"Quaderni di Ricerca in Didattica", Numero speciale n. 8, 2020
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In-service teachers' training on the integrated use of digital tools through the usage of e-learning platforms: "Introspectum" project

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Abstract. L'introduzione di nuovi strumenti di interazione quali e-book, video lezioni, esperimenti virtuali, app interattive (p.e. Geogebra), e di produzione (podcast, video) integrate all'interno delle varie piattaforme digitali, dalle più strettamente e-learning (cfr. Moodle) a quelle con caratteristiche più gestionali (cfr. GSuite- Google), richiede una modalità di formazione insegnanti in servizio sull'utilizzo di tali strumenti attraverso il loro stesso uso. Da questa idea nell'anno 2018/19 è nato il progetto "Introspectum" che si propone di effettuare un percorso di formazione di docenti di Matematica di Scuola Secondaria di II grado attraverso la progettazione di attività didattiche di matematica, l'osservazione e l'analisi, del processo di insegnamento-apprendimento, utilizzando piattaforme digitali sia in fase di formazione sia in fase di intervento didattico.

1. Introduction

In recent years, internationally, much research in mathematics education has offered a range of theoretical perspectives attempting to provide different and interrelated frameworks and perspectives to the study of the role and use of digital technologies in/for teaching and learning mathematics (Hoyles & Lagrange 2010; Drijvers at al. 2010; Drijvers at al. 2013). On one hand, recent research shed some light on changes in classroom practices, closely linked to the role of the mathematics teacher in the digital era (Clark-Wilson at al. 2013). On the other hand, other studies have focused more on the ways in which digital technologies can influence the learning of mathematics, as well as on the way technology affect or even change the nature of the mathematical objects and relations. Students' interactive engagement with mathematics, their motivation and their level of interest have also been part of the wide landscape (Calder & Campbell 2016; Hegedus & Tall 2016; Faggiano at al. 2017).
It should not be underestimated how the widespread use of technologies in everyday life has forged changes in the ways people interact and communicate beyond their knowledge, implying, in return, a wide range of possibilities of ways of thinking.
The use of digital environments in teaching practices require the teacher to be fully aware of the goals of the learning to be promoted and, consequently, of the ability and responsibility to choose digital resources related to specific mathematical knowledge and appropriate digital environments, such as platforms and e-books.
In this context, the student has the potential to take on the constructive role of new digital tools such as apps and synthetic communication methods, like the one operated on Twitter.
If on one hand the tools of traditional mathematics teaching have proved obsolete, on the other hand, the presence of digital resources and the analysis of the interactions between teacher, students, discipline and digital tools, lead to rethinking the design of the secondary school mathematics curriculum.
In this paper, we present the design of "Introspectum" project that aims at carrying out a training course for secondary school mathematics teachers through the design of didactic mathematical activi-

ties, the observation and analysis of the teaching-learning process, by using digital platforms both in the training and in the didactic intervention phase.
The project aims to investigate the following research questions:
Q1 How can the in-service teachers grasp the potential of the technological resources correlated with mathematics?
Q2 How and to what extent do in-service teachers re-design the mathematics curriculum using digital resources?
Q3 Which can be the links between traditional teaching and teaching integrated with digital resources?
Q4 Can the observation and analysis of interactions among students and teachers as well as among students themselves in a digital environment highlight strengths and weaknesses of the above-mentioned interactions in order to define a teacher training activity useful for the re-design of the classroom practice?

