Using error correction exercises with primary children

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Introduction:

This paper reports on the exploratory phase of a project that is currently underway to investigate the potential of using error correction exercises with children in the primary school. In these exercises the children are given the completed work of another "unknown child". The work of the "unknown child" has been created so that it demonstrates error patterns that can be linked to specific misconceptions and/or common errors. In the exercises the children are asked to:

a. mark the work and establish which examples are incorrect ;
b. suggest why the "unknown child" might have made the errors;
c. suggest ways in which they would help the child overcome the problems they are exhibiting.

The main objectives of the exploratory phase have been to:

- identify aspects of the educational potential of the approach;
- test the viability of the approach in a variety of teaching situations;
- establish issues for further exploration in the subsequent stages of the projects.

The research is purely qualitative in nature and the data reported in this paper is restricted to an early analysis of the comments made by children in response to one of the exercises. At this stage, however, we feel that the nature of the discussion that has been generated by the exercises, which we hope is illustrated by the episodes described below, suggests the approach warrants further investigation.

The paper will begin with a brief summary of the variety of research findings and published opinion that underpin the rationale for the project. I will then go on to describe the research approach in more detail, using the example of one exercise and discuss the main findings up to this point. To conclude the paper there will be a summary of possible implications for teaching and a brief description of the intentions for the next phase of the study.

Background

The project is founded on three major premises about the effective teaching of mathematics, each supported by considerable research evidence.

The first of these is that "*Learning is more effective when common misconceptions are addressed, exposed and discussed in teaching*" (Askew and Wiliam, 1995). The Diagnostic Teaching Project – Nottingham University Shell Centre (Bell 1993) reported improvements in achievement and the long-term retention of mathematical skills as a result of using teaching packages that were designed to elicit and address pupil's misconceptions. The importance of identifying and correcting children's misconceptions has also been given prominence in key official documents that deal with mathematics education in the UK. The report of the Numeracy Task Force (DfEE 1998a), the Framework for Teaching Mathematics (DfEE 1999), and the National Curriculum for Mathematics for initial teacher training (DfEE 1998b) all place the recognition and remediation of pupil misconceptions at the centre of effective practice

The Shell Centre research (ibid.) has suggested that the beneficial effect on learning was greater when children encountered misconceptions through their own work than when teachers drew attention to potential errors in their introductions to topics.

Set against the suggestion that the greater benefit was to be gained through dealing with misconceptions in context we recognised that there are difficulties in using personal errors as a starting point for a discussion about mathematical ideas. Koshy (2000) reports that when primary school children were asked how they felt about making mistakes they expressed strong feelings of anger, frustration and disappointment. We felt that by using the device of the "unknown child" we might be able to escape the emotional backwash that might contaminate attempts to use the children's own errors as the basis for a discussion of misconceptions. Underpinning our thoughts in this area is research that shows a strong correlation between self-esteem and school achievement.[1]

The third and final element of our rationale is based on the connections that have been made between effective learning and children's articulation of their mathematical ideas. E.g. Askew et al. (1997) reported that in one of the schools in their study that was found to be amongst the most effective in terms of children's learning of mathematics there was an expectation that children would explain their methods from the age of 5 years. Nickson (2000) in a meta-analysis of predominant theoretical perspectives in mathematics education reports an emphasis on the social character of mathematical learning in which the classroom is seen as a "mutually constructive situation where pupils learn both from the teacher and their peers" (p.176)

Description of the exercises

All the work in the first phase has been carried out in one primary school and has involved groups of children in years 5 and 6 (ages 9-11). Exercises involving the Year 5 children (9 and 10 year olds) were conducted in groups of 4 who had been withdrawn from the classroom and have been recorded on video tape. Exercises with the year 6 children were carried out with the whole class and were linked to the preparation of those children for the national Standardised Assessment Tests (SATs) in mathematics for children at the end of their primary education.

