

Non-verbal communication in thinking about arithmetic problems

Andreas Moutsios-Rentzos & Panagiotis J. Stamatis

Department of Sciences of Pre-School Education and Educational Design,
University of the Aegean, Rhodes, Greece

amoutsiosrentzos@aegean.gr; stamatis@rhodes.aegean.gr

Received 24 October 2012

Abstract. In this study, we investigate the relationships between the students' thinking about mathematics and non-verbal communication, as identified through the students' eye movements. Forty Grade 3 (8-9 years old) students were verbally presented with simple (with respect to the students' expected mathematical ability for this Grade) arithmetic problems. Three categories of mathematical questions were used in ascending level of information load: simple operation questions, word problems with necessary information, and word problems with unnecessary information (information overload). In all three categories, both addition and subtraction questions were included, while the word problems can be classified as problems of 'change'. The findings of this study suggest that during the students' reasoning about the mathematical questions most of their eye movements were right, suggesting the activity of the left hemisphere. Moreover, both the verbal information load and the type of the operation included in the question (addition or subtraction) appeared to affect the students' reasoning. The pedagogical implications of these findings are discussed. **MSC 97C30**

Abstract. In diese Forschungsarbeit wird die Beziehung zwischen der Prozedur des schulischen Gedankens und des nonverbale Kommunikation der Grundschul Kinder, wie die durch den Bewegungen der Augen bestimmt wird. Vierzig Schüler der 3. Klasse (Alter zwischen 8 und 9 Jahre alt) haben in dieser Forschung teilgenommen und beantworteten mündlich in einfachen Fragen der Mathematik, verglichen mit der zu erwarteten Leistung im Mathematik. In der Forschung wurden drei Kategorien mathematischer Aufgaben gestellt die zunehmend mehr Angaben erhielten. Beginnend mit ganz einfachen mathematischen Berechnungen, danach zu einfach formulierte mathematische Aufgaben mit nur den nötigen Angaben erhalten und abschließend zu Aufgaben mit komplexe Formulierung die überflüssigen Angaben erhielten die in eine Reihe von Sätze formuliert waren. In alle 3 Kategorien von „Wechsel“-Probleme, waren Aufgaben die Additionen und Extraktionen erhielten. Die Ergebnisse der Forschung zeigen, dass während der Zeit wo die Schüler beim überlegen sind um die Aufgabe zu lösen, die Augenbewegungen der Schüler orientierten sich nach rechts und links was uns verrät dass der linke Gehirnteil aktiviert ist. Zusätzlich, sowohl die Art der Aufgabenstellung, als auch die Art der mathematischen Berechnung die nötig für die Lösung waren, zeigten, dass die das Denkvermögen der Schüler beeinflussten. Die pädagogische Wichtigkeit dieser Befunde, wird zum Diskussionsthema in dieser Forschung. **MSC 97C30**

1. Introduction

Mathematics education researcher have investigated the students' reasoning with mathematical problems from a variety of perspectives including: the students' understanding of symbolic and word mathematical problems, their solving and proving strategies, and the different representational systems involved (Carpenter, Moser & Romberg, 1982; Christou & Phillipou, 1998; Geary, 1994; Kaput, 1989; Mayer & Hegarty, 1996; Moutsios-Rentzos, 2009; Siegler & Shrager, 1984; Vergnaud, 1982). Moreover, mathematics educa-

tors have realised the importance of non-verbal communication when thinking about mathematics, focussing, amongst others, on the students’ gestures (Radford, 2003) and on their eye-movements (Andrà et al, 2009).

Left hemisphere activity has been found to be important in various cognitive processes, including in logico-mathematical reasoning and problem-solving (Bear, Connors & Paradiso, 2007), time perception, language-related functions (including speech, reading, writing), logical reasoning, processing of acoustic stimuli, and abstract information and others (Gazzaniga, Ivry & Mangun, 2009). Though non-verbal communication is considered to be primarily controlled by the right hemisphere (Joseph, 2011), eye movements have been linked with both left and right hemisphere brain activity (Garrett, 2008; Smith & Kosslyn, 2007). In Greece, the current Unified Interdisciplinary Curriculum Framework –according to which the curriculum of each school course is designed– describes the purpose of school mathematics with cognitive functions that have been linked with left hemisphere activity, thus, indirectly adopting the aforementioned findings from cognitive psychology.

Within this framework, we argue that the study of eye movement during the students’ dealing with mathematical questions may help in gaining deeper understanding of the students’ thinking, as it reveals aspects of brain activity, including cerebral lateralisation (Gluck, Mercado & Myers, 2007). Such knowledge is especially useful for an effective teaching process (Stamatis, 2011), as it allows the teachers to utilise non-verbal communication in the synchronous assessment of their teaching, thus enabling them to offer an appropriately differentiated pedagogy in terms of, for example, task presentation, allowed response time etc. Hence, we posit that the teachers’ awareness of the links between eye movement (and non-verbal communication in general) and mathematical thinking substantially contributes towards the implementation of more effective pedagogical practices. Furthermore, in this research project we build on the preliminary findings reported in Kodakos, Stamatis & Moutsios-Rentzos (2012).