2. Theoretical Framework

Some scholars have highlighted that various teaching and instrumental factors that foster digital technology integration can be identified. According to Pepin et al. (2017), a crucial aspect of teachers' work (both individually and in collectives) is their design capacity that can be enhanced with the use and interaction with digital curriculum resources. Certainly, the teachers require increased design expertise because of the changing nature of the resources.
Assude at al. (2010), for instance, pointed out that among these factors, one concerns the didactical transposition, and another one, the problem of management in the classroom. In addition, the relationship between instrumental and conceptual mathematics must be taken into account. Pierce & Ball (2009) highlighted that the knowledge of, the experience with and the views on mathematics education and the role of technology within classroom activities guide the process of a teacher developing instrumental orchestrations. In particular, these often follow implicit guidelines, such as teachers' personal knowledge and skills concerning the integration of technology as well as their concerns about time constraints, behavioural control, participation and cooperation through forums (Faggiano at al., 2018).
In this paper, we will use the term "orchestration" to refer to Trouche's metaphorical theory of instrumental orchestration. It grew within the framework of the instrumental approach, according to which the "instrumental genesis" term has been coined to reflect the long and complex process during which a student turns an artefact into an instrument (Vérillon and Rabardel, 1995). The instrument is the result of a complex construction, on a basis of a given artefact, which involves the co-emergence of mental schemes and techniques for using the artefact to develop specific types of tasks. In such schemes, technical knowledge and mathematical meanings and understandings are intertwined and embedded. However, the instrumental genesis needs to be monitored by the teacher. Pointing out the necessity for the teacher to rely on the external steering of the students' instrumental genesis, Trouche (2004) introduced the notion of "instrumental orchestration" in order to describe the teacher's management of the individual instruments in the collective learning process.
An instrumental orchestration, indeed, is defined (Trouche, 2004) as the teacher's intentional and systematic organization and use of the various artefacts available in a learning environment in a given mathematical task situation, in order to guide the students' instrumental genesis. It is based on the combined action of two elements: first, a "didactic configuration", that is an arrangement of the artefacts in the environment according to the teaching purposes set in advance; secondly, an "exploitation mode", that is the set of decisions on the roles that artefacts, teachers and students should play as well as the selection of technologies and procedures to develop (with regards to the didactic configuration) (Mariotti & Montone, 2020; Montone at al. 2019). Drijvers at al. (2009) highlighted that an instrumental orchestration is partially prepared beforehand and partially created 'on the spot' while teaching. For this reason, they felt the need for a third component reflecting the actual performance. Their model of instrumental orchestration included a third element, that we deem necessary to highlight, the "didactic performance", which involves the ad hoc decisions taken (while teaching) on how to actually perform the enacted teaching in the chosen didactic configuration and exploitation mode.

In agreement with Lagrange at al. (2003), we believe that the teacher can strongly stimulate the student's educational process by promoting the creation of meanings through an orchestration process. The metaphorical theory of instrumental orchestration could be used as a lens to analyse our research, with the aim to seek the crucial factors influencing the awkward process of integrating technology into math class and to identify further outcomes to be considered and developed in our teachers' training research program.

3. The project "*Introspectum*"

The Project "*Introspectum*" was created with the aim of reducing the difficulties connected to the teaching-learning processes of Mathematics through the observation of students' afternoon activities such as homework, learning videos, on-line forms and forums on e-learning platforms in the same way these are usually used during classes. The goal of the project is to carry out a training course for secondary school mathematics teachers focusing on the design of mathematics teaching activities, the observation and analysis of the teaching-learning process, by using digital platforms both in the training teachers phase and in the educational intervention phase.
The project took place throughout two school years (2019 to 2021) and involved around 10 secondary school mathematics teachers, and 10 secondary school classes (about 250 students).
It was developed over 64 hours, divided into two phases and different types of activities whose details are described below.

School priorities	Improve the quality of the teaching-learning process by enhancing the didactic quality of their educational offer through the analysis of the afternoon activities of content reworking performed by the students.
Objectives	• reduction of the problematic aspects connected to the assessment of learning with reference to mathematics teaching; • improvement of the methodological and didactical awareness of teachers; • improvement of the educational service for the students; • experimentation and validation of university research for the qualitative enhancement of materials proposed to the students; • elaboration of a final digital product gathering educational materials to propose to the students.
Current state of affairs	The Project addresses the need to correctly guide the orchestration of learning.