The work in the school was carried out during the Summer Term. Through consultation with class teachers the mathematical themes covered were chosen to suit the planning priorities and the syllabus for each class. The start of the Summer Term for the year 6 children is dominated by a programme of preparation for the SATs. For this group we targeted some of the items that had been reported as causing the most problems in the previous year's tests (QCA 2000). Items included; the interpretation of calculator displays in the context of money and problems related to the reading of scales. For the year 5 children we selected items that had been part of the syllabus during the previous term and were associated, by the class teacher and by authors on the subject of misconceptions, with a high incidence of errors and misconceptions. The areas covered included:

- Multiplication by 10 and 100 – over-generalisation of the rule of adding a zero and applying the rule to the multiplication of decimals.
- Dividing by 10 and 100.
- Errors in notation when adding 1, 10, 100 and 1 000.
- Ordering of decimals – the idea that e.g. 2.17 is larger than 2.4.
- Rounding numbers to the nearest 10 or 100 – focussing on errors that occur when the nearest 10 is also a complete 100 e.g. 496
- The understanding of the terms area and perimeter.
- The interpretation of remainders within arithmetical word problems.

We wanted to create a structure for the communication of the children's ideas that would serve situations when the children were working independently and provide prompts for a teacher working with smaller groups. The following writing frame[2] is the latest version.

1. Put an example of one question they got wrong in this box.
2. I think they went wrong because…

[1] E.g. As reported in White (1991)

[2] Rawson (1998) reports on a project to investigate the use of writing frames for mathematics with children in primary schools. He suggests that frames, by providing a structure of starting points and connectives allow the children to concentrate on what they wish to communicate.

3. To get this type of question right you need to know about….
4. Can you think of anything that you can use to help this child? Can you show them or tell them something? Can you think of any equipment that might help them?
5. Can you think of a situation in real life where you might use these mathematical skills?

The intentions behind the inclusion of each item were as follows:
Item 1: We wanted to use the writing frame to help children structure a report that could be made during a plenary session to other children in the class who had not been involved in the exercises. The report would therefor begin with the demonstration of one example of the errors that had been made by the "unknown child".
Item 2: Here we would record the children's speculation about the reasons the error had occurred.
Item 3: The purpose of this item was twofold; firstly it provided an opportunity to elicit a further articulation of the reasons for the error in terms of the mathematical knowledge or understanding that was missing. Secondly we intended to investigate the children's awareness of how ideas or skills, drawn from different areas of the subject could contribute to the solution of a mathematical problem.
Item 4: Through this question we wanted to probe children's awareness and understanding of the mathematical models and apparatus that had been used by the teacher in teaching the relevant topic and/or were available in the classroom and/or models that were invented by the individual children.
Item 5: The final item was included to test the extent to which children could make a connection between the mathematics of the classroom and the situations in everyday life in which the skills or knowledge could be applied.
In our construction of the writing frames we became increasingly aware of one of the caveats provided by Rawson (ibid.) when he suggests that the frames can begin to proscribe the children's thinking. He suggests that the frames should be seen as performing a role in an early part of a process that has as its objective a move to greater independence in children's communication of their thinking.

Following one example: Multiplying by 10 and 100
The example shows the "unknown child" over-generalising the rule off adding a zero when multiplying by 10.[3] This particular exercise was completed by two groups, working outside the classroom. (nb. A video record of the two groups was made and it is intended that edited versions of this record would be used during any presentation of this paper.) Each group worked in the same way. They started by working alone to go through the worksheet and identify the errors, after marking the work they compared their findings with others in the group and then entered into a discussion based around the writing frame described above. The first group consisted of two boys and two girls, one 9 years old and the others 10 and considered, by their class teacher, to be of the highest mathematical attainment group. . The second group offered an identical mix of gender and age and contained children who were considered, by their class teacher, to be amongst the lower attainment groups for the class. The comments of each group are recorded separately (see appendix 2), under four headings:
1. those that relate to the children's understanding of the concepts covered in the exercise;

[3] The frequency with which this misconception is encountered might be explained in part by the dilemma faced by the teacher in introducing the topic without being able to introduce examples for which the pattern of adding a 0 will not work. In the Framework for Teaching Mathematics (DfEE 1999) multiplication by 10 and 100 is included in the syllabus for year 3 whilst working with mixed decimal numbers occurs for the first time in year 5.

