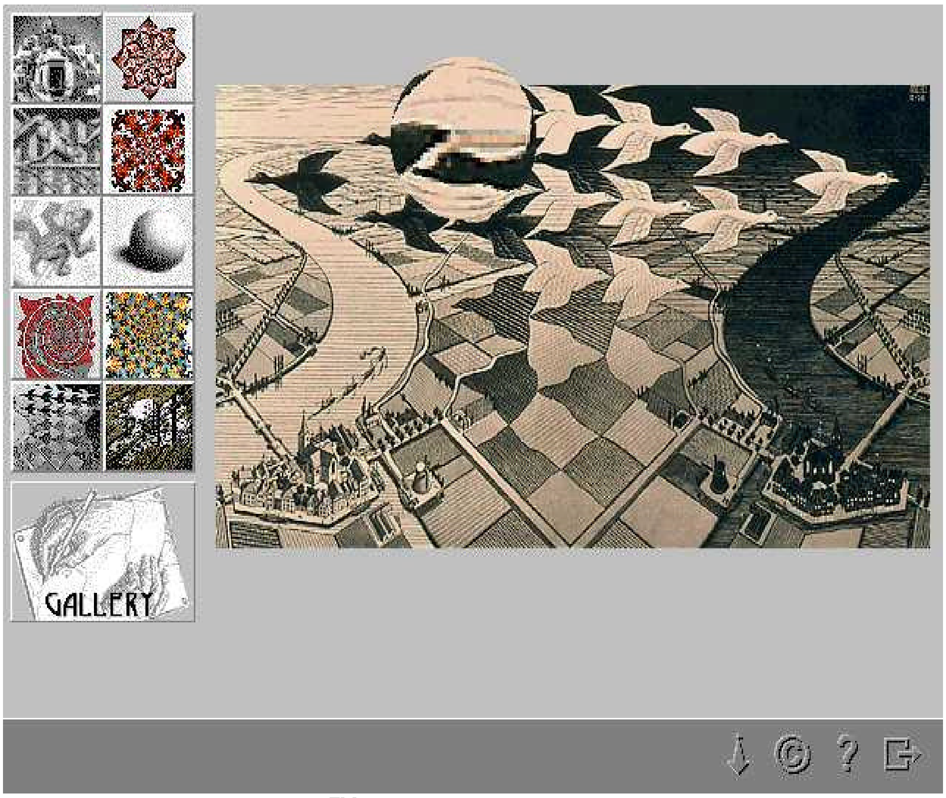

Figure 2 Escher Interactive™ : the section dedicated to the spheres

3. THE GOLDEN SECTION

The golden section (also called the Divine Proportion), said by J. Kepler (1571 - 1630) to be "*one of the two treasures of geometry*", appears repeatedly in growth patterns in nature and has fascinated mathematicians and artists for centuries. The golden section is a certain length that is divided in such a way that the ratio of the longer part to the whole is the same as the ratio of the shorter part to the longer part. Even from the time of the Greeks, a rectangle whose sides are in the "golden proportion" (1 : 1.618 which is the same as 0.618 : 1) has been known since it occurs naturally in some of the proportions of the Five Platonic Solids. This rectangle is supposed to appear in many of the proportions of that famous ancient Greek temple, the Parthenon, in Athens, Greece. In arts and architecture there are other examples of the use of the golden section (e.g. in the Santa Maria Novella's church in Florence, in the Constantino's Arc in Rome). I have searched some Internet sites to propose in the students' learning path, e.g. at the Internet address: http: // www.mcs.surrey.ac.uk/Personal/ R.Knott/ Fibonacci/fibInArt.html there is a site dedicated to the golden section in Art, Architecture and Music. The students have navigated in this hypertext, in the laboratory activities, and they have visited the Uffizi Gallery's Web site in Florence, Italy, where there is a virtual room of Leonardo da Vinci's paintings (the pictures are links to the Uffizi Gallery site and the they are copyrighted by the Gallery).