Consequently, in this study, we address the fundamental question: *What is the nature of the relationship between the eye movements of primary school students and their thinking, when they deal with arithmetic (addition and subtraction) problems?*

2. Mathematical thinking and eye movement in arithmetic problems

2.1. Eye movement and mathematical reasoning

The importance of left hemisphere activity in logico-mathematical reasoning and problem-solving has been acknowledged within cognitive psychology (Bear, Connors & Paradiso, 2007; Rayner, 1998), as it has been linked with time perception, language-related functions (including speech, reading, writing), logical reasoning, processing of acoustic stimuli, and abstract information and others (Gazzaniga, Ivry & Mangun, 2009). For example, research conducted with fMRI revealed that the performance of ‘exact arithmetic’ (such as a single arithmetic operation) is linked with the language-based system, whereas ‘approximate arithmetic’ is linked with activity bilateral areas (Dahaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999).

Argyle (1998) notes that in a question and answer situation, the respondent –during the process of thinking about the answer– usually avoids eye-contact with the questioner, whereas the respondents’ looking up and right may indicate intense or deep thinking. Moreover, the respondent usually looks straight to the eyes of the questioner, when reaching to a conclusion and uttering the response (ibid).

Considering mathematical thinking and the students’ eye movements, research has been conducted in a variety of mathematical activities including reasoning in arithmetic word problems (Hegarty, Mayer, & Monk, 1995; Hegarty, Mayer, & Green, 1992) to reading and evaluating mathematical proofs by professional mathematicians (Inglis & Alcock, 2012). These studies investigated the participants’ eye fixations when dealing with a task, drawing upon the eye-mind hypothesis (Just & Carpenter, 1980), which links eye fixation with cognitive processes. This research paradigm allows the researchers to gain deeper understanding about the employment of the mental activities when dealing with a task. For example, in the number bisection task (the identification of the numerical middle of two numbers) Moeller, Fischer, Nuerk, and Willmes (2009) suggest that multiplicativity is activated at early processing stages, whereas parity appears to be involved in later processing stages.

2.2. Arithmetic problems

Numerous research projects have focussed on the students' thinking about arithmetic (symbolic or word) problems (Bebout, 1990; Carpenter, Moser & Romberg, 1982; Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993; Christou & Philippou, 1998; De Corte & Verschaffel, 1993; Fuson & Briars, 1990; Kamii, Lewis & Kirkland, 2001; Riley, Greeno & Heller, 1983; Siegler & Booth, 2004; Vergnaud, 1982). Various factors have been identified as being crucial in the primary students' successful dealing with word arithmetic problems, including the students' ability of executing an operation, their short-term memory, and their familiarity with the mathematical language (Geary, 1994).

Amongst the various differentiations of word addition and subtraction problems, the most widely cited is Greeno's categorisation of *change*, *combine* and *compare* word problems (Riley, Greeno & Heller, 1983). This categorisation is based on the differentiation of the nature of the relationship between the quantities that are added or subtracted. A 'change' problem refers to dynamic situations within which the initial quantity changes, whereas a 'combine' problem describes static relationships between quantities. In a 'compare' problem one quantity is compared with another, taking also into consideration their difference. The following are examples of these three types: '*Nick has 4 pencils. Then George gave him 5 more pencils. How many pencils does Nick have now?*' ('change'); '*Nick and George have 9 pencils altogether. George has 4 pencils. How many pencils does Nick have?*' ('combine'); '*Nick has 5 pencils. George has 3 pencils. How many more pencils does Nick have?*' ('compare').

The basic categorisation scheme can be further elaborated into a 14-category scheme (Riley, Greeno & Heller, 1983) taking into account the identity of the unknown quantity, the direction of change (increase or decrease) and the nature of the comparison (more or less). Carpenter and Moser (1982) suggest an additional category, the *equalise problems*, which refer to dynamically expressed comparative relationship of two quantities; for example, '*There are 3 red balls and 7 blue balls in this box. How many more red balls should I put in the box, so that there will be as many red balls as the blue balls are?*'. Moreover, Vergnaud's (1982) six-category scheme roughly corresponds and expands Greeno's. Nevertheless, it appears that these categories do not sufficiently describe the range of the empirical data of word problems (Fuson, 1992).

2.3. In this study

Following the above, in this study, we investigate the students' thinking about addition and subtraction problems, both symbolic and word problems, as indicated by their eye movements. Furthermore, we focussed on addition and subtraction word 'change' problems. Drawing upon the links between complexity and cerebral activity, we investigated different types of mathematical questions, with respect to their information load. Moreover, we considered the students' thinking in a verbal (rather than a written) setting. Consequently, the main research question of this study can be operationalised in the following questions:

- *What are the students' eye-movements when they listen to, think about and answer a verbally expressed arithmetic problem (addition and subtraction)?*
- *What is the nature of the relationship between eye-movements and the information load of the mathematical question?*
- *Is this relationship affected by the operation (addition or subtraction) required for answering the question?*

3. Methods and procedures

3.1. Sample

In this paper, we consider forty Grade 3 (8-9 years old) students (N=40). The sample characteristics are summarised in Table 1, including the students' gender, their mathematical attainment both as indicated by their teacher and as perceived by the students themselves, as well as the students' positive or negative attitude towards mathematics. We gathered information regarding the participants' handedness, since it is linked with cerebral lateralisation (Haken, 2008). Furthermore, the students' mathematics attainment –both as indicated by the teacher (Smith, Jussim, & Eccles, 1999) and by the students (Mason & Scrivani, 2004)– and their affective disposition (Grootenboer & Hemmings, 2007) towards mathematics have been linked with the participant's performance in dealing with mathematical tasks.