2. those that provide insight into the children's awareness and understanding of models and apparatus that could be used to support understanding of the concept in question;
3. those that illustrate the children's awareness of how the mathematics covered in the exercise relates to other mathematical topics and to applications outside the classroom;
4. Comments that provide insights into children's perception of the didactical contract[4] in operation in their classroom.

Analysis of the project so far and proposed future developments

1. In analysing the outcomes of the exercises, together with the class teachers, we identified different forms of assessment information including; examples of when children appeared to have a good understanding, where confusion still existed, the use of mathematical language and how the children understood the mathematical models that had been used to scaffold their understanding.

2. It needs to be noted that much of this information was obtained through having the opportunity to view the video recording. In normal classroom situations similar information was still available, however, through examination of the thinking recorded in the writing frames and through hearing children report back to the rest of the class in plenary sessions.

3. In all of the exercises the teachers identified that there were particular ideas that they hoped to hear articulated. The idea of movement of digits to register a change in value was one such "ideal articulation"[5]. There was a feeling that such articulations were not only proof of understanding but also the audible record of that understanding being refined and reinforced.

4. The danger that the notion of "ideal articulations" might lead to children being steered towards particular trains of thought and the corollary that other ideas might be dismissed without proper consideration has been noted.

5. It was felt that whilst children have opportunities to show they can use models and apparatus they seldom are given the chance to describe their understanding of how they work. In effect the opportunity to describe their understanding of e.g. the models of number (appendix 3&4) proved elusive as in most cases the children, quite sensibly, chose to demonstrate how the models could be used by working through an example. More structured questioning might result in an improvement in this area.

6. No comparison of the children's approach to tasks can be offered to gauge how their behaviour in these exercises was affected by the de-personalising of the errors but it can be reported that the children contributed freely to the discussion. It is also interesting to detect that the thought that someone else could make errors that they felt they would avoid themselves generated a certain amount of hubris. A member the 2nd reported sample group, the group that initially exhibited the same error, stated at one point; " *…We've only done this for two days but we know if you go up or down."*

7. In the recorded episodes we had children who exhibited the same misconception as the "unknown child" in that on first view of the work they were unable to identify errors. Reassurances that there were errors (in later exercises we gave the number of errors that we expected them to be able to find) led to closer inspection of the answers and through discussion within the group and in all cases the successful identification of the errors in question.

8. The children were unable to offer many examples of how the mathematical topics covered could relate to applications in real life. The suggestion made by one of the

[4] Douady (1997) suggests that the didactical contract can be viewed as "the implicit or explicit agreement between teachers and pupils about what it means to learn mathematics and what role this learning plays in their relationship…."

[5] It was interesting to note that the children were able to use models (appendix 3 & 4) in which the direction of movement to denote a change in value was both horizontal and vertical.

children in the reported example that knowing about how to multiply by 10 would be useful when you were asked to do worksheets or work from maths textbooks is indicative of the majority view of the main purpose of learning mathematics. It is thought, however, that by asking the question we might hope that the issue is gradually introduced to the agenda for thinking about the mathematics that is being learnt.

9. Our instincts are that these exercises should be used sparingly and are perhaps more naturally located within the assessment part of the teaching cycle.

10. The issues to be investigated further in the next stage of the project will include:

- Using the exercises to prepare children to act as peer tutors;
- Investigating the use of similar exercises with younger age groups;
- Further refinement of the writing frames;
- Investigation of the learning outcomes, for both the audience and the presenters, of plenary presentations of the results of exercises.

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Appendix 1:

Examples used in the "unknown child's" worksheet on multiplying by 10 and 100.
(n.b. copies of the worksheets used will be made available when the paper is delivered)

7 x 10 = 70
4 x 10 = 40
7.9 x 10 = 7.90
4.5 x 10 = 4.50
45 x 10 = 450
67.9 x 10 = 67.90
4 x 100 = 400

39 x 100 = 3900
709 x 100 = 70900
3.8 x 100 = 3.800
9.8 x 100 = 9.800
34.7 x 100 = 34.700

Appendix 2:
N.B. The comments have not been attributed to individual children, as doing so was not thought to serve the intention of this report to provide a flavour of the type of discussion that was generated by the exercises.

