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Developing linguistic competence for mathematics learning: online bridging courses at the beginning of university

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Abstract. Il contributo è legato alla normativa emanata per il Sistema Universitario Italiano negli ultimi anni, volta a migliorare la qualità dell'insegnamento senza disporre di tutte le risorse umane e materiali necessarie. Le università sono state fortemente sollecitate a rispettare le raccomandazioni europee sulla formazione permanente e in particolare sulla costruzione di competenza. Qui si cerca di sostenere che l'idea di competenza, al di là delle distorsioni e delle interpretazioni riduttive delle burocrazie universitarie, rappresenta ancora un passo avanti verso il miglioramento della qualità dell'insegnamento, rispetto alla programmazione basata solo sui contenuti. Vengono presentati alcuni item del test iniziale di verifica delle competenze del Dipartimento di Scienze e Innovazione Tecnologica dell'Università del Piemonte Orientale. Infine, vengono delineate le caratteristiche di un corso di piattaforma realizzato successivamente con l'obiettivo di aiutare gli studenti del primo anno del corso di Scienze Biologiche dello stesso dipartimento a superare alcune difficoltà in matematica, in particolare quelle relative alle competenze trasversali come ad esempio quelle linguistiche.

1. Introduction

The introduction of the University Quality Assurance System (https://www.anvur.it/attivita/ava/riferimenti-normativi/) and the related regulations that require greater transparency and clarification in the description of university courses, apart from the usual narrow interpretations by bureaucracy, are an opportunity to improve the quality of university teaching and also to enhance the contribution of experts in disciplinary education. The traditional 'programme' consisted in a list of contents which was normally poorly informative due to the variety of interpretations, is replaced by a more complex description, named 'syllabus', which includes, among other things, the list of expected learning outcomes, contents and learning objectives, the prerequisites and the explanation of the methods for verifying learning. Beyond the negative aspects, such as the need to fill complex forms and the poor efficiency of the IT support tools, the new guidelines have contributed to shifting attention from content to competence. Some colleagues, fond of the content-oriented teaching, resist the change and sometimes interpret the idea of competence in a restricted way, associating it with a list of contents equipped with some examples of exam tests. My university has developed a training program for its teachers on issues related to the Quality Achievement System, namely the competence-oriented teaching and the use of the platform as a tool to achieve it.

2. Competence

According to the Recommendation of the European Parliament and of the Council of 23 April 2008 'competence' is defined as the proven ability to use knowledge, skills and personal, social and/or methodological abilities, in work or study situations and in professional and personal development. In the context of the European Qualifications Framework, competence is described in terms of responsibility and autonomy.

There are different definitions of competence in literature, but all refer, in a more or less explicit way, in addition to the knowledge and skills also to the attitudes and metacognitive aspects that can emerge in the performance of a task in a given context.

Competence therefore manifests itself as knowing how to act in specific situations and for this reason it cannot be reduced to knowledge and skills only, but is the result of an amalgamation of aspects related to motivations, attitudes, social role, self-image, awareness, sensitivity to the context, commitment, etc.

Pellerey (2004) defined competence as "ability to cope with a task, or a set of tasks, managing to set in motion and orchestrate one's internal, cognitive, affective and strong-willed resources, and to use the external resources available in a coherent and fruitful way."

The Council on 22 May 2018 has adopted a Recommendation on Key Competences for Lifelong Learning based on a Commission proposal. For the purposes of this Recommendation, competences are defined as a combination of knowledge, skills and attitudes, where:

- knowledge is composed of the facts and figures, concepts, ideas and theories which are already established and support the understanding of a certain area or subject;
- skills are defined as the ability and capacity to carry out processes and use the existing knowledge to achieve results;
- attitudes describe the disposition and mind-sets to act or react to ideas, persons or situations.

Key competences are those which all individuals need for personal fulfilment and development, employability, social inclusion, sustainable lifestyle, successful life in peaceful societies, health-conscious life management and active citizenship. They are developed in a lifelong learning perspective, from early childhood throughout adult life, and through formal, non-formal and informal learning in all contexts, including family, school, workplace, neighbourhood and other communities.