Table 1. The participants of the study.

Mathematical question		Boys	Girls	<i>p</i>	Total
Handedness	Right	18	18	<i>ns</i>	36
	Left	2	2	<i>ns</i>	4
Mathematics attainment (teacher)	Above average	16	16	<i>ns</i>	32
	Below average	4	4	<i>ns</i>	8
Mathematics attainment (perceived)	Above average	18	18	<i>ns</i>	36
	Below average	2	2	<i>ns</i>	4
Mathematics attitude	Positive	16	18	<i>ns</i>	34
	Negative	4	2	<i>ns</i>	6
Total		20	20		40

3.2. The mathematical questions

The presented mathematical questions varied in the amount of information contained in the verbal expression of each task. For each operation (addition and subtraction) we differentiated amongst three types of mathematical questions. The first type, the *simple operation question*, refers to a question enquiring the result of a simple operation (addition and subtraction). These questions are in essence a translation of the symbolic expression of an operation between two numbers to the natural language. For example, a translation from the symbolic expression ‘ $12 + 9 =$ ’ to the question ‘How much is twelve plus nine?’ (see Table 2).

The second type and third type correspond to two variations of word ‘change’ problems: a) *word problems with necessary information*, referring to a word ‘change’ problem that employs natural language including only the necessary for answering the question information, and b) *word problems with unnecessary information (information overload)*, referring to a word ‘change’ problem that employs natural language including more than the necessary for answering the question information.

Recall that we focus on Grade 3 primary school students (8-9 years old) and, therefore, we expected that addition and subtraction problems with small numbers would be within the students’ mathematical abilities, since as suggested by the curriculum. In Table 2, we present the six mathematical questions that the students were asked to verbally answer, along with their characteristics.

Table 2. The mathematical questions included in the study.

Mathematical question	Abbreviated title	Characteristics
“How much is twelve plus nine?”	‘12+9’	Simple operation
“How much is twenty three minus six?”	‘23-6’	Simple operation
“Mary has five pencils. Kostas gave Mary six pencils. How many pencils does Mary have now?”	‘pencils’	Word problem necessary information
“George had twenty five stickers. He gave to Helen twelve stickers. How many stickers does he have?”	‘stickers’	Word problem necessary information
“Anna is a Grade 3 student. She likes painting, while listening to soft music. She uses twenty four markers of a variety	‘markers’	Word question

of colours. Her father gave her a box containing twelve markers. How many markers does Anna have in order to paint more colourful pictures, while listening to soft music?”		information overload
“George usually plays football at a court close to his house. Yesterday, he took fifteen euros with him and went to play with his friends. George stopped at a kiosk to buy soft drinks for the whole team. The soft drinks cost him 7 euros. How many euros did George have left in his pocket, as he was in his way to play football with his friends at the court that is close to his house?”	‘soft drinks’	Word problem information overload

3.3. Procedures

Drawing upon the methodology suggested by Babad (2005), the data collection consisted of three phases, *listening*, *reasoning* and *response*, in accordance with the three phases of the students’ listening, thinking about and answering each question. In the first phase, ‘listening’, each of the six questions was uttered by one researcher of the three-member data collection team in a clear steady voice, medium speed and good enunciation, with the purpose to investigate the students’ reasoning and responses when a task is spoken only once. We focussed on the students’ responses when the task is uttered only once, because, on the one hand, we were interested in the students’ reasoning unaffected by cognitive process that may be related to the repetition of the utterance of a task and, on the other hand, this information is crucial for this study since we focus concentrate on identifying ways that may help the teacher to appropriately differentiate their pedagogy. The second phase, ‘reasoning’, refers to the participants’ thinking about the answer of the task, while the third phase, ‘response’, consists of the participants’ uttering the response.

During these phases, the second member of the data collection team was keeping notes regarding *time information* including: the participants’ *listening time* (the time they were listening to the question), their *reasoning time* (referring to the time that each participant spent to think about the answer), and their *response time* (referring to the time that they spent to utter the answer). The third member of the data collection team focussed on *eye movement information*, referring to the direction of the participants’ eye movements (right, left, up, down and their combinations) during all three phases of each question. Furthermore, both time and eye movement information was noted in a log especially designed for the purposes of this study.

The analysis of the data was quantitative (both descriptive and inferential), conducted with SPSS 17 (SPSS, Inc., Chicago, IL). The collected data required the implementation of non-parametric statistical tests. For the comparisons of two groups the Mann-Whitney *U* test was used (ordinal data) or the Fisher’s exact test (categorical data), while for the identification of change in two or more than two variables we considered respectively the Wilcoxon’s signed rank test or Friedman’s ANOVA (Sheshkin, 2004).

4. Results

First, we investigated whether or not there were any statistically significant differences within our sample. The Mann-Whitney test and the Fisher’s exact test suggested that no such differences were evident in our sample when comparing boys with girls, with respect to handedness, mathematics attainment (both as indicated by the teacher and as perceived by the student) and mathematics attitude (see Table 1). Focussing on the students’ listening, reasoning and responding time, we confirmed that the same listening time for each question was achieved for all the participants. Moreover, the same response time was recorded for all students and for all the questions (about 1 sec). Furthermore, all the students provided us with the correct answer, thus supporting our conjecture that their mathematical ability would be sufficient for their successfully coping with these questions. The Mann-Whitney tests did not reveal any statistically significant differences when comparing the boys’ recorded times with those of the girls in all questions and for all phases.

