Alive Maths: Living Mathematics on the Web

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The world wide web is becoming an intriguing place/tool in mathematics education. It combines a variety of modes linked with learning. It is a reference library that is always growing and always open. It provides ways of sharing and communicating visually, textually, and symbolically. And, it is capable of running programs that, like some software programs, can help students grasp sophisticated concepts through dynamic interaction and feedback with multiple representations of mathematical ideas. Although there are many interactive environments currently on the web (mostly Java applets) that allow students to run simulations and explore relationships, there are very few that push students to express themselves mathematically within the environment.

This paper describes the preliminary design of Alive Maths, a collection of bilingual interactive learning environments for engaging middle school learners (grades 7-9) in mathematics. Alive Maths will be accessed from Canada's national education website SchoolNet, and will be available to all classrooms and homes via standard web technology utilizing Java. Alive Maths will feature six microworlds, developed sequentially over the 2000-01 school year.

The first goal of the Alive Maths site is to design a learning environment for middle school students that is "alive", where students are encouraged by the mathematical activities on the site to explore, pose problems, conjecture, and express themselves mathematically. The second goal of the Alive Maths site is to investigate the web's communication modes in the context of the first goal. That is, what is the potential of the web as a tool for learning when it can also be a place for communicating and sharing? I will begin by presenting the rationale for these two design goals and then discuss some initial research conducted with one of the Alive Maths microworld.

Design of Computer-based Learning Environments

Soundly designed computer-based learning environments are particularly well suited for student exploration, manipulation and representation. Students can act on mathematical objects -- much as they might on physical manipulatives -- and obtain immediate feedback of their actions. Although there are many interactive environments currently on the world wide web (mostly Java applets) that allow students to run simulations and explore relationships, there are very few that push students to express themselves mathematically within the environment. It is precisely this functionality that helps students generalize and formalize their mathematical understandings (Noss & Hoyles, 1996).

There are certain activities that lend themselves particularly well to student mathematizing. I will call such activities Rich Mathematical Situations (RMS) and draw upon Higginson's work (1973) in order to characterize them. A RMS is a problem space that leads to open, sophisticated and multi-dimensional mathematical investigation. Openness can refer to the multiple solutions or solution paths to a given problem, as well as the potential for generating more questions and problems. Open-ended problems have been found to be most effective in fostering both student mathematical thinking as well as student motivation in mathematics activity (Nohda, 1997).

The RMS must first be accessible: students can approach the situation from a variety of ways, using a variety of tools (some of them concrete) and pose a range of problems related to the situation. Secondly, the RMS is fertile: its richness in terms of patterns, be they visible or inherent, can lead to sophisticated mathematics. Thirdly, it is connectable: students can make links with previous experiences from both inside and outside of the mathematics classroom. This

continual connection to past achievements, to more profound understandings of mathematics rather than to just more mathematics
underscores the notion that we know and experience things in many different ways, none of them final or absolute.

A fourth characteristic of the RMS is that of symbolizability. Higginson (1973) describes this characteristic as follows: "in working on the problems generated from the given situation the student finds it convenient to invent and modify terms and symbols for the concepts identified" (p. 61). Lastly, the fifth important feature of the RMS: it is compelling. There are several ways in which a situation can be compelling for a student. For instance, it can appeal to an aesthetic sense: students can derive pleasure out of working with the tools and materials; students can engage in actions of transforming, patterning and categorizing within the situation; and, students can create pleasing mathematical products such as designs or explanations or theorems. It can also be body syntonic, that is resonate with body knowledge (Papert, 1980) and thus be inviting in the way that it allows learners to use their knowing in novel situations and connect it with established mathematical ways of knowing.

The six microworlds that will compose the Alive Maths site will embody these principles of RMS while mapping explicitly onto curriculum outcomes as they are stated in the various provincial documents. It is hoped that this will facilitate the integration of these activities into the classrooms of middle school students across Canada. Each microworld will attempt to focus on different strands of the middle school curriculum.

Learning on the Internet

The world wide web is particularly well-suited for communication and sharing[1]. The majority of educational materials currently available make limited use of this functionality. The second goal of the Alive Maths site is to investigate the potential of this functionality in the context of mathematics education. An important component of mathematics learning is communication; indeed, this component has been stressed in current curriculum documents (eg. Ontario Ministry of Education and Training, 1999).

Researchers have found that writing mathematics is a powerful way for students to reflect on their understanding and communicate their mathematical meanings to their peers and teachers (Phillips, 1996). Indeed, encouraging students to write solutions to mathematics problems has been an ongoing goal of the popular mathematics website, The Math Forum (http://www.mathforum.com), where students submit solutions to weekly mathematics problems via email to mentors. Alive Maths would like to take this practice one step further and allow students to write about their mathematical experiences in each microworld on a web page (called My Album). This will allow students to include artifacts of their work (images, applets, etc.) as well as text-based reflections and to review or update their work as a result of further interactions. Moreover, teachers will have access to these web pages and thus an enhanced means of assessing their students' work.