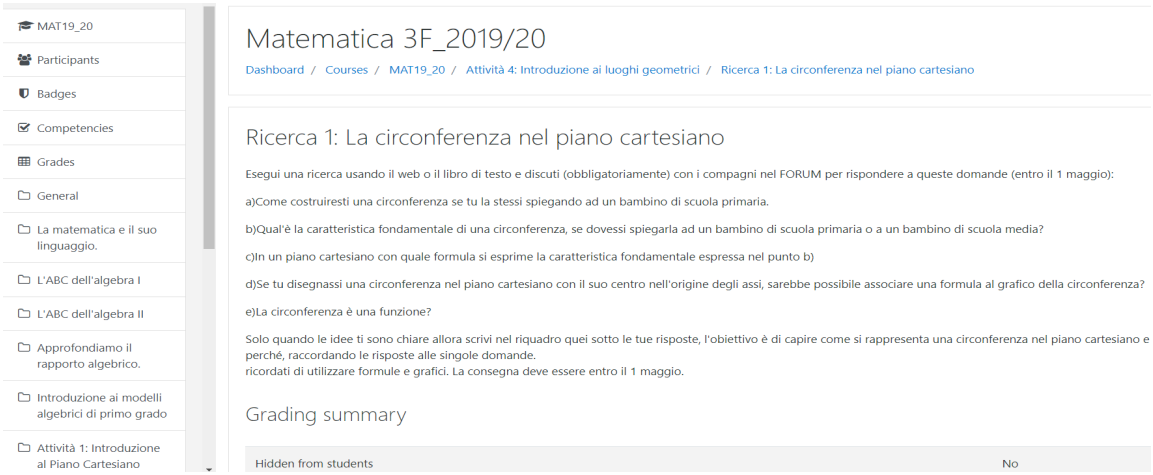
Forecasted activities	Phase 1: February 2020 to May 2020 - February 2020, one-hour face-to-face meeting: the teachers will meet the university trainer and define the observational parameters valid until June 2020. - March-April 2020, one-hour meeting per month: the teachers involved in the project will be connected online to the e-learning platform Moodle; they will identify strengths and weaknesses of the interactions among students and analyse in a qualitative way the answers and the active participation of students. - 10 teachers will meet their selected class of students via the e-learning platform on a weekly basis (roughly) for a total duration of 16 hours. During these online meetings, the teachers will evaluate the whole process through the platform: starting from the materials provided and going through the questions (audios and videos recorded by the students themselves) raised via the chat as well as the completed exercises, they will identify any hurdles, misunderstandings, lack of prerequisites and the need to provide other types of materials. - June-July 2020 (dates to be agreed), 4 meetings of a total duration of 10 hours: using an evaluation grid to assess the online interactions with the students, the teachers will design new learning materials. Phase 2: through the entire school year 2020/2021 - September 2020, one-hour face-to-face meeting: the teachers will meet the university trainer and define specific learning units that were re-designed in June 2020 and that will be re-proposed to the students and assessed once again. - December 2020-April 2021, two meetings of one hour each: the teachers will meet via the e-learning platform; they will identify strengths and weaknesses and briefly analyse the partial observational data collected. - 20 teachers will meet their students online on a weekly basis (roughly) for a total duration of 30 hours. During these online meetings, the teachers will evaluate the whole process through the platform: starting from the materials provided and going through the questions (audios and videos recorded by the students themselves) raised via the chat as well as the completed exercises, they will identify any hurdles, misunderstandings, lack of prerequisites and the need to provide other types of materials. - May 2021, two face-to-face meetings: the teachers will analyse the results and design a final product.
Logistics and organisational resources	• Computer Labs for face-to-face meetings • G-Suite and Moodle Platforms belonging to the teachers' schools
Outputs expected by the end of the school year	The teachers are expected to provide final reports and/or prepare e-books with learning materials to be proposed to the students.

