Evaluating the levels of problem solving abilities in mathematics

NOOR AZLAN AHMAD ZANZALI
FAKULTI PENDIDIKAN, UNIVERSITI TEKNOLOGI MALAYSIA, 81310 UTM SKUDAI **azlan@fp.utm.my**
LUI LAI NAM
FAKULTI PENDIDIKAN, UNIVERSITI TEKNOLOGI MALAYSIA, 81310 UTM SKUDAI lnlui@tm.net.my

ABSTRACT Currently, there is a general agreement among mathematics educators that students need to acquire problem solving skill, learn to communicate using mathematical knowledge and skills, develop mathematical thinking and reasoning, to see the interconnectedness between mathematics and other disciplines. Based on this perspective, this research looked into the levels of problem solving ability amongst selected Malaysian secondary school students. A sample of 242 Form Four science and non-science students from four schools in an urban district participated in this research. The respondents were asked to solve several mathematical problems. The students' level of abilities in using basic knowledge, standard procedures and problem solving skills were evaluated from their written responses. The evaluation was done based on Polya's problem solving model. Data were gathered through questionnaires and interviews. These data indicated that students have limited exposure to problem solving instruction. Research findings also showed that students have fairly good command of basic knowledge and skills, but did not show the use of problem solving strategies as expected. Generally, these students have a low command on problem solving skills. Most of the students were unable to use correct and suitable mathematical symbols and vocabulary in providing reasons and explanations for certain problem-solving procedures. It is hope that these findings will serve as a reference for educators in improving the learning and teaching of mathematics in general and problem solving instruction in particular.

Background

There is currently a general agreement among mathematics educators that students need learn and often, different mathematics. They stress the importance of students being actively involved in their learning. That is they should "construct, modify and integrate ideas by interacting with the physical world, materials and other children" (Romberg, 1992). Given this background, we argue that the mathematics curriculum should at least include, among others, the opportunities for students to solve problems, learn to communicate using mathematics ideas and symbols, acquire mathematical reasoning, see the connectedness of mathematics {National Council for the Teachers of Mathematics (NCTM), 1989}. Through these, students need to acquire new skills in various relevant mathematical topics (NCTM, 1989).

The research reported here is part of a bigger study that attempts to look at sudents' levels of understanding in several selected topics as related to the four strands mentioned above. The research group realized that this would require a tremendous amount of effort and time. As an initial step, we decided to look into the levels of understanding in problem solving skills, which in Malaysia, are important components of both the primary and the secondary mathematics curriculum (Kementerian Pendidikan, 1989). Further, the emphasis of problem skills in the learning and teaching of mathematics is well documented. This will further followed by studies that will attempt to look at the other three strands.

"Problem" in mathematics has been interpreted in various ways by mathematics educators. There is a general agreement, however, that mathematical problems refers to a situation that requires one to make decisions. The individual does not have an immediately clear or a spontaneous solution to the problem at hand (Polya, 1945; Krulik & Rudnick, 1980; Newell & Simon, 1972; Burns 1992). Problem solving is a process of achieving a solution goal (Polya, 1981). In his book, *How To solve it*, Polya (1945) describes the processes involved, in terms of the heuristics, in problem solving. The four processes involved are understanding of the problem, planning the solution, carrying out the solution and looking back. In this study, the Polya's model of problem solving, modified to suit the research questions generated, is used as a guideline in determining the students' levels of problem solving abilities (see Appendix 2).

Assessment of the levels of problem solving skills

The current literature on problem solving has mostly concentrated on looking at the different kinds of problems, the students' thinking processes in problem solving, the various types of problems, thinking processes, the suitable teaching methods and so on. Previous research, however, did not give particular focus on the levels of problem solving skills attained by students. The research conducted focus particularly on the levels of problem skills attained by the students. Based on the School Achievement Indicators Programme (SAIP) conducted by the Ministry of Education, Canada, a standardized measurement of problem skills levels was used as the basis for measuring or gauging the problem solving skills of students. This standardized indicator states that the levels of problem solving skills can be categorized from levels 1 to 5 (see appendix 2), depending on the complexity of the problem.(SAIP[1], 1997; SAIP[2], 1997). These levels are then linked to the heuristics of problem solving suggested by Polya (1945). In addition to the assessment of problem solving skills this research also look at the difficulties faced by students in attaining the levels.