Another motivation behind My Album is to personalise the experiences of students both in their mathematical classrooms and on the web. Creating their own web pages should give them a sense of pride and ownership in their work and perhaps help motivate their interest in and attitude towards mathematics. They will be able to share their web pages with their families and

[1] The internet also has the potential of creating a more level educational playing field, providing affordable access to learning by all Canadians, regardless of geographic and schedule constraints. Furthermore network software such as web browsers obviate the need for installation and maintenance of special-purpose software which is often a major impediment to technology use in schools.

can continue their explorations at home without being concerned about purchasing special software or accessing school intranets.

Teaching with Alive Maths

Teachers are faced with difficult choices when trying to use the web in their mathematics classrooms. They must choose between adapting existing materials for the needs of their students and curricula and creating their own materials. The latter is unfeasible for most. And the former is certainly not facilitated by the design of existing web materials. Alive maths has incorporated a functionality that will enable teachers to create their own paths through the microworlds with minimal amount of work and/or expertise. This will allow teachers to add and/or delete text and select particular sequences of materials for their students. Thus, students will be able to login to the Alive maths site as part of a particular teacher's classroom and engage with preset materials.

It is well known that teachers often have difficulty in shifting their practices in order to mediate the kind of learning opportunities offered by exploratory and interactive environments. In order to assist teachers in their use of Alive Maths, there will be a collection of classroom narratives contributed by teachers that describe possible ways of structuring classroom activities using Alive Maths. One area of research that I will undertake is to determine the extent to which such narratives can support classroom teachers.

Initial Feedback

I have conducted two pilot studies using "Lulu", the first microworld on the Alive Maths website. Lulu is a microworld in which the student can move around on a grid and try to meet with lulu (the green circle in Figure 1), who is moving in response to the student's movements, according to a variety of movement rules. The first study was conducted at a distance where the students were receiving instruction from their regular classroom teacher, while the second was conducted at a local bilingual school in Kingston where I took over the role of teacher for three classroom periods.

First Pilot

During the week of March 13th-17th, the Lulu microworld was tested with two classes of middle school students. The first grade 7 class (n=22) has had more than average experience with internet use in the mathematics classroom and is taught by an internet savvy teacher, Suzanne Alejandre who has done extensive development of web materials, especially through the Math Forum. The second class of grade 8 students (n=17) has had little experience with computers in the mathematics classroom and is taught by the same teacher.

Both groups found the site design and layout easy to navigate and the activity itself easily accessible. This was encouraging as it is often difficult to design sites which include advanced functionalities but do not alienate beginning users. Since many of the SchoolNet potential users are likely to have little on-line experience, this feedback is crucial.

Feedback was solicited from the teacher on two aspects of the Lulu microworld. The first aspect concerns the degree to which the set of activities provided rich mathematical situations for the students in which they felt motivated and encouraged to explore. In particular, the students were asked to describe how they had moved through the set of activities, whether the questions had helped them in their conjectures and explorations, and what they thought of the activities in general. Each student submitted feedback via an online mail form and the teacher provided daily reports on their activities.