The Reference Framework sets out eight key competences:

- Literacy competence,
- Multilingual competence,
- Mathematical competence and competence in science, technology and engineering,
- Digital competence,
- Personal, social and learning to learn competence,
- Citizenship competence,
- Entrepreneurship competence,
- Cultural awareness and expression competence.

We report the detailed description of the first three key competences, as the most directly involved in teaching mathematical competence. They are useful to understand the extent to which content-oriented education is inadequate to the needs of society.

Literacy is the ability to identify, understand, express, create, and interpret concepts, feelings, facts and opinions in both oral and written forms, using visual, sound/audio and digital materials across disciplines and contexts. It implies the ability to communicate and connect effectively with others, in an appropriate and creative way. Development of literacy forms the basis for further learning and further linguistic interaction. Depending on the context, literacy competence can be developed in the mother tongue, the language of schooling and/or the official language in a country or region. Essential knowledge, skills and attitudes related to this competence This competence involves the knowledge of reading and writing and a sound understanding of written information and thus requires an individual to have knowledge of vocabulary, functional grammar and the functions of language. It includes an awareness of the main types of verbal interaction, a range of literary and non-literary texts, and the main features of different styles and registers of language. Individuals should have the skills to communicate both orally and in writing in a variety of situations and to monitor and adapt their own communication to the requirements of the situation. This competence also includes the abilities to distinguish and use different types of sources, to search for, collect

and process information, to use aids, and to formulate and express one's oral and written arguments in a convincing way appropriate to the context. It encompasses critical thinking and ability to assess and work with information. A positive attitude towards literacy involves a disposition to critical and constructive dialogue, an appreciation of aesthetic qualities and an interest in interaction with others. This implies an awareness of the impact of language on others and a need to understand and use language in a positive and socially responsible manner.

According to the Council recommendation of 22 May 2018, multilingual competence defines the ability to use different languages appropriately and effectively for communication. It broadly shares the main skill dimensions of literacy: it is based on the ability to understand, express and interpret concepts, thoughts, feelings, facts and opinions in both oral and written form (listening, speaking, reading and writing) in an appropriate range of societal and cultural contexts according to one's wants or needs. Languages competences integrate a historical dimension and intercultural competences. It relies on the ability to mediate between different languages and media, as outlined in the Common European Framework of Reference. As appropriate, it can include maintaining and further developing mother tongue competences, as well as the acquisition of a country's official language(s).

In the same recommendation, mathematical competence is defined as the ability to develop and apply mathematical thinking and insight in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought and presentation (formulas, models, constructs, graphs, charts).

The adoption of the construct of competence is not an easy enterprise in Italian universities. It is known that most courses require an assessment of initial preparation of the students, in order to help them in choosing an appropriate course. The CISIA (consortium for integrated access systems) provides the test to most of Italian universities. The general description of their test (https://www.cisiaonline.it/area-tematica-tolc-cisia/cose-il-tolc/) and all of their syllabi (for example, https://www.cisiaonline.it/area-tematica-tolc-scienze/struttura-della-prova-e-syllabus/) are focused on knowledge rather than competence. So we had to work hard in order to explain the idea of competence to our colleagues and to convince them that it was not a useless addition to the list of contents. It is also true that the lack of competence sometimes may depend on the lack of knowledge, so we cannot totally neglect knowledge. Moreover, we had to show that linguistic competence is a basic starting point to scientific education. For linguistic competence there are different interpretations too. If you do a search online, at the top of the list you are likely to find definitions related to Noam Chomsky's approach, in which linguistic competence is opposed to performance. For example, a definition that appears frequently reads: "Linguistic competence is the system of linguistic knowledge possessed by native speakers of a language. It is distinguished from linguistic performance, which is the way a language system is used in communication." Of course, Chomsky's definition of linguistic competence is inconsistent with the definition discussed above or with the interpretation of Italian National Guidelines for the curriculum of kindergarten and the first cycle of education (http://www.indicazioninazionali.it/wp-content/uploads/2018/08/Indicazioni_Annali_Definitivo.pdf).