4. THE FRACTAL GEOMETRY

Fractal geometry is the study of mathematical shapes that display a cascade of never-ending, self similar, meandering detail as one observes them more closely [4]. The self-similarity is a property, in which small parts of an object are similar to larger parts of the object, which in turn are similar to the whole object. The fractal dimension is a mathematical measure of the degree

of meandering of the texture displayed. To introduce the fractal dimension, the self similarity, the fractal objects, and the connection between the fractal geometry and the dynamical systems I have proposed to my students an interesting scientific documentary produced by Le Scienze™ (Italian version), which also contains some interviews to Mandelbrot and Lorenz and some animations [9, 10]. After this phase I have studied in depth this subject using the Web as an archive of information. At: http://www.saltspring.com/brochmann/Fractals/fractal-0.00.html
for example there is a hypertext online on the fractals. At this address there are some Web pages (by Robert L. Devaney): http://math.bu.edu/DYSYS/FRACGEOM2/FRCGEOM2.hmtl which explain the fractal geometry of the Mandelbrot set and its connection between the Fibonacci sequence.
The author forgets for the moment about the rotation numbers and he concentrates only on the periods of the bulbs (the denominators). He calls the cusp of the main cardioid the ``period 1 bulb.'' The largest bulb between the period 1 and period 2 bulb is the period 3 bulb, either at the top or the bottom of the Mandelbrot set. The largest bulb between period 2 and 3 is period 5.
And the largest bulb between 5 and 3 is 8, and so forth. The sequence generated (1, 2, 3, 5, 8, 13,...) is, of course, essentially the Fibonacci sequence [5]. Inside this hypertext there are some links to some web pages dedicated to the fractal theory.
During the lectures I have observed that architectural forms are man-made and thus very much based in Euclidean geometry, but we can find some fractals components in architecture, too. I have divided the fractal analysis in architecture in two stages [12]:

- on a little scale analysis (e.g., an analysis of a single building)
- on a large scale analysis (e.g., the urban growth).

In the analysis on a little scale we can find:

- the building's self-similarity (e.g., a building's component which repeats itself in different scales);
- the box-counting dimension (to determine the fractal dimension of a building).

I have explained these subjects using also the new technologies. For example to introduce the basic concepts of the fractal geometry I have created a hypertext using HTML language. It is available at the Internet address: http: // www. arch.unisi.ch / fractals/fract1e.htm
inside the hypertext there is a page dedicated to the fractal architecture with the building's self-similarity (e.g., the fractal Venice, an Indian fractal temple, the fractal floor of the church of Anagni, Italy) (Figure 3).

Figure 3. The floor of the Anagni's Cathedral (Italy), first example of fractal art.

In the fractal analysis on a large scale we can find:

- the fractal geometrical description of real built-up area;

- the simulation of urban growth using fractal algorithm (e.g. using the Diffusion Limited Aggregation (DLA) model).

A historical analysis has demonstrated that the growth of a town can be thought like a set of living units that are "little copies" of a big structure (from the point of view of the spatial and relational organization) [12].

There is an interesting syllogism:

- the house like a part of the town, and it is little image of the town, too;
- the town is constituted from copies of itself (this is an example of self-similarity).

Other approach of mine it has been to find the connections between the structure of the towns and the fractal sets.

The geographer Michael Batty suggested that the fractal geometry can describe the urban growth: "*The morphology of cities bears an uncanny resemblance to those dendritic clusters of particles which have been recently simulated as fractal growth processes*" [3] .