4. The research process

The suggested research process will be illustrated using the metaphor of *grafting*.
To begin with, we will observe the features of available plants and design the new plant we want to grow. The mother plant is represented by the didactic dynamics in separate groups of classes (teacher-student-class) while the scion is represented by the adequate processing of the results of the research in teaching practices. What follows are some initial observations made on the mother plant.
Currently, digital platforms such as Google Classroom, Moodle, etc. are used as a tool to observe the interactions among the teacher, student and classroom, through the analysis of the answers to the different tasks and the reading of the forum chat. The researcher has a direct access to the platform and observes the whole process, mainly focusing on those interactions that do not involve face-to-face classes. Before analysing the results of the activity, the teacher is requested to answer a semi-structured interview that aims at understanding the expectations and the level of awareness of the didactical choices made with respect to the goals of the curriculum selected in accordance with the departmental one. The interview also aims at assessing the micro-objectives expected by the teacher and identified during the school year.
Although predictably, the observational results track in a peculiar way the process that each student has followed and enable the production of qualitative statistics. As a matter of fact, taking into account how teachers choose the activities to propose to their students and the learning objectives, several sheets (one for each activity) will be designed to summarize the data collected, not only in correlation with the peculiarities and delivery method of such training, but also with the expectations of the teacher, outlined during the interview. A systematic organization of the results will be the rootstock for the implementation of the research outcomes in mathematics didactics during the second year of activities.

4.1 A preliminary case study

Below is the description of one preliminary case study, the starting point of the project.
In a secondary school, a teacher proposes to his/her 11th grade class an activity to review the Pythagorean theorem using a series of videos from the website "Schooltoon" (also a YouTube channel) that present the Cartesian plane as well as the formula to calculate the distance between two points, justified by the Pythagorean theorem. The videos include some completed exercises, some questions to assess the level of understanding and some exercises for the application of the formula. The videos wrap up with the assignment to come up with two exercises on the topic. The choice of the topic is based on the educational design of the teacher.
This review is a preparatory activity for the introduction of the notion of geometric locus through an initial calculation of the circumference equation. The calculation is expected to be done in an exercise including a series of questions automatically triggering an algorithmic reasoning that should allow the student to autonomously justify the formula of the circumference equation to calculate the distance between two points, provided that the students clarify the transition of one of the two points from parameter to variable. Below is the assignment sent to the students.



Research 1: the circumference on the Cartesian plane

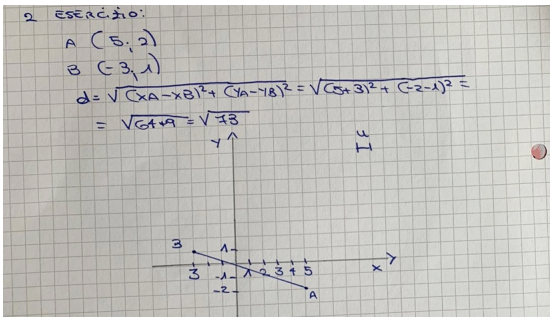
Perform a research on the Internet or in the schoolbook and discuss (compulsory) with your classmates via the FORUM the answers to these questions (by the 1st of May):

a) How would you build a circumference, if you had to explain that to a primary-school child?
b) Which is the fundamental characteristic of a circumference, if you had to explain that to a primary-school or middle-school child?
c) In a Cartesian plane, which formula would you use to explain your answer to question b)?
d) If you had to draw a circumference on the Cartesian plane with its centre in the origin of axes, would it be possible to associate a formula to the circumference drawing?
e) Is the circumference a function?

Please write below the answers to these questions only when you are fully convinced about them – the goal of these questions is to understand how to represent a circumference on a Cartesian plane and why, by linking the answers to the single questions.
Remember to use formulas and charts. The deadline is set for the 1st of May.