3. Methodology

Strictly speaking, this is not an empirical study. The aim of this paper is to show how the university platform DIR, which is a version of Moodle, can be used to assess and promote students' linguistic and content-related competence. The data on the percentages of correct answers for a selection of items are aimed at showing that difficulties cannot be ascribed to lack of knowledge only, but heavily involve competences (which include beliefs and attitudes too). Students' attitudes seem closely related to their difficulties. This has prevented us from designing longitudinal studies, as students' attendance is highly variable and only the most diligent ones are available to engage in any kind of extra activity. Most students even refuse to be interviewed in order to explain their written answers. Anyway, I think it is my duty to teach (and to struggle to improve my teaching) to all students, not just the most diligent or the very few available for extra activities.

In our university the regular Verification Test of Initial Competence (VTIC) was administered in two sessions, at the end of September and at the end of October. The participants were distributed in two consecutive days, according to the town where they take their courses. Recovery sessions were kept in January and February and, if necessary, later. The attendants of the regular sessions were from 300 to 400 for each day, whereas the recovery sessions had a lower attendance. Failure in the test would not prevent students from enrolling but from taking exams only. According to the guidelines of my department, the test must focus on the basic linguistic competence required to productively attend the classes. The test includes 20 items, of which 10 are focused on general scientific language and 10 to the specific representation systems of mathematics. The score required to pass the test is 50,01%. The test was followed by a recovery course reserved for people not passing or not attending the test. An online bridging course has been planned (beyond the recovery one) with the purpose of provide some help to the freshman Biology students, most of whom were known to meet with difficulties in mathematics. We proposed both routine, calculation problems and more critical questions. The meetings between students and tutors were planned both in presence and online.

4. Some examples of items

4.1 Testing the use of scientific language

We show some example of items related to general scientific language.

The following kind of item (figure 1) consists in a piece of text, concisely describing life and main achievements of the great chemist Amedeo Avogadro, and some question requiring the understanding of its basic meanings.

Nacque in una famiglia di antica nobiltà piemontese, da Anna Vercellone di Biella e da Filippo Avogadro, conte di Quaregna e di Cerreto, il quale fu Senatore del Regno di Sardegna e alto magistrato.

Amedeo fu uno studente brillante; si laureò in diritto canonico a vent'anni, nel 1796, e iniziò a praticare. Ben presto, però, si dedicò allo studio della fisica e della matematica, le sue scienze preferite, e nel 1809 cominciò a insegnarle al collegio di Vercelli (dove la sua famiglia aveva dei possedimenti).

Durante la sua permanenza a Vercelli scrisse una memoria nella quale formulò un'ipotesi che viene oggi chiamata Legge di Avogadro e che si è abituati ad esprimere nella forma:

« volumi uguali di gas diversi, alla stessa temperatura e pressione, contengono lo stesso numero di molecole »

La Legge di Avogadro implica che le relazioni tra i pesi di volumi identici di gas differenti (a parità di condizioni di temperatura e pressione), corrispondono alle relazioni tra i rispettivi pesi molecolari. Quindi, i pesi molecolari relativi possono essere calcolati dal peso dei gas.

Avogadro sviluppò questa ipotesi dopo che Joseph Louis Gay-Lussac aveva pubblicato la sua legge sui volumi (e i gas combinati) nel 1808 (la Prima legge di Gay-Lussac). La principale difficoltà che Avogadro dovette risolvere fu la grande confusione che regnava al tempo su atomi e molecole: uno dei più importanti contributi del lavoro di Avogadro fu quello di distinguere gli uni dalle altre, ammettendo che anche particelle semplici potessero essere composte da molecole, e che queste ultime fossero composte da atomi. John Dalton, ad esempio, non considerava questa possibilità. Avogadro, in realtà, non usò la parola "atomo", in quanto i termini "atomo" e "molecola" erano usati in maniera quasi indistinta. Egli considerava l'esistenza di tre tipi di "molecole", comprese le "molecole elementari" (i nostri "atomi"). Oltre a ciò, diede una particolare attenzione alla definizione di massa, come distinta dal peso.