METHODOLOGY

Research sample

The research sample consisted of 242 students in Form IV (approximately 14 years) of which 118 are from the science stream and 124 from the non-science stream. These students were chosen from 4 schools in an urban area. The schools are ordinary schools and not the premier ones. That is, the students are of average abity group and can thus represent the "average" students in Malaysia. These students are chosen and classified as high, average and low achiever based on their performance in the public examination.

Research instruments

Three main instruments used in this research are shown in the following table:

No,	Type of task	Type of question	No of questions	Assessment aspect
1	Content questions	Structure	6	Level of mastery of concepts and skills
2	Objective questions	Multiple choice	20	Application of strategies in solving problems.
3	Structured questions	Structure	6	Levels of problem solving skills which include understanding the problem, the ability in planning solution, and the ability to carry out the plan.

Types of instruments used

All the questions are constructed based on Polya's Model of Problem solving processes (Polya, 1945). The content of the questions is based on the school syllabus (Kurikulum Bersepadu Sekolah Menengah, KBSM) and several questions chosen from Krulik & Rudnick, (1980); Ohio Department of Education, Columbus, 1980; SAIP[2], 1997). The contexts of the questions were modified so as to be suitable with the Malaysian students.

All the questions were evaluated by the heads of the mathematics department and senior mathematics teachers (more than five years teaching experience) who were willing to participate in this research. A pilot study was also conducted using a set of sample from a class that did not

participate in the actual study. The levels of problem solving skills were ascertained based on the criteria specified by the SAIP[1] (1997). The levels of problems solving skills were categorized in five levels; level 1 being the most simple and level 5 as the most complex.

The students were asked to answer the questions distributed to them. Their answer scripts were checked, following which 16 students were chosen to be interviewed. The aim of the interviews was to gather deeper and information of the solving processes used by the students. All the interviews were audiotaped.

RESEARCH FINDINGS

Performance of content questions

The answers of the content questions were analyzed and showed that 80.35% of the questions were answered correctly by the science students. Among the science students 72 (61.02%) and 49 (41.53%) did not answer question 1c and question 3 correctly respectively. For the other questions less than 21 (17.80%) did not answer correctly.

The performances of the non-science students were found to be low compared to their science counterparts. Overall, 59.09% of the questions were answered correctly. The data indicated that students from the non-science stream attained lower performances in answering questions 1c and 3. Only 8 (6.45%) and 37 (29.84%) of the students managed to answer questions 1c and 3 respectively. 35 (28.23%) and 44 (35.48%) did not attempt questions 5a and 5b respectively.

Performance on the objective tests

Both sets of students (science and non-science) did not perform well in the objective test questions. It was found that 32.80% of the objective questions were answered correctly by the science students while only 20.20% of the objective questions were answered correctly by the non-science students. Majority of the students was inclined to choose the answers given, without indicating the procedures involved, even though there were specifically requested to show how the answers were obtained. Only one student from the science stream managed to answer question 14 correctly, while none of the non-science students manage to answer this question correctly. In addition, it was observed that the questions that were not answered correctly were questions 16 and 18.The most frequently questions answered correctly by both groups of students were questions 1,2 and 20.

Analyses of the questions indicated that the students did not use other solution strategies, except to draw diagrams and tables. The use of manipulatives for questions 4. was very limited, even though there were told that they could use any material as aids. Among the other strategies used by the students, among others, include identifying patterns (in questions 2, 3, 16, and 18), listing in some order (questions 1, 6, and 15), working backwards (questions 10 and 14), listing all possibilities (questions 6 and 15), simplify related problem (questions 5 and 17), and looking a problem from a different perspective (questions 17 and 18). The common approach used by students was to solve problems by using standard calculations that involve the application of certain algorithms and procedures (questions 2, 3, 5, 7, 14, 19 and 20), and counting (questions 1, 3, 6, and 15). A small number of students solve certain problems by using algebraic expressions or equations (questions 5 and 13). Reasoning process was not observed (questions 12 and 14).