The Diffusion-Limited Aggregation (DLA) and Dieletric Breakdown Model (DBM) might form an appropriate baseline for models of urban growth. The form of the cities can be visualized in very different ways at many levels of abstraction. Diffusion-Limited-Aggregation (DLA) was introduced by T. A. Witten and L .M. Sander in 1981. The rules of the model are quit simple. One start with a seed particle at the origin of a lattice. Another particle is launched far from the origin and is allowed to walk at random (e.g., diffuse) until it arrives at a site adjacent to the seed particle. Then it is stopped and becomes part of the growing duster. A third particle is then introduced and undergoes a random walk until it also becomes incorporated into the growing cluster. This procedure is repeated until a cluster of sufficiently large size is formed. To explore the Diffusion Limited Aggregation model I have used two hypermedia available at these Internet sites:

- http:// polymer.bu.edu /~trunfio/java/dla2/dla.html (which contains an Applet Java which explore the DLA model),
- http:// apricot.ap.polyu.edu.hk /~lam/dla/dla.html.

Second hypermedia contains an Applet Java which illustrates the fractal growth. There is the presence of the button "Grow slowly" which initiates an illustration of the algorithm. We launch walkers from a "launching circle" which inscribes the cluster. They are discarded if they wander too far and go beyond a "killing circle". During launching or killing, the corresponding circle is shown in red or blue respectively. The diffusion is simulated by successive displacements each of one-tenth of the particle diameter in an independent random direction. After every step, all particles on the cluster are checked to detect any overlapping with the walker which would form an attachment (see figure 4).

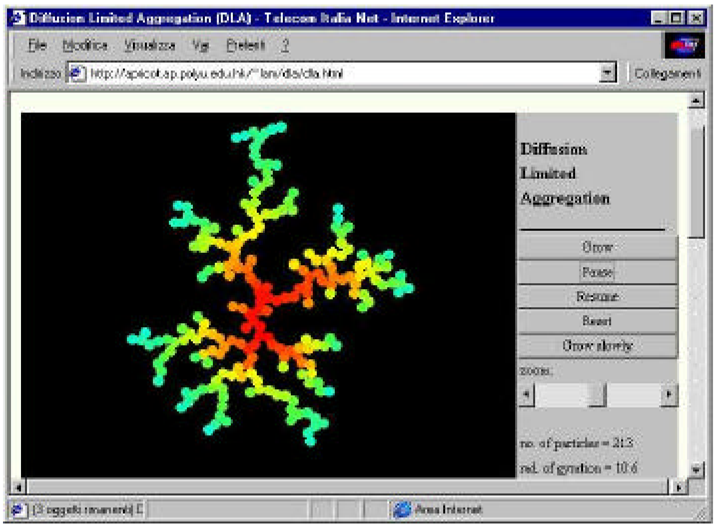

Figure 4. Applet Java which illustrates a Diffusion Limited Aggregation.

5. CONCLUSION

The diffusion of technology throughout an educational institution cannot be seen as separate from the learning process that all members of the organization go through as they learn about their new roles in relation to the technology, as they struggle to transform their perspective toward technology in general, and as they begin to appreciate the value that it can add to the teaching/learning process [13]. The new technologies and the World Wide Web are revolutionizing the ways in which we share information. In particular, it is affecting the ways in which we teach and learn [8]. Web-based instruction is an approach that can engage a global audience using the World Wide Web as a medium. It involves creating a learning environment where resources are available and collaboration is supported, where Web-based activities are incorporated into an overall learning framework, and where novices and experts alike are supported [13]. Some researchers, such as Spiro and Jehng (1990), have emphasized the active role learners must play in order to learn in hypertext-based learning environments. The experience of mine is only an example where the new media have been used, in support to the traditional educational methods, to analyze the connections between mathematics, arts and architecture. This approach can make more interesting and interactive the lessons instead of the traditional educational methods. During the students' navigation in hypertext I have observed that some students can only interact with pages passively, by reading and clinking the links. For this reason in the laboratory activities it is important the presence of the assistants professor because they can illustrate the correct and active navigation inside the hyperdocuments. Using this approach, the role of the teacher is evolving from that of a giver of information to that of a facilitator of student learning. For this reason new technologies can help the teachers to complete this evolution [11].

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