Figure 1. An example of text to be interpreted

I do not believe that an English translation may be much helpful in order to understand the difficulties of Italian students in interpreting Italian texts. Anyway, to give a rough idea of the kind of texts presented the English translation of the content of figure 1 might be of some use:

He was born into a family of ancient Piedmont nobility, from Anna Vercellone of Biella and Filippo Avogadro, count of Quaregna and Cerreto, who was Senator of the Kingdom of Sardinia and a high magistrate. Amedeo was a brilliant student; he graduated in canon law at the age of twenty, in 1796, and began to practice. However, he soon devoted himself to the study of physics and mathematics, his favourite sciences, and in 1809 he began teaching them at the college of Vercelli (where his family had possessions). During his stay in Vercelli he wrote a memoir in which he formulated a hypothesis which is now called Avogadro's Law and which is used to express in the form: "Equal volumes of different gases, at the same temperature and pressure, contain the same number of molecules". Avogadro's Law implies that the relationships between the weights of identical volumes of different gases (under the same conditions of temperature and pressure) correspond to the relationships between the respective molecular weights. Hence, the relative molecular weights can be calculated from the weight of the gases. Avogadro developed this hypothesis after Joseph Louis Gay-Lussac published his law on volumes (and combined gases) in 1808 (the First Gay-Lussac Law). The main difficulty that Avogadro had to solve was the great confusion that reigned at the time about atoms and molecules: one of the most important contributions of Avogadro's work was to distinguish one from the other, admitting that even simple particles could be composed of molecules, and that the latter were composed of atoms. John Dalton, for example, did not consider this possibility. In reality, Avogadro did not use the word "atom", as the terms "atom" and "molecule" were used almost indistinctly. He considered the existence of three types of "molecules", including "elementary molecules" (our "atoms"). In addition to this, he paid particular attention to the definition of mass, as distinct from weight.

Questions like the following (figure 2) require nothing more than an accurate reading of the excerpt.

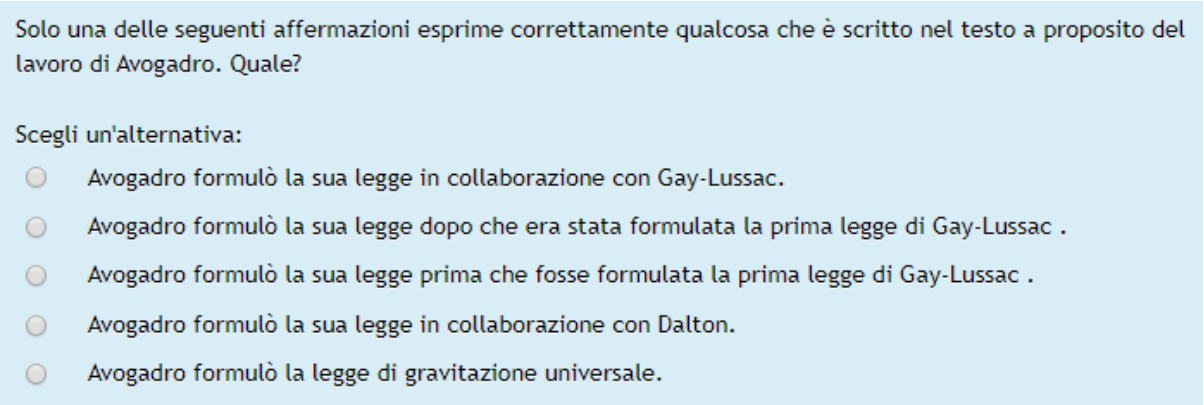


Figure 2. An example of multiple-choice question related to the text in figure 1.

Here an English translation of the question in figure 2:

Only one of the following statements correctly expresses something that is written in the text about Avogadro's work. Which?

- *Avogadro formulated his law in collaboration with Gay-Lussac.*
- *Avogadro formulated his law after the first Gay-Lussac law was formulated.*
- *Avogadro formulò la sua legge prima che fosse formulata la prima legge di Gay-Lussac.*
- *Avogadro formulated his law in collaboration with Dalton.*
- *Avogadro formulated the law of universal gravitation.*

The average number of correct answers for items of this kind varies from 55% to 75%.

Items like the following (figure 3) require some basic knowledge of the use of connectives. The average number of correct answers for items of this kind varies from 80% to 98%.