Performance of structured questions

48.92% and 28.66% of the structured questions were answered correctly by the science and non-science students respectively. Overall, the performance was rather low for questions that have high levels (level 3 onwards) of difficulty. The number of science students that answered questions 2d, 3d, 4d, 5d and 6f (level difficulty 5) are 2, 2, 5, 1, and 21, respectively. That is less

that 18% of the students could answer them correctly. From the 124 non-science students, the performance for the above five questions is less those 2% or 3 students. The performance of the non-science students for other questions such as 3c, 4c, 5c and 6e (level of difficulty 4) is also low, that is less than 28 (22.58%) had answered the questions correctly.

Overall, the percentage of respondents that were able to answer questions of higher difficulty drops as the level of difficulty increases. For example, 69.83% manage to answer questions of level 2 type, while 3% at for questions at level 5. The findings indicated that the mastery of problem solving among the students up to level 2 is satisfactory (69.8%). The achievement at level 4 and 5 is rather low (16.22% and 3.00% respectively). The percentage of correct reponses is much higher among the science students (including questions of higher difficulty levels) as compared to the correct responses by the non-science students.

The interviews

Among the aspects given particular focus in the interview questions was the ability

1) of students to describe the problems in their own words
2) to determine the data and the information need to be found
3) to plan solutions
4) carry out the solution
5) to use alternative solutions in a flexible manner.
6) the ability to evaluate and look back on the solution.

All the students interviewed could understand and were able to explain the problem at hand. The students of low ability group faces some difficulties in explaining the problems presented to them. This group of students also did not show flexibility in trying out alternative strategies in finding the solutions to the problems if their first attempt failed.

Students from the high ability group showed they are capable of solving problems of higher levels. They are also capable of stating the solution process (algorithm and procedures) clearly. These students could explain accurately and effectively about the conclusion, ideas and basic mathematical reasoning by using the appropriate symbols and terminologies.

Students from the lower achievement group, when interviewed, stated that they do not show their "workings" in solving the problems, because they basically guess when answering the objective questions. This group of students could not state the ideas of mathematics verbally and in written form and using the mathematical terminologies and symbols accurately and effectively.

Students from the science stream indicated that that were not exposed to the problem solving topics, even though there is a specific topic on problem solving in the addition mathematics syllabus. Almost all of the non-science students stated that they were not exposed to almost the entire problem solving strategies, except for drawing diagrams and constructing tables.

Discussion of research findings

Based on the analyses of the data generated the following conclusion can be made:

The Overall performance of the students

The result from the study indicated that students form four science stream students showed better grasp as compared to the non-science students in all the three aspects related to the mathematical content (80.35% compared to 59.09% for the non-science students), the ability to apply problem solving strategies (32.80% compared to 20.20%), and the levels of problem solving abilities (48.92% compared to 28.66%). The differences in performance may be the dueto the students' background and the mathematics curriculum they are studying. Students from

the science stream, overall, attain better performance in mathematics at the PMR[1] (90.68% of the students obtained A grade) as compared to the non-science students (29.03% of the non-science students obtained A grade). The additional mathematics curriculum taught to the science students require higher levels of thinking and abilities as compared to the mathematics curriculum taught to all secondary school students. Further research need to be conducted to identify the variables that can result in differing performances among the two groups.

Mastery of basic facts and concepts (content questions).

Responses obtained from the sample suggest that the science students have a good understanding of the basic facts and concepts in the selected topics, while the mastery level of the non-science is satisfactory. The selected topics include those related to the calculation of volumes of cuboid, number and operations, the conversion of measurement of lengths and mathematical operations involving fractions.

Research findings also showed that the level of mastery does not influence the students achievement in answering questions that attempt to test students in applying problem solving strategies (objective questions) and the ability of the students to plan and carry out the plans (structured questions). The failure to obtain solutions to the given problems is not the result of the lack of understanding in basic skills and understanding of mathematical facts and the ability of students to plan and carry out the plans in problem solving (structured questions). Hence, the ability to understanding the content of the content is important but not a necessary condition in attaining good problem solving skills.

