EVALUATION OF GEOMETRIC PROBLEM BY APPLYING THE STATISTICAL IMPLICATIVE ANALYSIS

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TITLE

Évaluation d'un problème géométrique Par application de l'analyse statistique implicative

ABSTRACT

The aim of this article was put to determine all the possible strategies that the selected group of students could use in solution one geometry problem. The theoretical framework is put to the theory of didactic situations. The authors defined didactic variables in a – priori analysis as construction levels in solutions. Results as relations among the didactic variables were displayed in graphs such as similarity tree, implicative tree and implicative graph by using software C.H.I.C. The interpretation of the results brings a few interesting suggestions in practice which are oriented in questions such as students' problem how to solve elementary construction tasks not so serious to geometrical knowledge, the choice of solution strategy or finally an ability how to use method circle inversion.

Keywords : *construction task, circle method inversion, geometry, problem, student, strategies, evaluation, variables, theory of didactic situation, C.H.I.C., a-priori analysis*

RÉSUMÉ

Le but de cet article était d'identifier toutes les stratégies possibles que le groupe sélectionné d'étudiants pouvaient utiliser dans la résolution d'un problème de géométrie. Le cadre théorique est basé sur la théorie des situations didactiques. Les auteurs ont défini les variables didactiques dans l'analyse *a priori* des étapes de construction des solutions. Les résultats d'une des relations entre les variables didactiques sont présentés dans les tableaux et les graphiques issus du traitement avec le logiciel CHIC tels que l'arbre de similarité, le graphe implicatif, le graphe cohésitif. L'interprétation des résultats fournit des suggestions intéressantes sur la pratique orientée vers des questions relatives aux problèmes des étudiants face à des tâches de construction élémentaire sans difficulté sérieuse du point de vue des connaissances géométriques, sur le choix des stratégies de résolution ou finalement sur les compétences relatives à l'utilisation de la méthode d'inversion du cercle.

Mots-clés : *problème de construction géométrique, méthode de l'inversion du cercle, stratégies de résolution, théorie des situations didactiques, logiciel CHIC, analyse a priori*

1 Introduction

The modelling through statistic argumentation gives to the research in mathematics education a greater possibility of transferability of the experience. However the statistic

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argumentation would not have some weight, without an accurate theoretical reflection from the viewpoint of the didactic and the epistemology of the mathematical contents. Spagnolo (1998)

Statistical implicative analysis is a data analysis method created by Régis Gras, which has a significant impact on a variety of areas ranging from pedagogical and psychological research to data mining. (Gras, Suzuki, Guillet, Spagnolo, 2007). The implicative statistical analysis aims at giving a statistical meaning to expressions like: "if we observe the variable *a* in a subject of a set *E*, then in general we observe the variable *b* in the same subject". (Gras, Peter, Briand, Philippé, 1997). The implicative analysis is a powerful tool. It allows a clear visualization of relations of similarity and implication among the variables or classes of variables of the situation-problem, through the graphs elaborated by the software C.H.I.C.

C.H.I.C. is a software tool that allows implementation of Statistical Implicative Analysis by offering an effective interface for easy use. Software allows different treatments:

- The building of a hierarchy of similarities according to IC Lerman´s method;
- The building of the implicative graph of variables and of the implicative tree of classes;
- The designation of subjects who contribute the most the paths of the graph or to the classes of the tree;
- The comparison between the implicative graph and an inclusive graph, which modeless at the best the inclusion of classes of subjects to a given threshold. (Gras & Peter 1999).

Statistical Implicative Analysis is a method mainly used by didactics as a profitable and heuristic method of data analysis. The results of this study are organized into two parts based on the method of analysis. The similarity analysis is a classification method, which aims to identify in a set *V* of variables, thicker and thicker partitions of *V*, arranged in an ascending order. (Lerman 1981)

2 Theoretical framework

An analysis of particular didactical problem in educational process from different aspects is a research objective for many didactic schools in the world. One of them is the French didactic school represented by Guy Brousseau, Yves Chevallard and Anna Sierpinska.