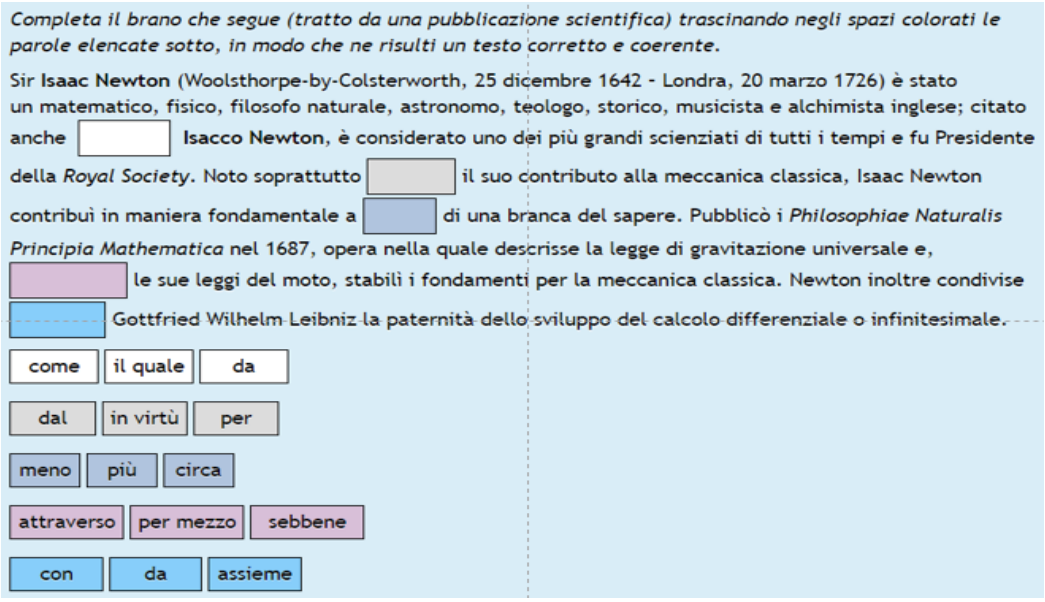
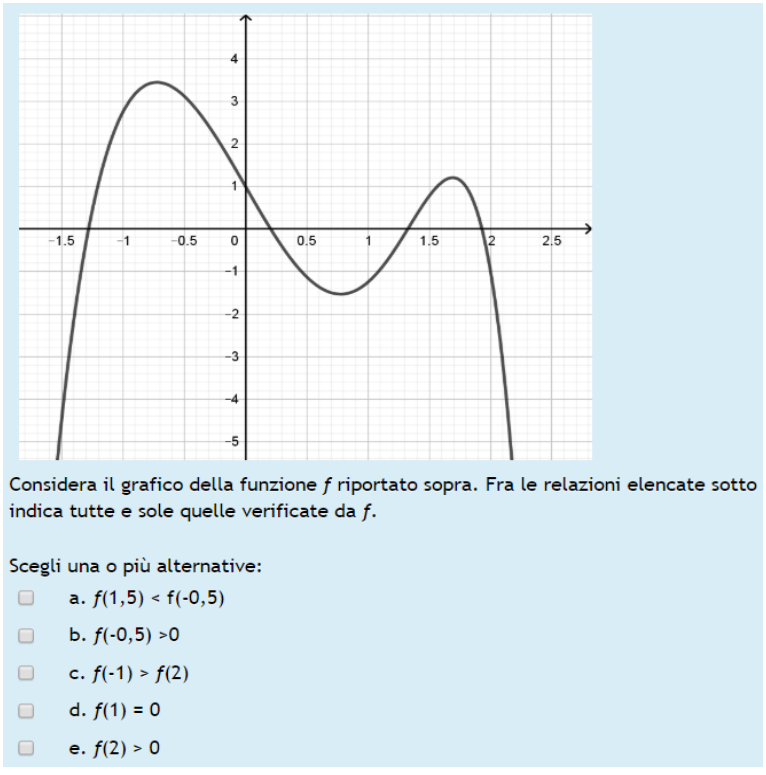


Figure 3. A drop-in-text item focused on connectives.

4.2 Testing the use of mathematical notations

Here are some examples are shown of items involving the representations of mathematics. In all of these examples I have added an English translation.



English translation:

Consider the graph above. Among the relations listed below mark all and only those satisfied by f.

Choose one or more alternatives:

...

Figure 4. An item focused on the plain reading of a graph.

