LEARNING HERITAGE RESTORATION, LEARNING MATHEMATICS

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Introduction

On one occasion, the Spanish architect Antonio Gaudi showed to one of his visitors a range of geometrical models and he remarked, with excitement in his eyes:

Wouldn't it be beautiful to learn geometry in this way?

Without any doubt, mathematical education for architecture students will be more effective and pleasant if all the theoretical knowledge is explained with real architectonic examples. It is well known the relationship between classic architecture and mathematics, relationship that is extended along the history of architecture though illustrious names like Wright, Botta, Kahn, Nervi, Calatrava or Le Corbusier.

Architecture, like no other scientific discipline, can be used as a never ending source of numerical, algebraic, geometric, analytic or topologic problems, amongst others fields of mathematics. A modern conception of architecture must necessarily include mathematics for its comprehension. Reciprocally, the education of mathematics in architecture needs to be based on the constructive event to be effective.

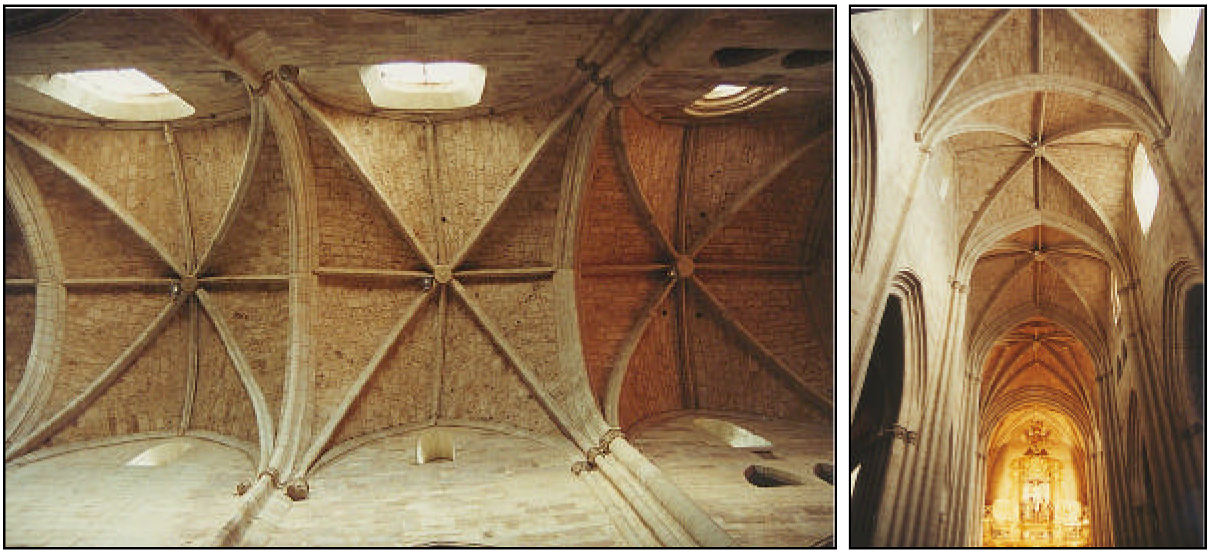

Fig 1. Main nave of the Church of Santa Maria la Real, in Najera, Spain. A geometric analysis of the church shows the vaulting system completely distorted. Arches and vaults are deformed, the walls and columns are bended and cracks appear on the aisles. Geometry is the first test to verify the structural state of a gothic construction.

Interdisciplinary education stimulates positively teachers and students, providing a much more persistent and interesting training. It is obvious that the mathematical knowledge acquired inside an architectural environment will be more easily used by future architects once finished their studies in the university. As an example of this way of teaching mathematics, in this paper we will show some ideas and mathematical experiments related to one of the more complex branches of architecture: restoration, repair and maintenance of architectural historical heritage.

Geometry of a heritage building

Describing the geometry of a building is the first phase of a heritage restoration work. A precise geometric model must be conceptually simple, but representative enough of the structural and constructive system of the building.

In order to create an accurate survey, there exist two basic techniques to acquire the geometrical data that defines a heritage building: photogrametry and topography.