Following these, it was justifiable to consider the whole sample in our subsequent analysis about the students’ reasoning time. Regarding the operation aspect of the questions, it appears that subtraction was more difficult for the students to calculate either in the form of simple operations or word problems (see mean times in Table 3). Nevertheless, it should be stressed that when comparing the two operations (addition-

subtraction) for each phase of all three question types, statistically significant difference were found only between the simple operation questions ($U = 213, p < 0.01, r = -0.4$) and the word questions containing unnecessary for the answer information ($U = 185, p < 0.001, r = -0.4$), but *not* between the word questions containing only the necessary information ($U = 370, ns, r = -0.1$).

Furthermore, we investigated whether or not the time spent in the reasoning phase differed for each type of tasks. Friedman’s ANOVA revealed that there was a statistically significant increase in the students’ reasoning time as the information load of the question increased ($\chi^2(2) = 51.6, p < 0.001$). We followed these results with Wilcoxon signed rank tests (with Bonferroni corrections applied, thus reporting at a 0.0167 level of significance), revealing a statistically significant increase when comparing the simple operations questions with word problems both with and without necessary information (respectively: $T = 157.5, z = -5.578, p < 0.001, r = -0.5$ and $T = 42, z = -6.427, p < 0.001, r = -0.6$). Nevertheless, the reasoning time increase between word problems with and without necessary information was not found to be statistically significant ($T = 649.5, z = -1.778, ns, r = -0.2$). Similar results were found when the analysis was conducted separately for the addition questions and for the subtraction questions.

Table 3. The participants’ eye movements and mean time during ‘listening’, ‘reasoning’ and ‘response’.

Question		Eye Movements											Time ⁴ (sec)	
		S-A ¹	U	D	L	R	U-D	L-R	U-R	U-L	D-R	D-L		Dom ³
‘12+9’	Lis ²	100%											S-A	2
	Rea	25%	10%	15%		15%	20%	30%	45%	30%	30%	5%	R	12.95
	Res	100%											S-A	1
‘23-6’	Lis	100%											S-A	1
	Rea	25%	10%	15%		15%	20%	30%	45%	30%	30%	5%	R	19.60
	Res	100%											S-A	1.10
‘pencils’	Lis	100%	5%					10%	5%				S-A	5
	Rea	35%	10%	35%	5%	45%	10%	40%	60%	45%	65%	25%	R	36.05
	Res	100%											S-A	1
‘stickers’	Lis	100%	5%					10%	5%				S-A	5
	Rea	40%	15%	40%	5%	55%	15%	45%	60%	40%	65%	20%	R	45.90
	Res	95%						5%					S-A	1.10
‘markers’	Lis	100%		25%		10%	5%	5%	10%	5%	20%		S-A	20
	Rea	35%	20%	45%	15%	45%	35%	35%	50%	40%	60%	35%	R/D-R	44.10
	Res	100%											S-A	1.10
‘soft drinks’	Lis	100%		25%		10%	5%	5%	10%	5%	20%		S-A	20
	Rea	35%	20%	45%	15%	45%	35%	35%	50%	40%	60%	35%	R/D-R	68.45
	Res	90%		5%	5%	5%			10%		5%		S-A	1.35

Note. The percentages refer to the percentage of the participants whose eyes moved towards a direction.

¹: ‘S-A’: Straight-Ahead, ‘U’: Up, ‘D’: Down, ‘L’: Left, ‘R’: Right, ‘U-D’: Up-Down, ‘L-R’: Left-Right, ‘U-R’: Up-Right, ‘D-R’: Down-Right, ‘D-L’: Down-Left.

²: ‘Lis’: listening phase, ‘Rea’: reasoning phase, ‘Res’: response phase.

³: ‘Dom’ refers to the eye-movement direction most frequently identified considering all the participants for each phase of each question.

⁴: This the mean time each participant spent for each phase of each task.

Subsequently, we focussed on spread of the students’ eye movements during the listening, reasoning and response phase for each type of mathematical question. For this purpose, we calculated the percentage of eye movements of each participant for a phase of a question (rather than the percentage of the whole sample looking towards a direction as we do in Table 3). Regarding the listening phase, the students were looking straight at the questioner’s eyes following the rule: the less information contained in a question the more focussed were the students’ eyes towards the questioner. Thus, all the students were looking straight-ahead in the simple operation questions, followed by 91% in the word problem with necessary information and 60% in the word problems with information overload. Finally, considering the response phase all the participants were looking straight at the questioner’s eyes while uttering the response. Moreover, the situation was different when looking into the reasoning phase of the different types of questions. In the simple operation questions, during the reasoning phase of both the ‘12+9’ and the ‘23-6’ questions most of the participants (64%) looked *rightwards* (including right, down-right and up-right). Regarding the word problems containing only necessary information, during the reasoning phase of both the ‘pencils’ and the ‘stickers’ question we identified low spread in the direction of the students’ eye-movements with the majority of the students (61%) looking rightwards (up or down). On the other hand, in the reasoning phase of both the ‘markers’ and the ‘soft drinks’ questions (word problems with information overload), there was a higher distribution in the directions of the participants’ eye movements, though the majority of the recorded eye-movements was rightwards (67%; right or down-right).