Group 1: These children identified the errors immediately and were quick to point out that the errors were consistent in that the "unknown child" had an incorrect answer for all the examples that involved the multiplication of mixed decimal numbers. Their comments included:
1. **Children's understanding of the concepts** *"they just added a 0 and with decimals it's just the same number"* *"they have got to move it up one space"* *"they got the whole numbers right, like 709 x 100 is 70 900 but if it was a decimal they would have just put two zeros."* *" they need to know that when you multiply by 10 each number goes up…for example if you 0.4 x 10 you would put the 4 into the units"*
2. **Children's awareness and understanding of models and apparatus** " *Using this chart (a whole number chart: appendix 3) say you had 4.1 x 10. You find the 4 and go up one row and it makes 40 and then .1 becomes 1 by moving up one row. If it's x 100 just take two jumps"* *" In this (pointing to a place value chart; appendix 4) when you multiply by 10 you go to the next column. Ten thousands become hundred thousands, hundred thousands millions. Say if it was x 1000 it would go up twice."*
3. **Children's awareness of how the mathematics covered in the exercise relates to other mathematical topics and to applications outside the classroom** *"They need to realise that with 0.4 the 4 is tenths"*
4. **Insights about children's perception of the didactical contract**[6] *"Someone has just told them to add a 0 on the end but it just doesn't work. They might have asked someone…but I think you should make sure it's right before they say they'll help you because they are telling the person the wrong answer."*

Group 2 Three members of this group may have exhibited the same error as the "unknown child" in that they were initially unable to identify the errors. They needed to be reassured that there were some errors but after discussion within the group they agreed that "they're getting the point ones wrong". Their comments included;
1. Children's understanding of the concepts *" Normally it's OK (to add a 0 when multiplying by 10) but with decimals you don't just add a 0."* *" S. You said that when add (sic) 10 or 100 you always add a 0 but you don't always."* " *She said add one 0 but like 10 x 10 is 100 so you add two zeros"* *" You don't just add one zero all the time, you add 2 or 3, 4 or 6 or whatever"*

" They need to know if you need to move up or down, we've only done this for two days but we know if you go up or down."
2. Children's awareness and understanding of models and apparatus *" With this board (appendix 3) we move up 1 for 10 times, 100 times moves up 2 and divide goes back down."* *"We have this chart (appendix 3)on the OHP so we can all do it together with Mrs H* *"We have a decimal line (appendix 5) that goes around the classroom" 2.*
3. Children's awareness of how the mathematics covered in the exercise relates to other mathematical topics and to applications outside the classroom *" They need to understand decimals"* *" This (knowing how to multiply by 10) would be useful if you were doing sheets or in a book"*
4. Insights about children's perception of the didactical contract "Maybe they thought that if they just added a zero or two nobody would notice" " Mrs H is a good teacher, we had another teacher and she did different stuff. One day we were doing decimal points, then times, then adding but Mrs H does it one week on fractions so we learn better". " We have maths partners, sometimes we agree and sometimes we disagree but it's much easier than doing it on your own…we can help each other. "We have a top group, a middle group and a bottom group but we don't call them that because Mrs H doesn't like saying people can't do as much as others".

Appendix 3

The whole number chart

9 000 000 ◄	5 000 000	4 000 000	3 000 000	2 000 000	1 000 000
900 000 ←	500 000	400 000	300 000	200 000	100 000
90 000 ←	50 000	40 000	30 000	20 000	10 000
9 000 ←	5 000	4 000	3 000	2 000	1 000
900 ←	500	400	300	200	100
90 ←	50	40	30	20	10
9 ←	5	4	3	2	1
0.9 ←	0.5	0.4	0.3	0.2	0.1
0.09. ←	0.05	0.04	0.03	0.02	0.01

Appendix 4

Place value chart

Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths
6	**4**	**4**	**3**	**2**	**1**	**8**	**0**	**7**

Appendix 5 Decimal number line

0 ↑
0.6
0.7
0.8
0.9
1.0
1.1
1.2 ↓
3.0