The average number of correct answers for items of this kind (figure 4) varies from 40% to 60%.
Next item (figure 5) is one of the most critical.

Nel diagramma è rappresentato (in rosso) il punto *A* di coordinate *(x; y)*. Trascina l'etichetta B (che si trova in basso) in modo che corrisponda (col suo circolino in altro a sinistra) al punto di coordinate $(2x ; -y)$.

English translation:

In the diagram the point A of coordinates (x; y) is represented (in red). Drag the label B (located at the bottom) so that it corresponds (with its circle at the top left) to the point of coordinates (2x; –y).

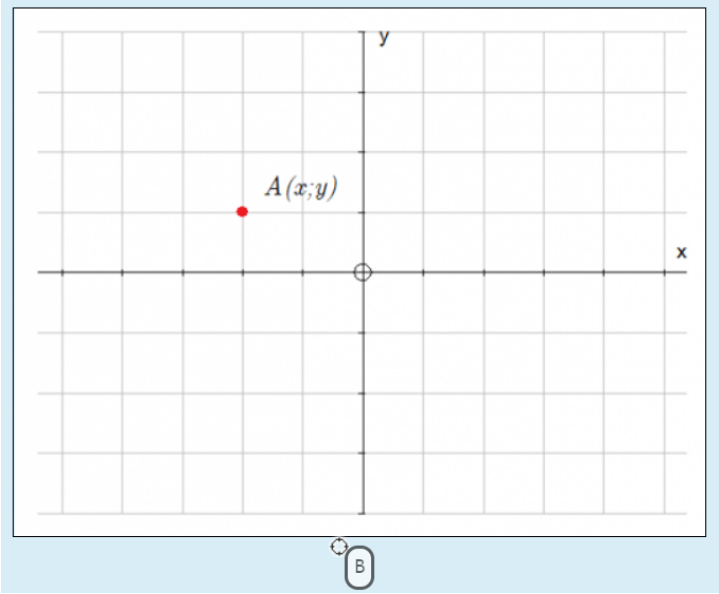


Figure 5. A drag-and-drop item

In the first four editions of the academic year 2019/20 clones of this item had success rates ranging between 34% and 58%. The average success rate on over 1000 subjects was 45%.

The most common of the wrong answers place the point B in the first or fourth quadrant. The wrong answers cannot be totally ascribed to lack of knowledge, as they seem to depend on the initial position of the point A. Actually, if A is placed in the first quadrant (positive *x*'s and *y*'s) the percentage of correct answers significantly increases. It is possible that the students are misled by the belief that an expression associated to a negative number should display a sign '–'. Of course, this behavior is related to the use of mathematical notations rather than to mathematical content.

The last item related to mathematical notations is the following (figure 6).

Il valore di un immobile è triplicato nel corso di un anno. Di quanto è aumentato in percentuale?

Scegli un'alternativa:

- a. del 20%
- b. del 300%
- c. del 200%
- d. del 3%
- e. del 30%

English translation:

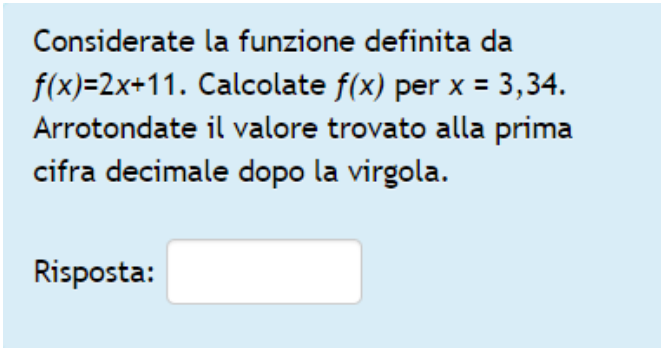
The value of a property has tripled over a year. By how much did it increase in percentage?

Figure 6. Converting words into percentages.

This item requires to associate a percentage to a verbal expression. The average number of correct answers for items of this kind varies from 18% to 25%. The most preferred wrong answers are those including a digit '3'.

4.3 An online course

This is an example of a calculated numerical answer item (fig.7). The value of *x* is automatically changed at each trial.