Research base of the theory of didactic situation (TDS) issued from this didactic school is analysis of problem in particular levels of didactic situations. Brousseau (1998), Chevallard (1992), Sierpinska, (2001)

The basic notion of this theory is the didactic environment. Brousseau (1990) following the Piaget´s theory the environment is source of contradictions and non-steady states of learner (subject) by process of adaptation (by Brousseau, 1986, it is assimilation and accommodation).

The environment is specific for each of knowledge. Interactions between the subject (student) and the environment form particular levels of didactic situations with corresponding didactic environments. In our work we used the structure of didactic situations and environments by Margolinas (1994). A framework to this approach is based on works Brousseau (1998), Földesiová (2003), Kohanová (2006), Trenčanský et al. (2001, 2003).

3 Methodology of the research

This paper deals with some geometry research which was realized in teaching of subject Geometry 4 in CPU Nitra. The probe was focused to analysis of the students' solutions. The group of 20 math students solved one famous Pappus constructive task. Two different solutions were evaluated by means of TDS.

Further to previous shortly presented theory in the theoretical framework we formulated the following research questions: *Can we determine all the possible strategies that the students could use. Which kind of strategy student have used at most? Did student use only one strategy that has came him to a good result or it was necessary to change strategy, if in the first choice one failed? In an evaluation of results we are also concerned with coherencies of the construction steps limited solutions?*

We remark that the students worked individually, we did not allow them to consult books, notes or any computer software. We know that the count of the tested students is not sufficient to critical and rigorous research and due to we talk only about probe. This count represents the sample of all students in 3-rd class of the math teacher bachelor study.

The students solved this construction problem.

Given two circles k_1 and k_2 with centers O_1 and O_2 and radii r_1 and r_2 have external tangency in a point T. A circle k_3 with center O_3 and radius r_3 is lying outside circles k_1 and k_2 and $r_3<r_1$, $r_3<r_2$. Construct a circle k which tangents the circles k_1, k_2, k_3.

There are three standard approaches there how to correct solve this famous Pappus problem. We are concerned with two ones due to absence of the third solution's variant in students' elaborations. In coming analysis we describe the solution strategy, geometry details of the solutions are left to the readers. [3]

Student's strategy A.

Student fixed a center of circle inversion in the point of the tangency of the given circles and its radius put perpendicular to circle k_3.Under circle inversion student mapped the circles k_1, k_2 into two parallel lines and the circle k_3 left as an invariant. Student constructed a circle which tangents to the lines and simultaneously to the invariant circle. A student could construct also a parallel line to image lines with simultaneous tangent point to invariant circle. Student mapped this circle/line the under circle inversion and obtained one of the six solutions.

[3] Unfortunately, here is no place to present the fundamentals of circle inversion mehod. We note that the reader can find the nature of this method in famous books such Coxeter et all (1967) and Pedoe (1998).

Student's strategy B.

Student constructed concentric circles to the given circles k_1, k_2 with radii $r_1 - r_3$, $r_2 - r_3$ and fixed center of circle inversion in the point O_3. The radius of inversion circle defined perpendicular to one of the concentric circle. Student constructed the images of the circles and consecutively found their common tangents. These four common tangents mapped under inversion into four different circles. The student extracted two of them as concentric circles with the final solutions.

Results were interpreted with help of the statistical software *C.H.I.C.* (Classification Hiérarchique Implicative et Cohésitive)[4] which permits some quantitative analysis of student strategies aimed at clarifying relations among individual variables or entire classes of variables. The didactic variables were defined in a – priori analysis as construction levels and classified by binary value with 0 or 1. Relations among the didactic variables were well displayed in graphs such as similarity tree, implicative tree and implicative graph.

3.1 Didactical variables of problem (a-priori analysis)

The didactic variables are based on the students' strategies described above. We use the dichotomic variables into classes marked with labels *A* and *B*. The strategy *A* preferred 13 students and the strategy *B* was selected in 7 cases.