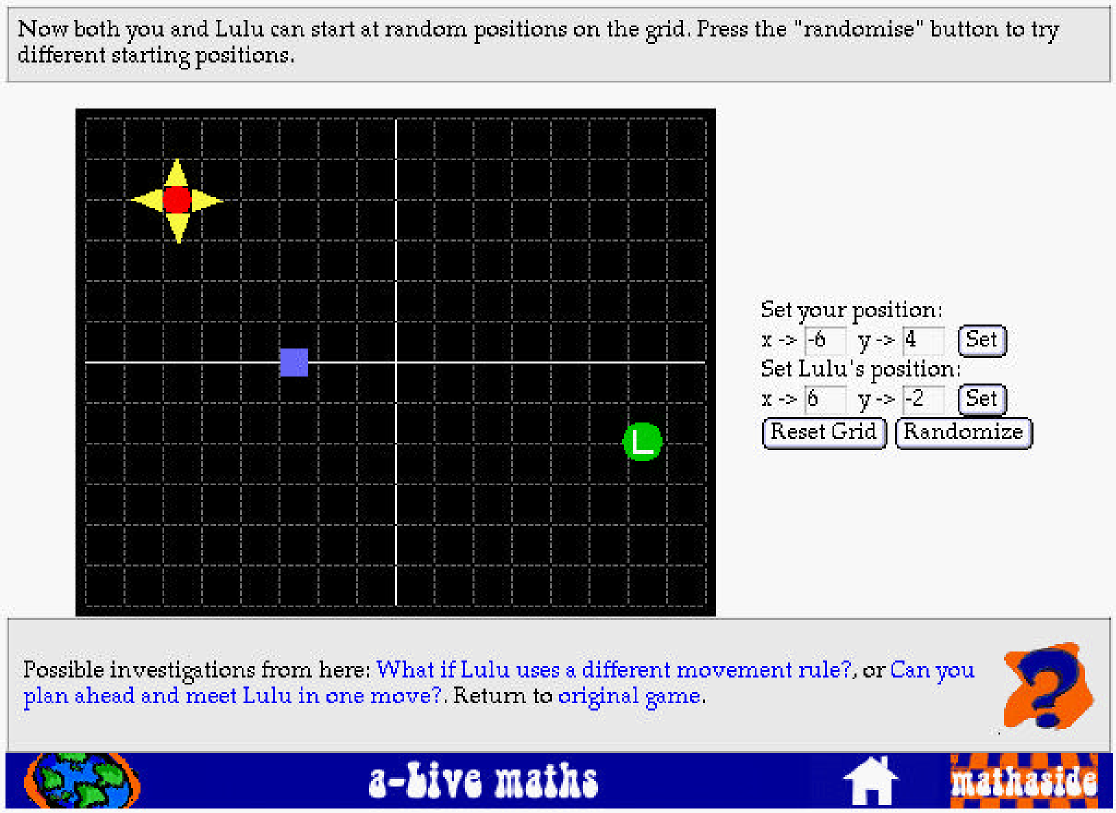

Figure 1: Screenshot of the Lulu microworld.

Both the teacher and the students reported that the Lulu activities were "fun", "cool", a "great way to teach us about coordinate graphing", etc. Many students "wandered" through the different links at their own pace and according to their own interests, saying that they were happy that you didn't have to stick to one way of working through the activities. The teacher commented that the students were very engaged in the activity and that it was so exciting to see them working hard to try to figure out different aspects of the Lulu problems.

The second aspect of feedback concerns the mathematical learning of the students. The questions provided in each activity are designed to motivate certain mathematical thinking - relationships, movement, coordinate graphing and invariance in particular. The Lulu Microworlds is not intended, however, to "teach" these concepts but to provide a relevant setting for them. The teacher reported that the questions did indeed get the students thinking about these concepts in an exploratory and engaging environment and that she was able to draw out the students' mathematical thinking through classroom discussions and additional prepared materials. Moreover, she noted that students were able to construct creative solutions to these questions using their inchoate understandings. This is very encouraging.

The teacher pointed out that the Lulu Microworld activities might not be enough in and of themselves to achieve desired learning outcomes. Indeed, individual teachers will have to also conduct classroom discussions and perhaps prepare additional materials while using these Microworlds. As pre-testing continues, examples of these will be posted in the MathAsides section of Alive Maths to provide support for less experienced teachers.

Second Pilot

The second pilot was conducted with a group of ten grade 8 students attending a bilingual school in Kingston, Ontario. The pilot was spread out over three days, with almost 4 hours of work between May 22nd-24th . The students were described by their classroom teacher as being slightly more advanced in mathematics than their peers.

The students' interaction with the Lulu microworld provides three important insights. The first is that the students easily make the scaffolded transition from intuitive manipulations to mathematically precise and expressive ones. This was demonstrated in two domains of mathematics: (a) the transition from moving in four directions using arrows on a grid to moving on the coordinate plane using coordinate positions and vectors; and (b) the transition from creating movement rules using descriptive language such as "when I move horizontally, Lulu moves vertically, twice as far in the opposite direction" to using algebraic language such as "x = -2y".

The second insight is the way in which the microworld provides students with many opportunities to pose and answer their own problems. For example, on the second day many students began to attend to the trace on the grid left by the two players. They started investigating the relationships between the two traces, investigating questions such as "If I trace a square, what shape will Lulu trace, and how do the two shapes relate?" They also investigated how the movement rules would change the relationships between the two shapes, thus entering the domain of transformational geometry. Other students began creating designs with the two traces, and still others investigated whether the entire grid would be traced using any of the movement rules.

The third finding relates more to affective and aesthetic aspects. The students were very engaged over the course of the three days and were perseverant in their explorations. In addition, they were proud of the work they were doing and expressed enjoyment in a variety of ways as they worked, either as they solved problems or created interesting designs or actions. In particular, the connection to geometry made through the traces was both surprising and compelling. Although the aesthetic aspect is one which is rarely considered in children's mathematics, it would seem that it not only makes mathematical activity more enjoyable and rewarding for children, but it also motivates them to work mathematically, that is, to attend to and discern relationships and patterns and to represent and express them.

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