We followed these results with a comparison of the students’ eye-movements in the reasoning phase of each question type. We considered in our analysis both ‘right’ and ‘rightwards’ eye movements, focussing on whether or not a participant looked towards a direction when thinking about each question. The results of this analysis are diagrammatically outlined in Figure 1. Succinctly, it appears that the higher the information load of the question the more ‘right’ eye-movements were identified. When considering the broader set of ‘rightwards’ eye-movements it appears that the increase of rightward eye-movements is when comparing simple operation questions with word problems.

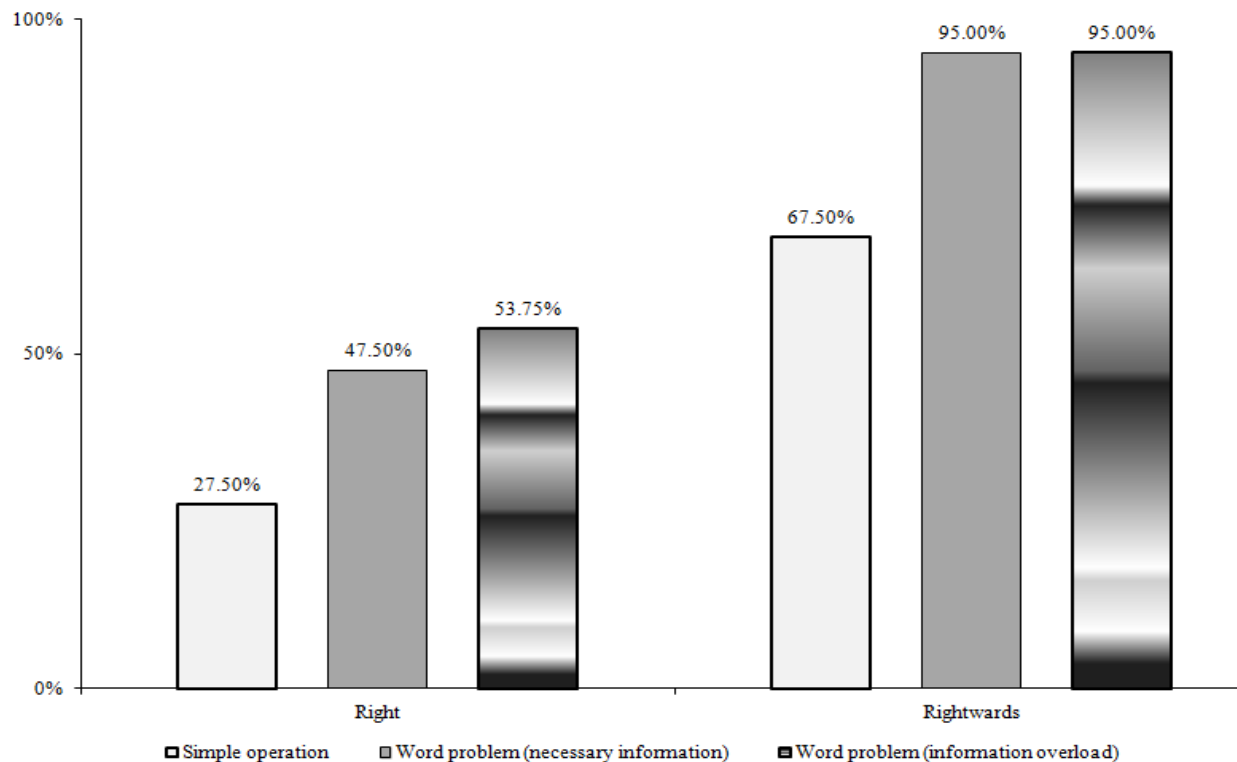


Figure 1. Eye-movements during the reasoning phase of the questions.

Drawing upon the identified link between the students’ reasoning time and the operation in each type of question (addition or subtraction), we investigated the relationship between the type of operation included in each question and the students’ eye movements during the reasoning phase. It appears that the higher time the students spent reasoning about the subtraction questions is accompanied with a higher percentage of ‘right’ eye-movements. This *absolute* difference is observable in all types of questions, but it appears that the *relative* difference decreases as the information load increases: 16.7% for simple operation question, 10% for word problems with necessary information, and 4.5 % for word problems with unnecessary information (with the absolute difference being respectively 5%, 5% and 2.5%). As with our previous analysis, when considering the ‘rightwards’ eye-movements the identified difference is noted only when comparing simple operation questions with word problems. In Figure 2, we diagrammatically summarise these findings.