Photogrametry is based on the principle of three-dimensional vision starting from two flats images taken slightly separated amongst them. Like we use our brain to represent the spatial depth using two flat images taken from our left and right eyes, a computer can reconstruct a three dimensional environment using two photographs taken from separated points directed toward the same focus. A pair of photographs of that type is known as stereoscopic image.

The geometric basis of photogrametry is the following: an object placed near the infinity line appears in the same position in the flat images, whilst objects near the observer vary their relative position in both images. Measuring the displacement of one object in both images, we are able to deduce the distance from the observer in the real world.

Topography is a well-known process to get spatial coordinates from any construction. The geometrical theory is extremely simple: one object looks bigger or smaller to us depending on the distance it is placed from the viewer. With an especial device, the size of the object is measured and therefore the distance of the point deduced. Tracing imaginary triangles from known coordinates, any visible point of the building can be measured with high precision.

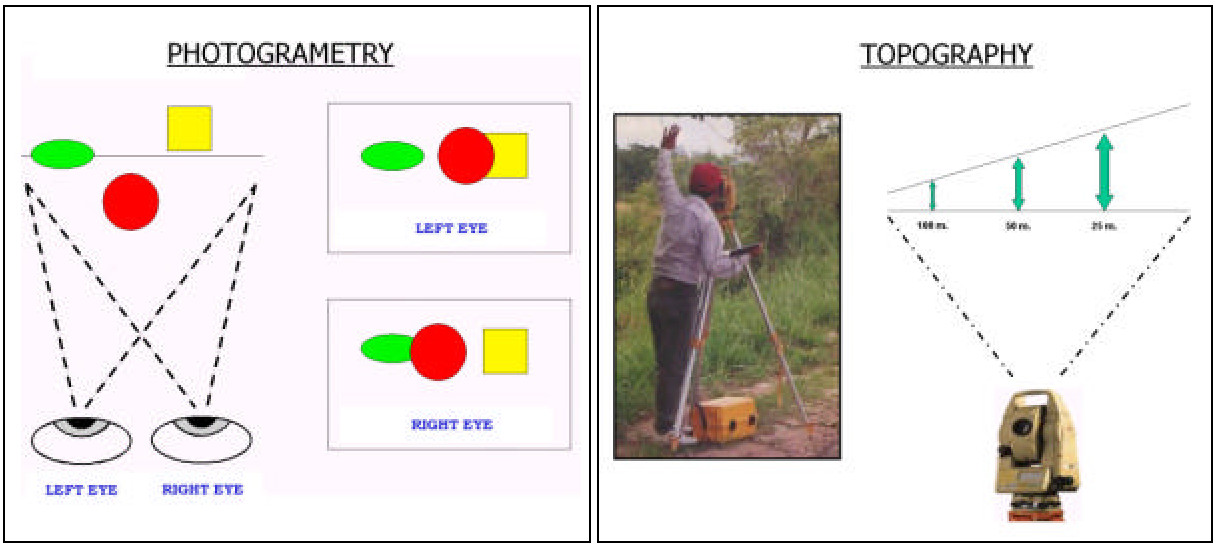

Fig 2. Photogrametry and topography are two techniques used to take precise coordinates from real world. Photogrametry is based on the measurement of an object in two photographs taken from separated points towards the same focus. Topography is based on the geometry of an object that looks smaller the further it is.


Fig 3. A Romanesque style portico (left) created by means of photogrametry and a gothic church main nave (right) measured using topography. Photogrametry is a technique usually used for high detail models with many ornaments while topography is generally used for big structures with less detail.

Constructing a three dimensional model

Once we have a database with enough points from the building, we proceed to the elaboration of a three dimensional solid model. Several shapes are used to create the final model: ellipsoids, cylinders, paraboloids, hyperboloids and many other shapes that might fit the coordinates from the database until the geometry is successfully completed.

Each and every part of a building has specific constructive, physical and structural characteristics that must be represented in the model. But not only the actual geometry need to be measured. Along time, bending, cracks and deformation of structural elements might have occurred and should be estimated during a representative period of time to get the best possible simulation of the building.

Fig 4. Interior of the Bilbao Cathedral. The making of a computer model is similar to the physical creation of the Cathedral. First, the columns and arches are constructed, then the vaults, piers and flying buttresses and finally the walls, windows and ornaments.