The findings also indicate students who did not answer the questions correctly did not attempt to answer certain questions. Further research need to be conducted to identify the difficult topics and at the same time identify the type of difficulties the students face

Ability to apply problem solving strategies (Objective Questions)

Based on the findings, it can be seen that the sample did not show good mastery of the problem solving strategies, as expected. The most commonly used strategy is the drawing diagram strategy. It is common to see students using the traditional approach, which involves skills that states mathematical expressions, applying the formulae directly and carrying certain operations and standard procedures. The students did not exhibit the ability to analyze facts, reasoning, processing and presenting data in an orderly manner and to make conclusions. A big number of the students used the basic arithmetic skills, that is using counting skills to arrive at the answers for problems 1,3,6, and 15. The more suitable strategies and effective were not being used in the problems solving process. For example, many students solve questions 2 and 14 (both objective questions) by applying ratio concept through certain algorithms and standard procedures, but reasoning strategy is more useful here.

Ability to plan and carry out the plan (structured questions)

The results of this research indicated that science stream students have attained higher level of problem skills than students form the non-science stream. The researcher, however, believes that the ability of students in problem solving of higher difficulty is rather low (16.22% for level 4 and 3.00% level 5) and thus we need to train students so that they attain ability to solve problems of higher levels of difficulty.

In the interviews, the students stated that their exposure to learning and practice of problem solving is limited. They had limited experience to plan and carry out their plans particularly those related to diverging problems that require higher order thinking.

The difficulties that the students faced in expressing mathematical ideas and symbols accurately are the result of lack of experience in answering problems that require students to

[1] PMR is a national examination for the 15- year olds

present their answers in systematic and logical manner using symbols and relevant terminologies accurately. The students stated that, they are not required to present their answers in systematic ways for they are only required to chose the correct answer from choices given.

Implications and suggestions for further research

The achievement of students in problem solving is unsatisfactory particularly those problems at difficulty as in levels 4 and 5. It can thus be concluded that the mastery of basic knowledge and skills is satisfactory among the sample. The performance of the sample in solving problems is below expectations particularly those related to questions that are of higher levels of difficulty as in levels 4 and 5. Thus in can concluded that this research had shown that the mastery of basic mathematics acts is not significantly related to the mastery of problem solving skills. We belief that problem solving abilities must be taught to students through planned teaching, either teaching to solve problems directly or teaching mathematics trough problem solving. Based on the above research, several recommendations can be generated:

a) Teaching and learning mathematics of mathematics should not only emphasize basic knowledge and skills. Students should be exposed and taught problem solving directly through teaching and learning activities in the classrooms.

b) The problem solving topic should be included in the mathematics curriculum (not limited to the additional mathematics) for the non-science students so that this group of students is given the opportunity to learn and thus master the appropriate skills.

c) Based on the information gathered through the interviews, teachers, generally regard the topic on problem solving as not important. Thus, mathematics assessment should reflect the importance of problem solving so that teachers will ultimately teach problem solving and students will attempt to master the related skills. The ability of students to express mathematical ideas effectively can be upgraded if the assessment procedures place great importance on the methods of solutions and problem solving processes. The students, however, should be given sufficient practice sessions to learn and master the ability to present solutions in a logical way, clearly presented using the correct terminologies and accurate symbols.

Conclusion

The research has presented a clear picture on the ability levels of problems solving skills among students. Generally, the levels of content mastery and the skills necessary to carry out certain standard algorithms are satisfactory. The mastery of problem solving skills , however, among the students is still at low. Efforts to upgrade and thus help students to mastery the problem solving skills should be planned and implemented. It is hoped that the data generated by this research can contribute towards the upgrading of teaching and learning mathematics in Malaysia.