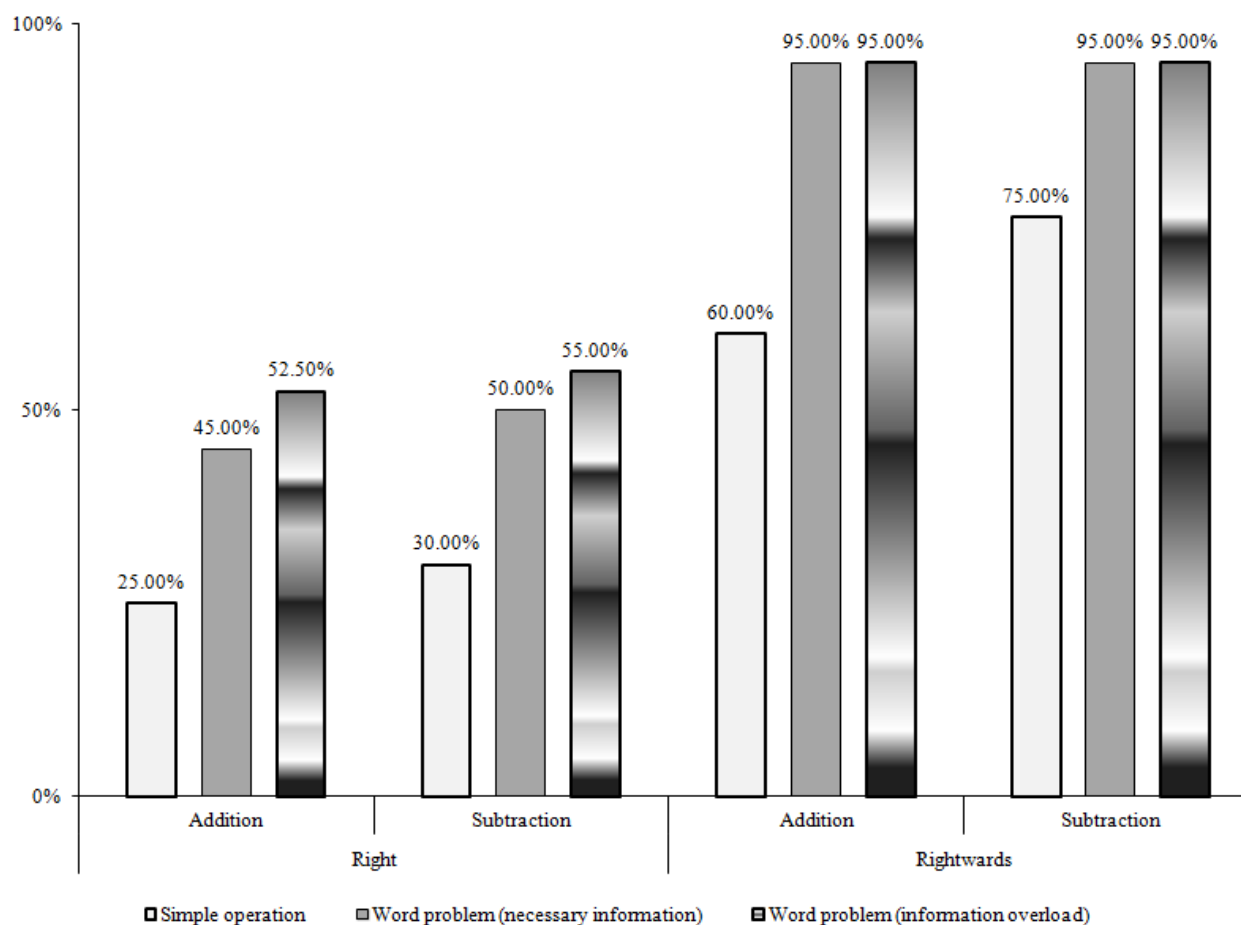


Figure 2. Comparing the eye-movements during the reasoning phase of addition and subtraction questions.

5. Discussion

The aforementioned results shed some light to the research questions posed in this study. With respect to the question about the students’ eye-movements when they listen to, think about and answer a verbally expressed arithmetic problem (addition and subtraction), a look in the findings of this study reveals clear discernible patterns and trends. First, during the listening phase, the participants, in their effort to understand better the task, they concentrate their gaze on the questioner. That is, they gather their attention to the questioner, as he is the one providing the data of the task. In cases with information overload (such as in the ‘markers’ and the ‘soft drinks’ task), as the data increase, the students turn their gaze to various directions, which may be interpreted as an effort to process the information (data and questions), in line with the listen-

ing process. This is in line with findings suggesting that in more complex activities both hemispheres are involved (Haken, 2008).

Regarding the reasoning phase, most of the students' eye-movements were identified as 'right' or 'rightwards' (right, up-right or down-right). Bearing in mind that the vast majority of the participants are right-handed (90%), these findings are in line with existing studies linking the left hemisphere activity with right body activity (Glannon, 2011). Moreover, it appears that reasoning with questions with greater information load is linked with a greater distribution in the directions of the students' eye-movement. This corroborates with the identified links between more complex arithmetic problems and higher left *and* right hemisphere activity (Dehaene et al, 1998).

Moreover, during the response phase, all the participants were looking straight at the questioner's eyes while uttering the response, which is in line with our expectation that in this phase, since the reasoning phase has been completed, the respondent would look straight to the eyes of the questioner (Argyle, 1988).

Considering our investigation about the nature of the relationship between eye-movement and the information load contained in the mathematical question, as well as about the effect addition or subtraction in this relationship, we conducted further analyses taking into consideration both the students' eye-movement and the time they spent in each phase. The initial analyses revealed that the higher the information load that a question contained, the greater variety of eye movements was noted, in line with studies that link the complexity of the information provided in a task with the activity of both hemispheres (Haken, 2008).