All this processes, known as monitoring, are completed with physical analysis of the soil and the materials used in the construction, mainly compression/deformation essays. The correlation of all the available essays, simulations, monitoring, weather conditions and soil prospecting may suggest hints to detect and solve the structural problems present in a building.

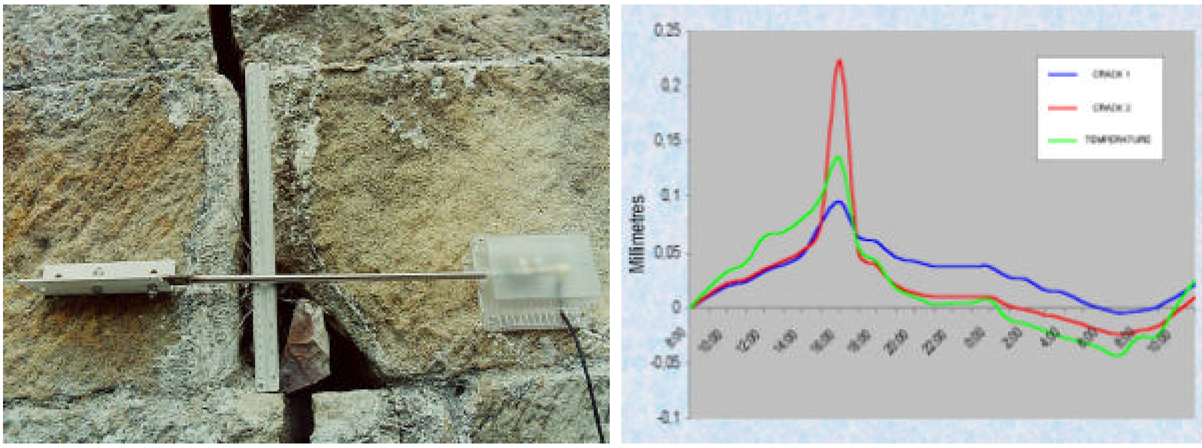

Fig 5. An example of a crack-monitoring device in a building (left) and the graph representing the movement of two cracks in 24 hours (right). The graph also includes a third measure showing the external temperature of the building. It is obvious that exists a clear correlation between the heat and the movement of the cracks, which work as dilatation joints.

Finite Elements Method

The Finite Elements Method is nowadays the unique methodology that gets satisfactory results analysing heritage architectural structures, especially gothic temples. The basis of this method consists on dividing a surface or volume into a reasonable number of small elements. The mechanical characteristics, as stress or displacement, of each and every element is calculated and transmitted to the neighbour elements.

The input for a Finite Element Method analysis is a computer model representing the geometry of the building. This model is not a simple drawing. Each geometric element must be perfectly defined and assembled with their neighbours. Also, it must have appropriate contour conditions and physical properties, like the Young and Poisson Modulus or density amongst others.

Once the geometric mathematical model is completed, it is meshed into small elements. In space, we usually use tetrahedra (four nodes) or brick (eight nodes) elements, although these elements can also be implemented extended to ten and twenty nodes respectively by including an extra node to each aristae.

All this information is translated to a system of equations in matrix form. The size of the matrix depends not only on the number of elements and nodes, but in the number of degrees of freedom. The degrees of freedom can be considered as the variables we want to solve for each node. A typical degree of freedom is six, that includes translation and rotation for each node along the three main axis X, Y and Z.