We show only the main results of our analysis.

TABLE 1 - *The variables to the strategy A*

A0	Student preferred strategy 1.
A1	Student put the center of inversion into a point of tangency of the given circles.
A2	Student constructed the circle of inversion perpendicular to the third given circle.
A3	Student put the radius of the circle of inversion arbitrary.
A4	Student constructed the correct images of the shapes under transformation.
A5	Student solved the elementary construction task – circle inscribed in two parallels and with simultaneous a tangent point to the invariant circle.
A6	Student constructed the image of the circle described above.
A7	Student solved the elementary construction task – parallel line with a tangent point to the invariant circle.
A8	Student constructed the image of the line described above. Student obtained the correct solution.
A9	Student obtained the correct solution.
A10	Student classified the correct count of all solutions.

[4]C.H.I.C (Version 3.1, 2003), Couturier *et al.* (2003)

TABLE 2 - *The variables to the strategy B*

B0	Student preferred strategy 2.
B1	Student put the center of inversion into the center of the third circle.
B2	Student constructed the auxiliary concentric circles.
B3	Student put the circle of inversion perpendicular to the one of auxiliary circle.
B4	Student constructed the correct images of the auxiliary circles under transformation.
B5	Student solved the elementary construction task – constructed the common tangents to circles' images by using the homothety method.
B6	Student mapped four common tangents under inversion into four different circles and extracted two of them as concentric circles with the final solutions.
B7	Student obtained the correct solution.
B8	Student classified the correct count of all solutions.

3.2 Evaluation of students' strategies by C.H.I.C.

The outputs of CHIC are tree types of graphs. Graphs offer comparison of similarity expressed of didactic variables (a-priori analysis), show the relations among cohesion variables.

Graphs also offer percentage of the probability of their implementation and express the probability of realization implications, respectively equivalences among variables.
We note that similarity tree represents the similarities among some arguments in analysis a-priori. Implicative tree represents the implications or the equivalencies among some arguments in analysis a-priori. By evaluation of experiment's results the most significant are the first two levels in the graph, the others are irrelevant. As the author states, in the implicative graph are the results of the experiment significant relationships between variables by 0.85.

These graphs are obtained.

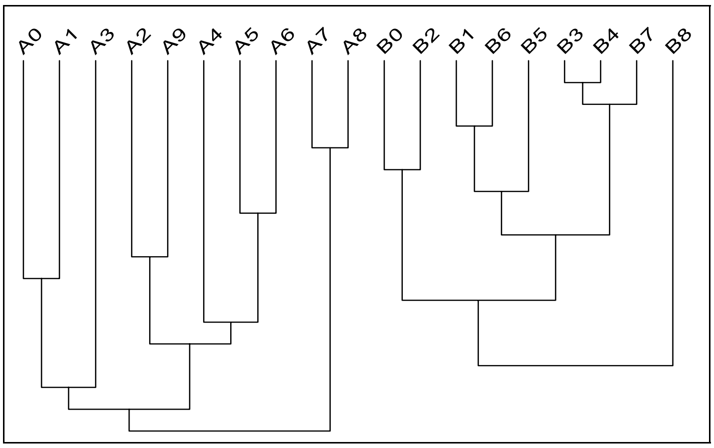

FIGURE 1 – Similarity tree by C.H.I.C.

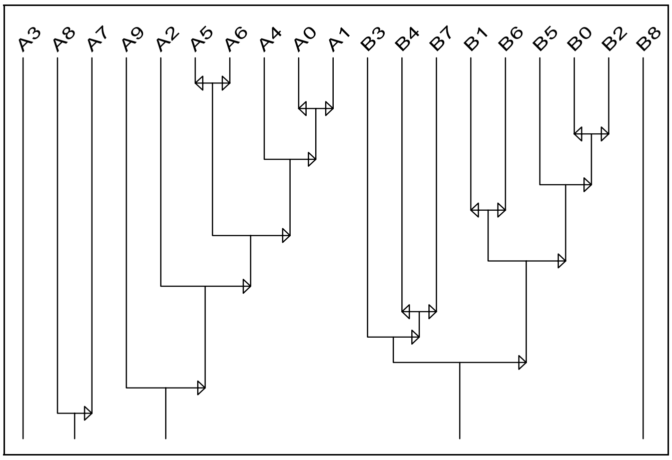

FIGURE 2 – Implicative tree by C.H.I.C.