Moreover, considering information load and reasoning time, we confirmed the reasonable hypothesis that the complexity of the question would be evident in higher reasoning time. Addition and subtraction with whole numbers, which are expected to be handled with ease by Grade 3 students (according to the curriculum) and their mathematical attainment (as identified by their teacher), the students allocating notably different amounts of time in order to perform them, depending on the information load of the question. Thus, it appears the more complex the question, the more reasoning time the students spent.

Nevertheless, it should be stressed that though this difference is significant when comparing the simple operation question with both of the word problems, the observable time spent difference between the two types of word problems was not found to be significant. We conjecture that though the students' reasoning was affected by the information load, the higher information load of the word problems with unnecessary information was partially counterbalanced by the low level of mathematical difficulty, thus resulting to a notable, yet not statistically significant difference.

Taking into consideration both the question type differentiation (simple operation, word problem with necessary information and word problem with unnecessary information) and the operation differentiation (addition and subtraction) helps us gaining deeper understanding of the students' reasoning about each question. The 'information loaded' questions were linked with more time spent and more 'right' eye-movements', implying a higher left-hemisphere activity. This was further amplified for the subtraction questions within each question type. Therefore, the students appear to think harder in order to cope with a subtraction question than with the addition question of the same type.

Following the above, we conjecture that the complexity of the questions, which in this study is realised as a mixture of information load and operation type (addition or subtraction), may account for these findings. The increased complexity of the mathematical question is positively linked with higher left-hemisphere activity (as indicated by right eye-movement) and higher reasoning time. Nevertheless, considering the low mathematical difficulty of the included questions, it appears that it is the type of question that mainly affects the complexity of the question, with the type of operation regulating, rather than determining, this relationship. Further research should be conducted to investigate the identified relationships in more mathematically challenging arithmetic problems (for example, the same questions could be asked to lower Grade students).

Considering the pedagogical implications of these results, we found that most of the students provided an answer in less than a minute (even for the more complex questions), which asks for the teachers' patience to allow for the students to deal with the question. Bearing in mind that the eye movements towards the right direction suggest the students' reasoning about the question or greater eye-movement distribution is evident in the students' thinking about more complex questions, we argue that the findings of this study provide valuable information that may help the teacher in deciding whether or not a student should be allowed a few

more seconds to answer. Overall, the findings presented in this paper highlight the importance of non-verbal communication and especially eye-movement in teaching mathematics, as it may provide crucial information regarding the students’ thinking processes while listening to or thinking about a question.

6. Concluding remarks

In this paper we discussed the students thinking about arithmetic (addition and subtraction) problems as indicated by their eye-movements. In our investigation, we also considered the time the students spent in thinking about each question. It appears that the more complex mathematical questions (in the sense of the amount of information they contain and, secondarily, the type of operation required to be answered) are linked with higher left-hemisphere and/or bilateral activity (as indicated by the students’ eye-movement) and higher reasoning time. Though research should be conducted to further investigate the identified links, it appears eye-movements can facilitate the teachers to draw more valid conclusions about the students’ thinking during everyday practice, thus facilitating a more effective pedagogy.

References

- Andrà, C., Arzarello, F., Ferrara, F., Holmqvist, K., Lindström, P., Robutti, O., & Sabena, C. (2009). How students read mathematical representations: An eye tracking study. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 49-56). Thessaloniki, Greece: PME.
- Argyle, M. (1988). *Bodily communication*. NY: Methuen & Co.
- Babad, E. (2005). Nonverbal behavior in education. In J. A. Harrigan, R. Rosenthal, & K. R. Scherer (Eds.), *The new handbook of methods in nonverbal behavior research*. NY: Oxford University Press.
- Bear, M. F., Connors, B. W., & Paradiso, M. A. (2007). *Neuroscience: Exploring the Brain*. PA: Lippincott Williams & Wilkins.
- Bebout, H. (1990). Children’s symbolic representation of addition and subtraction word problems. *Journal for Research in Mathematics Education*, 21, 123-131.
- Carpenter, T. P., & Moser, J. M. (1982). The development of addition and subtraction problem solving skills. In T.P. Carpenter, J.M. Moser & T.A. Romberg (Eds.), *Addition and subtraction: A cognitive perspective* (pp. 9–24). Hillsdale, NJ: Erlbaum.
- Carpenter, T. P., Moser, J. M., & Romberg, T. A. (1982). *Addition and subtraction: A cognitive perspective*. Hillsdale NJ: Erlbaum.
- Carpenter, T. P., Ansell, E., Franke, M., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children’s problem-solving processes. *Journal for Research in Mathematics Education*, 24, 427-440.
- Christou, C., & Philippou, G. (1998). The developmental nature of ability to solve one-step word problems. *Journal for Research in Mathematics Education*, 29, 436-442.
- De Corte, E. & Verschaffel, L. (1993). Some factors influencing the solution of addition and subtraction word problems. In K. Durkin & B. Shue (Eds). *Language in mathematics education* (pp. 118-130) Open University Press.
- Dehaene, S., Spelke, E., Pinel, P., Stanescu, R., & Tsivkin, S. (1999) Sources of mathematical thinking: behavioral and brain-imaging evidence. *Science*, 284, 970-974.
- Gluck, K. A. (1999). Eye movements and algebra tutoring. Unpublished doctoral dissertation, Carnegie Mellon University.
- Fuson, K. C. (1992). Research on whole number addition and subtraction. In D.A. Grouws (Ed.), *Handbook of research on teaching and learning* (pp. 243-275). Old Tappan, NJ: Macmillan.
- Fuson, K. C., & Briars, D.J. (1990). Using a base-ten blocks learning/teaching approach for first and second grade placevalue and multidigit addition and subtraction. *Journal for Research in Mathematics Education*, 21, 108-206.
- Garrett, B. L. (2008). *Brain & Behavior: An introduction to biological psychology*. NY: Sage.
- Gazzaniga, M. S., Ivry, R. B., & Mangun, G. R. (2009). *Cognitive neuroscience: The biology of the mind*. NY: W.W. Norton & Company.
- Geary, D. C. (1994). *Children’s Mathematical Development: Research and Practical Applications*. Washington, DC: APA.