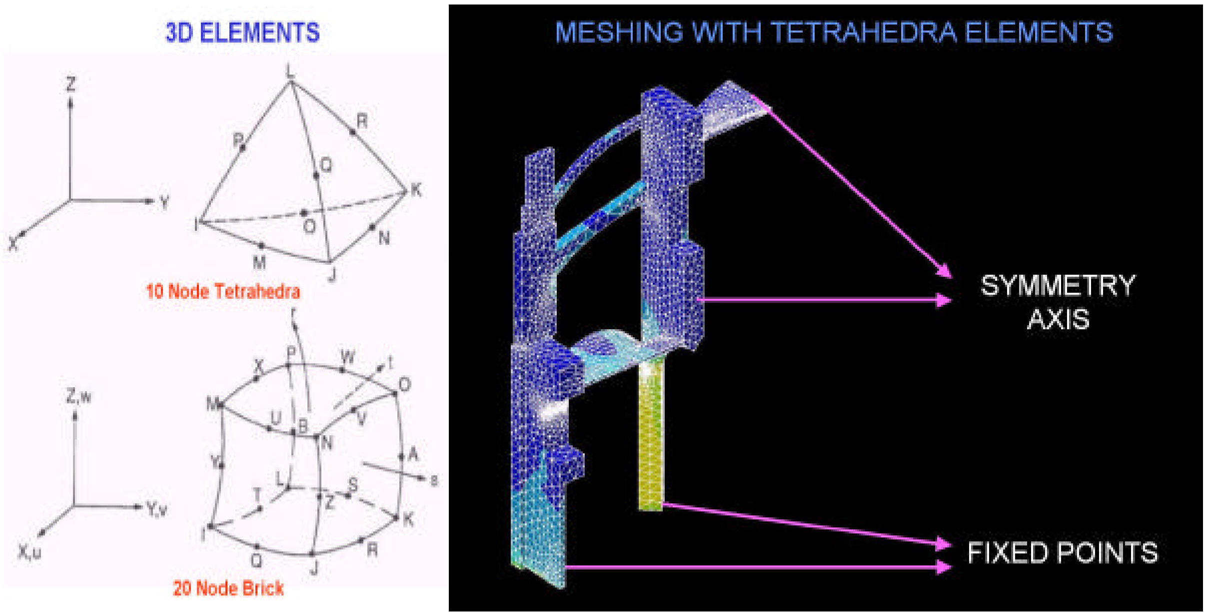

Fig 6. Meshing a section of the external contention system of a gothic church (flying buttresses, vaults, pier and column). The basic element selected for the mesh was the 10 node tetrahedra. Tetrahedra fit perfectly on complex geometries, like the gothic, with a reasonable number of nodes.

After the matrix containing the model is solved, stresses and displacements for each node can be easily estimated. The results are usually represented graphically with colour maps that represent magnitudes. This technique is widely used, as it allows a quick visual understanding and interpretations of the model.

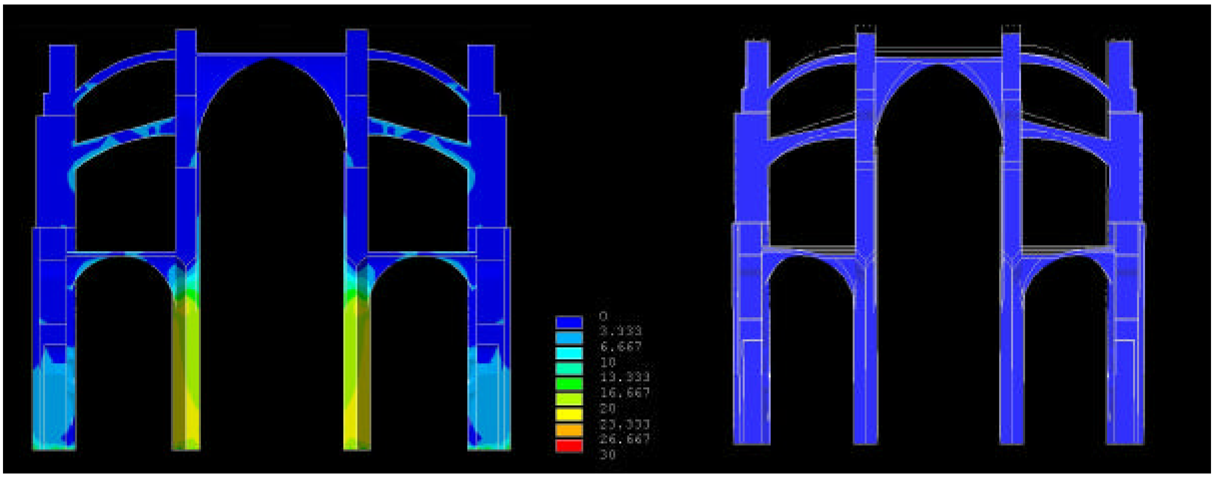

Fig 7. Church of San Antonio Abad in Bilbao. The picture of the left shows the equivalent stress on a section of the church. The picture of the right shows an estimation of the deformation suffered by the temple.

Acknowledgements

The University of the Basque Country and The Department of Spatial Planning, Housing and Environment of the Basque Country Government have sponsored this project.