English translation:
Consider the function defined by f(x) = 2x + 11. Calculate f(x) for x=3,34. Round the value you found to the first digit after the comma.

Figure 7. A computational item on functions

Of course, this item requires skill rather than competence. Items of this kind may be useful to detect the cause of difficulties, which sometimes lies even in the lack of basic skills.

Another item with automatic variation of the constants at each trial is the following (figure 8 - calculated multiple choice).

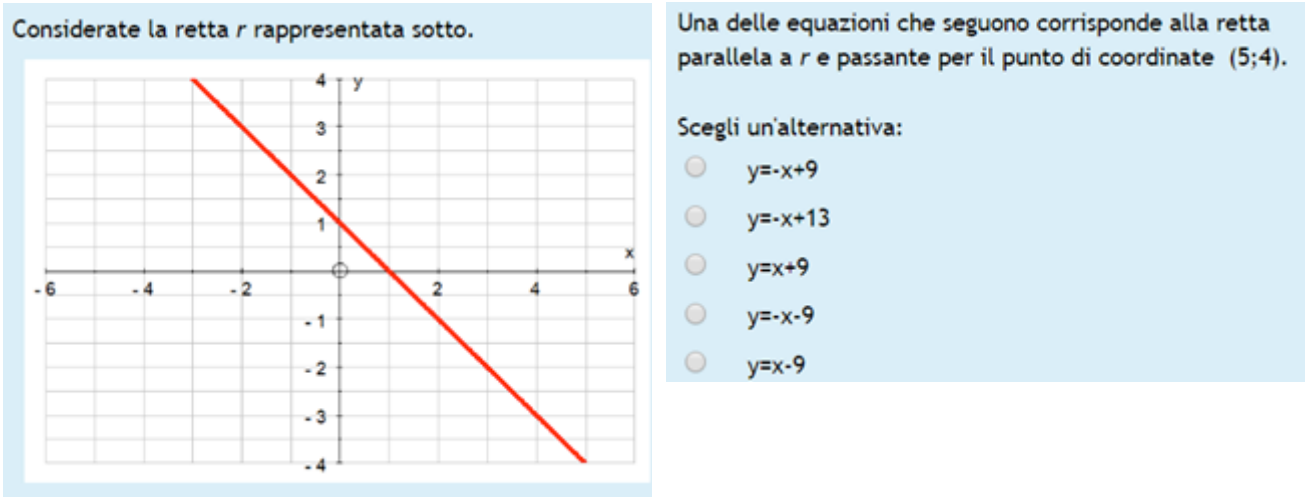


Figure 8. A multiple-choice item on parallel lines.

English translation: Consider the line r drawn below. ... One of the following equations is associated to the line parallel to r and passing through the point of coordinates (5; 4). Choose an alternative.

Items promoting reflection on mathematical notations have been included as well (figure 9).

Se una delle espressioni elencate sotto è equivalente a

"il prodotto della differenza dei quadrati di *x* e *y* per il quadrato della somma di *x* e *y*"

sceglila, altrimenti scegli la voce 'Nessuna delle espressioni proposte è equivalente'.

Scegli un'alternativa:

a. $(x^2-y^2)\cdot(x+y)^2$

b. Nessuna delle espressioni proposte è equivalente.

c. $(x-y)^2\cdot(x^2+y^2)$

d. $(x^2-y^2)\cdot(x^2+y^2)$

e. $(x-y)^2\cdot(x+y)^2$

English translation:

If one of the expressions below is equivalent to

"the product of the difference of the squares of x and y times the square of the sum of x and y"

choose it, else choose 'none of the expressions proposed are equivalent'.

Choose an alternative:

...