- Glannon, W. (2011). *Brain, body, and mind: Neuroethics with a human face*. NY: Oxford University Press.
- Gluck, M. A., Mercado, E., & Myers, C. E. (2007). *Learning and Memory: From brain to behavior*. NY: Worth Publishers.
- Grootenboer, P., & Hemmings, B. (2007). Mathematics performance and the role played by affective and background factors. *Mathematics Education Research Journal*, 19(3), 3-20.
- Haken, H. (2008). *Brain Dynamics. An introduction to models and simulations*. Berlin: Springer.
- Hegarty, M., Mayer, R. E., & Monk, C. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology*, 87, 18-32.
- Hegarty, M., Mayer, R. E., & Green, C. E. (1992). Comprehension of arithmetic word problems: Evidence from students' eye fixations. *Journal of Educational Psychology*, 84, 76-84.
- Inglis, M., & Alcock, L. (2012). Expert and novice approaches to reading mathematical proofs. *Journal for Research in Mathematics Education*, 43, 358-390.
- Joseph, R. (2011). *Right hemisphere, left hemisphere, consciousness & the unconscious, brain and mind*. NY: University Press.
- Just, M. A., & Carpenter, P. A. (1980). A theory of reading: From eye fixations to comprehension. *Psychological Review*, 87(4), 329-354.
- Kamii, C., Lewis, B.A., & Kirkland, L.D. (2001b). Fluency in subtraction compared with addition. *Journal of Mathematical Behavior*, 20, 33-42.
- Kaput, J. J. (1989). Linking representations in the symbol systems of algebra. In S. Wagner, & C. Kieran, *Research issues in the learning and teaching of algebra* (pp. 167-194). Hillsdale, NJ: Erlbaum.
- Kodakos, A., Stamatis, P., & Moutsios-Rentzos, A. (2012). Non-verbal communication in primary school mathematics: a case study focusing on eye movement. *International Journal for Mathematics in Education*, 4, 162-167.
- Mason, L., & Scrivani, L. (2004). Enhancing students' mathematical beliefs: An intervention study. *Learning and Instruction*, 14, 156-176.
- Mayer, R. E., & Hegarty, M. (1996). The process of understanding mathematical problems. In R.J. Sternberg & T. Ben-Zeev (Eds.) *The nature of mathematical thinking*. Hillsdale, NJ: Erlbaum.
- Moeller, K., Fischer, M. H., Nuerk, H. C., & Willmes, K. (2009). Eye fixation behaviour in the number bisection task: Evidence for temporal specificity. *Acta Psychologica*, 131, 209-220.
- Moutsios-Rentzos, A. (2009). *Styles and strategies in exam-type questions*. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 145-152). Thessaloniki, Greece: PME.
- Radford, L. (2003). Gestures, Speech, and the Sprouting of Signs: A Semiotic-Cultural Approach to Students' Types of Generalization. *Mathematical Thinking and Learning*, 5(1), 37-70.
- Rayner, K. (1998). Eye movements in reading and information processing: 20 years of research. *Psychological Bulletin*, 124, 372-422.
- Riley, M. S., Greeno, J. G., & Heller, J. I. (1983). Development of children's problem solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153-196). New York: Academic Press.
- Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, 75, 428-444.
- Siegler, R. S., & Shrager, J. (1984). Strategy choices in addition and subtraction: How do children know what to do? In C. Sophian (Ed.), *The origins of cognitive skills* (pp. 229-293).
- Sheshkin, D. J. (2004). *Handbook of Parametric and Nonparametric Statistical Procedures* (3rd edition). Boca Raton, FL: Chapman & Hall/CRC.
- Smith, E. E., & Kosslyn, S. M. (2007). *Cognitive Psychology: Mind and Brain*. NJ: Prentice Hall.
- Smith, A. E., Jussin, L., & Eccles, J. (1999). Do self-fulfilling prophecies accumulate, dissipate, or remain stable over time? *Journal of Personality and Social Psychology*, 77, 548-565.
- Stamatis, P. J. (2011). Non verbal communication in classroom interactions: A pedagogical perspective of touch. *Electronic journal of research in educational psychology*, 9(3), 1427-1442. Retrieved January 29, 2012 from <http://investigacion-psicopedagogica.org/revista/new/english/Conta+dor+Articulo.php?617>.

Vergnaud, G. (1982). A classification of cognitive tasks and operations of thought involved in addition and subtraction problems. In T. P. Carpenter, J. M. Moser and T. P. Romberg (Eds.), *Addition and Subtraction. A cognitive perspective*. Hillsdale NJ: Erlbaum.