In the semi-structured interview, the teacher outlined his/her expectation that the student would autonomously reorganize the results of the exercise. This is one of the micro-objectives that the teacher mentioned during the interview. The teacher had been working since the beginning of the school year and before the kick off of the project on the notion of variables and functions, on the importance of reasoning through mathematical models, and on the use of a symbolic language related to the conversational one. The teacher expected to obtain a structured and internally connected work. The answers to questions a), b) and c) were meant to trigger the reasoning towards the correct application of the formula to calculate the equation of the geometric locus of the circumference. While working on the assignment, the students were invited to use a Forum to collaborate.
The answer to questions a) and b) was a recollection of the knowledge acquired during middle school years. That is why the glass or round objects was mentioned by the student, while the distance between a generic point and the centre was not mentioned at all. Answer c) simply lead to the web results, but the student did not add any additional statements. The teacher had strongly recommended the use of the forum to his students in order to promote interactions and collaboration between them. Interestingly enough, the Forum was not used to ask for tips, explanations or clarifications. The students who used it simply shared parts of their work, showing their prompt readiness to satisfy the request of their teacher.
Here is a key example that shows the most common answer provided by almost all the students which did not meet the expectations: a fragment of the theoretical answer provided by student R. to the theoretical question about the distance between two points.

R: "A very important aspect to investigate further is indeed the distance between points, that is the length of the segment that separates two points; the formula to calculate the distance is very important: everything square rooted, $(x_2-x_1)^2+(y_2-y_1)^2$."
The answer above is not followed by any further graphic explanations and the first part of the equation is missing. When requested to come up with two exercises on the topic, student R. showed to be able to fill in this gap and wrote the following:



The same student R. wrote the following when handing in the assignment about the circumference:
A) If I had to show to a child how to build a circumference and could not use technical tools such as a goniometer or a compass, I would use the oldest method in the world, that is I would take a glass or a round object and draw it on a plane.
B) In my opinion, one of the fundamental characteristics of a circumference is that the ray, that is the distance between any given point and the centre, does not change, regardless of the direction, provided that you measure it from the centre.
C) The formula to indicate the ray is: r = everything square rooted, a^2 over 4+b^2 over 4-c
D) If I had to draw a circumference on the Cartesian plane, with its centre in the origin of axes, I would associate the following formula to the drawing: $x^2+y^2+c=0$ because A and B are 0.
E) The circumference is not a function. For a function to be considered as such, it must have a single y for any x. This is not the case in a circumference. If anything, if we divided the circumference into two parts hence obtaining two semi-circumferences, we would have two functions, one for the upper part and one for the lower part.
Student S. adds to a similar answer a simple image (not present here) of a circumference on a Cartesian plane without any additional notes.
25 students did the assignment: 70% of the students skipped the request fully relying on their memory of the web research; 25 % of the students used the formula of the distance, but did not elaborate on the answer; the remaining 5% provided a satisfactory answer to the assignment.

Conclusion

The preliminary case study described above seems to highlight the lack of efficacy of the educational design and the need to train the teachers on the selection and use of digital resources. The example of student R. mentioned above highlights the limited ability of the student to provide a meaninful answer. The answer to question A) is far from expressing the meaning of a circumference, but it rather refers to its graphic representation. In addition to that, the answer to question B) confirms the student's inability to link the features of a circumference with the circumference itself. Therefore a re-design of the mathematics curriculum is required in order to overcome the inefficacy of the teaching method: the use of digital tools like Geogebra could trigger a conceptualisation of the mathematical object. The potential of digital tools encourages their integration with traditional tools in order to enhance the learning process. The interview laid out to the teacher proved that the expectations were not in line with the outcome of the proposed activities. The hurdles faced by the students will be used to define the cornerstones of teachers' training in relation to the potential of technological resources linked to mathematics and to the re-design of the curriculum using those resources. Furthermore, it will be possible to analyse those elements that allow the fusion of traditional teaching with innovative teaching via the integration of digital resources. Also, teaching activities will be re-designed through the analysis of the interactions in digital environments and the use of free forums (among students) will be pro-

moted and led by the teachers in order to achieve the evolution of mathematical meanings. In comparison with a separate use of face-to-face training on one side and online training on the other, the synergy between these two types of training could produce more efficient outcomes.

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