none of the expressions proposed are equivalent

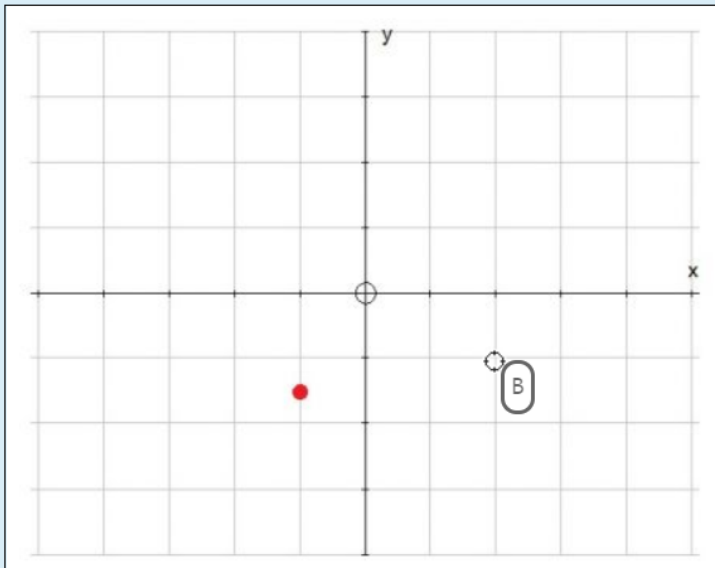
...

Figure 9. A multiple choice item on conversion from words to formulas

Reflection on critical items such as the one presented in figure 5 has been promoted as well. Here (figure 10) is a variation of that item, as the student is not required to answer the question but to evaluate the answer given by someone other.

Considera il problema descritto nel diagramma sotto. Uno studente ha collocato l'etichetta B nel punto che vedi. Secondo te la sua risposta è corretta?

Nel diagramma è rappresentato (in rosso) il punto di coordinate (x; y). Trascina l'etichetta B (che si trova in basso) in modo che corrisponda (col suo circolino in altro a sinistra) al punto di coordinate (2x; −y).



English translation: Consider the problem described in the diagram below. A student has placed label B in the place you see. Do you think his answer is correct?

In the diagram the point A of coordinates (x; y) is represented (in red). Drag the label B (located at the bottom) so that it corresponds (with its circle at the top left) to the point of coordinates (2x; –y).

Figure 10. A variant of the item of figure 5.

Another variation is given by the following question, based on the same diagram of figure 10.

Nel diagramma è rappresentato il punto A di coordinate *(x; y)*. Calcola approssimativamente il valore di *x* e *y*. Trascina l'etichetta B (che si trova in basso) in modo che corrisponda (col suo circolino in altro a sinistra) al punto di coordinate (2*x* ; –*y*).

English translation: The diagram shows point A with coordinates (x; y). Calculate roughly the value of x and y. Drag the label B (located at the bottom) so that it corresponds (with its circle at the top left) to the point of coordinates (2x; –y).

The requirement for approximately calculating the coordinates of A is aimed at promoting the conversion of semiotic systems. Associating a number to the letters *x* and *y* should help the students to focus on the sign '–' as a transformation of positive numbers into negative ones, and the other way around, rather than on the false interpretation of '–' as a sort of attribute of negative numbers.

Similar activities have been carried out with other critical topics, such as the reading and comparison of the values of a function from its graph, already presented in figure 4.

Also in this case the item has been varied in order to promote conversion, as the general equation defining the function has been provided.

We have had little opportunity of evaluating the effects of these experiences due to the troubles caused by the pandemic which has forced us to change the examination methods and to interrupt face-to-face tutoring. Anyway, as from now on it will be necessary to dramatically expand and improve online bridging and supporting activities, this experience, which has been suggested by the large number of students attending our freshman courses, might be a starting point for other activities, motivated by the need for managing distance courses.

References

ANVUR (N.D.). *Riferimenti normativi.* https://www.anvur.it/attivita/ava/riferimenti-normativi/

CISIA (N.D.). *Cos'è il TOLC?* https://www.cisiaonline.it/area-tematica-tolc-cisia/cose-il-tolc/

CISIA (N.D.). *TOLC-S. Struttura della prova e syllabus.* https://www.cisiaonline.it/area-tematica-tolc-scienze/struttura-della-prova-e-syllabus/)

COUNCIL OF THE EUROPEAN UNION (2018). Council recommendation of 22 May 2018 on key competences for lifelong learning. *Official Journal of the European Union.* (https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32018H0604%2801%29)

MIUR (2012). Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione. *Annali della Pubblica Istruzione* http://www.indicazioninazionali.it/wp-content/uploads/2018/08/Indicazioni_Annali_Definitivo.pdf

Pellerey, M. (2004). *Le competenze individuali e il portfolio*. Roma: La Nuova Italia.