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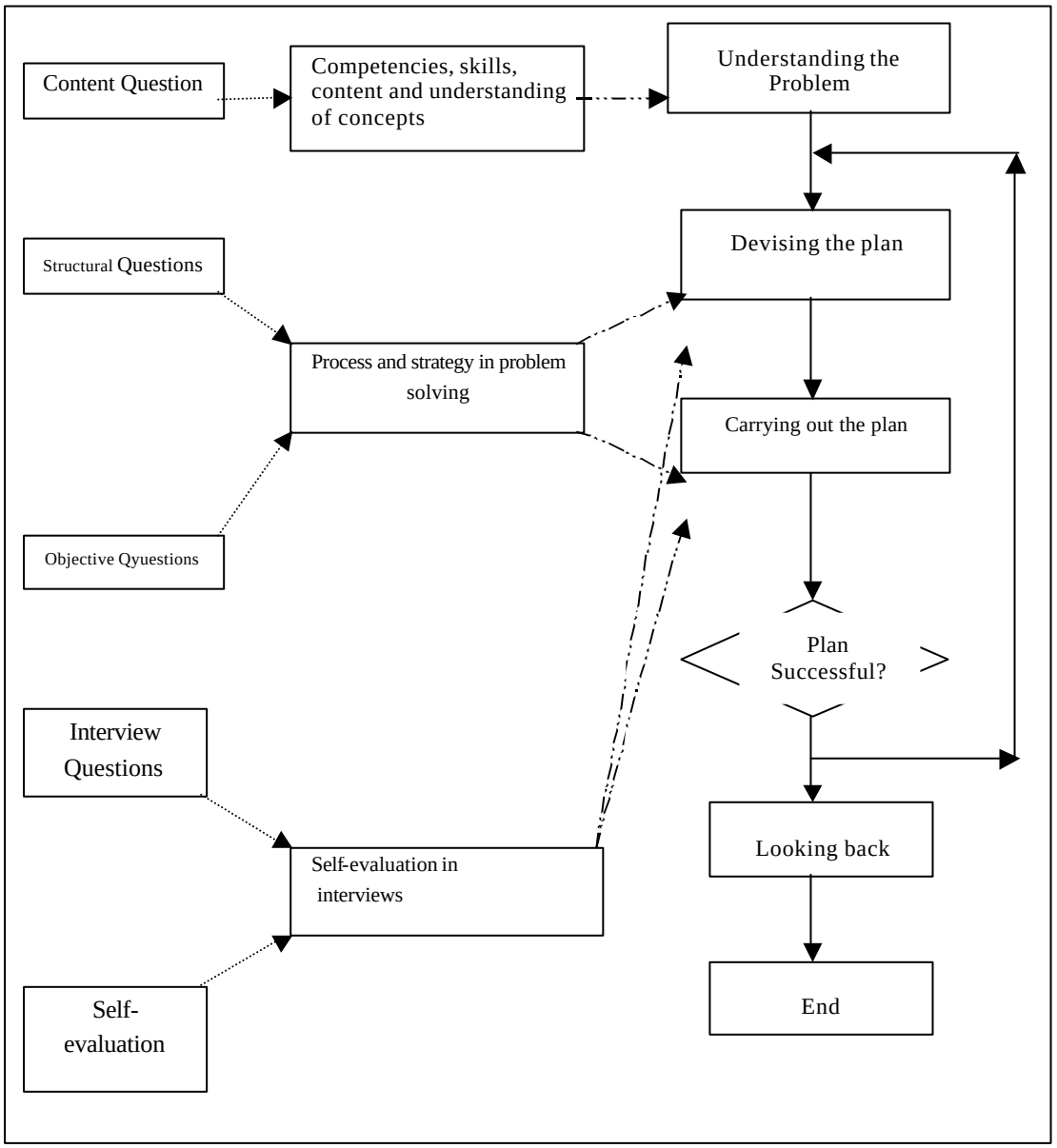
Appendix 1
Determining the levels of problem solving using the Polya's Model



Appendix 2

Summary of Levels for Mathematics Problem-Solving

The expectations of a student performing at each level of the mathematics problem-solving component are:

Level One

Find single solutions to one-step problems using obvious algorithms and a Limited range of whole numbers.

Uses one case to establish a proof.

Level Two

Makes a choice of algorithms to find a solution to :

a) Multi-step problems, using l imited range of whole numbers or

b) One-step problems, using rational numbers.

Uses more than one particular case to establish a proof.

Uses common vocabulary to present solutions.

Level Three

Choose from two algorithms to find a solution to a multi-step problems, Using limited range of rational numbers.

Uses necessary and sufficient cases to establish a proof.

Uses mathematical vocabulary, imprecisely, to present solutions.

Level Four

Adapts one or more algorithms to find a solution to a multi-step problems, Using the full range of rational numbers.

Constructs structured proofs that may lack some details.

Uses mathematical and common vocabulary correctly, but solutions may lack Clarity for the external reader.

Level Five

Creates original algorithms to find solutions to multi-step problems, using The full range of rational numbers.

Constructs structured proofs that provide full justification of each step

Uses mathematical and common vocabulary correctly, and provides clear and precise solutions.

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