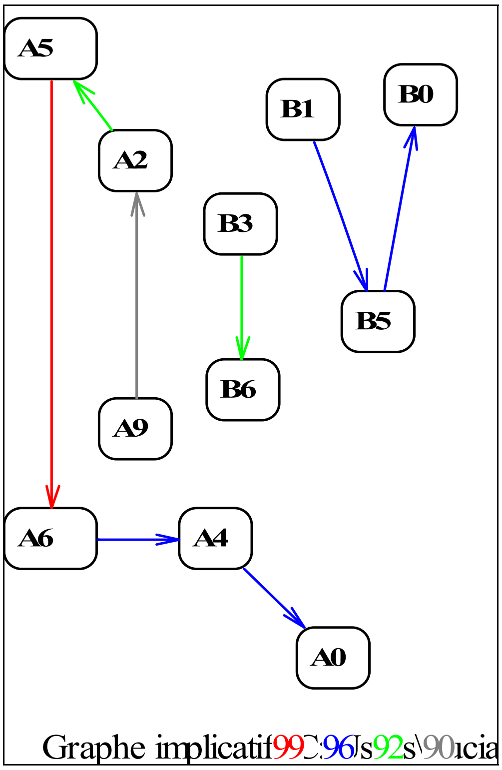

FIGURE 3 – Implicative graph by C.H.I.C.

From the graphs we identify these dependencies and facts:

- the similarity tree displays the strong similarity between variables *B3*, *B4* and at consequently in the second level, also significant level for evaluation, is evident the similarity between class of variables $\{B3, B4\}$ and variable *B7*;
- the implicative tree provides the strongest equivalence between classes of variables $\{A5, A6\}$ and $\{A0, A1\}$. The first mentioned group represents the most important equivalence; the second group has not so stronger mutual equivalence. Other equivalences and implications are irrelevant;
- the implicative graph displays relations among variables with the intensity in the level at least 0.90;

- between the variables *A5* and *A6* is 0.99 - intensity;
- the lower intensity at the quality 0.96 is among variables *A6, A4* and *A0.* The variables *B1, B5, B0* interact at equal intensity;
- intensity among variables *A2, A5* and *B3, B6* is 0.92.

4 Conclusion

Out of analyses of student test solutions we came to the conclusions that students used solely one strategy and nobody used various methods or their combination. If the student chose for one strategy, usually came to the result. These outcomes implied from simple analysis of the tests.

Depending on the strategy we evaluated the continuity of the geometry construction steps by using the software CH.I.C.

The interpretation of the results is following:

- the 0.99 intensity between variables *A5, A6* implies that the students had the scantest problem to solve the elementary construction task in strategy *A* and consequently to obtain the image of this circle. If the students had decided to solve the problem with the second strategy *B* than the situation was analogous. This fact illustrates the 0.96 intensity among variables *B1, B5* and *B0*;
- the importance of the 0.92 intensity among variables *A2, A5* and *B3, B6* indicates that the students has no problem to solve the elementary construction task if the correct construct the perpendicular circle of inversion. The implicative tree presents the strongest equivalence between variables *A5, A6*. The solution of the elementary construction problem was relevant to choice the strategy *A* what is evidently expected dependence. The corresponding variables *B3, B4* and *B7* have no such significant equivalence. This can be explained by non-evident situation what tangent line represent the image of the solution. The similarity graph indicates this conclusion.

These results lead us to conclusions that the main importance by using the circle inversion method is in premeditated analysis about an advantages of the strategies. The possible technical problems with constructions of the images under inversion are irrelevant. The validation of this assertion will be an object in our further research.

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