Formative peer-review practices in online environment to promote the undergraduate students' mathematical thinking

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Abstract. Il presente lavoro riporta le idee principali discusse in occasione della giornata di studio "e-learning e matematica nella formazione universitaria e post-universitaria: da buone pratiche a linee di ricerca". L'intervento ha avuto origine dalla discussione in merito alle difficoltà in matematica degli studenti nel passaggio da scuola ad università, attribuibili a vari motivi. Nello specifico, il lavoro vuole mostrare come un approccio misto basato su attività *peer to peer* online, in un contesto di valutazione formativa, possa essere una strategia pedagogica efficace per il superamento di tali difficoltà e la promozione di un pensiero matematico avanzato. L'attività, sperimentata in un contesto universitario, mira a potenziare le strategie di valutazione formativa e porta a sviluppare negli studenti continue riflessioni sui loro processi di apprendimento e, di conseguenza, ad una maggiore consapevolezza del proprio ragionamento. Il disegno dell'attività e l'analisi dei risultati sono presentati.

1. Introduction

This work is placed on the same panorama presented by some speakers in the plenary session in the occasion of the study day "e-learning and mathematics in university and post-graduate education: from good practices to lines of research".

Specifically, it takes into account the secondary-tertiary transition's problem. It is one of the key topics in research in mathematics education, even if the amount of research at the tertiary level is still modest (Selden & Selden, 2001). This is, probably, referred to the fact that the transition in mathematics is a complex problem. As De Guzman et al. write *"...the secondary-tertiary transition can be seen as a major stumbling block in the teaching of mathematics*" (De Guzman et al., 1998, p. 748). The authors distinguish between epistemological, cognitive, socio-cultural and didactical perspectives. It seems that although mathematics in elementary and high school has a predominant position in the curriculum, the knowledge and skills of undergraduate students may not echo this aspect (Artigue, 2001). One possible reason for this is that there are several changes in the transition to tertiary education, including those in teaching and learning styles, type of mathematics taught, conceptual understanding, procedural knowledge necessary to advance through the material and changes in the amount of advanced mathematical thinking needed. A large amount of research on advanced mathematics notes that students at the beginning of university or at the end of secondary school do not have the necessary skills to combine different reasoning, different ways of thinking.

For example, Sierpinska (2000) distinguishes between theoretical and practical thinking. Theoretical thinking is characterized by organized systems of concepts, and reflection on the semiotic means of representation. Practical thinking is the opposite: it uses archetypal examples, reasoning based on the logic of action. According to Sierpinska, mathematicians use both modes of thinking: most of the time, in a familiar context they think in practical way, and they use theoretical thinking when they find a new and difficult problem. Most of the difficulties that first year students meet can be understood as consequences of practical thinking. She observes for example students solving some linear algebra tasks who write statements such as: "linear transformations are rotations, dilations, projections, shears, etc. and their combinations with constant parameters" (p. 225). These students focus on the prototypical examples they know, and do not spring up a theoretical understanding of the concept of linear transformation.

These difficulties are more evident in contexts where the mathematics has a wide range of applications and gives the students the opportunity to learn how to make mathematical abstractions. In this context, it is not enough that students know how to operate, but that they become aware of what they are doing and why they are doing it. Attention to argumentative processes is also recognized by the National Indications (MIUR, 2012), which stress how mathematics can contribute for "developing the ability to communicate and discuss, to argue correctly, to understand the points of view and arguments of others" (p. 49). Although basic mathematics courses are typically taught in a traditional way, sometimes characterized by "chalk and conversation" (Weber, 2004), our aim is to show how a mixed approach based on peer to peer online activities, in a context of formative assessment, can be an effective pedagogical strategy for the promotion of advanced mathematical thinking and for overcoming the difficulties mentioned above.

So our research question is: how, in a context of formative assessment, peer to peer online activities can be an effective strategy for the promotion of mathematical thinking.

2. Theoretical framework

Starting point of our research is the definition of formative assessment (named FA in the paper) provided by Black and William (2009): "Practice in a classroom is formative to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (p.9).

The same authors underline as self- assessment and peer-assessment activities are fundamental elements of effective FA practices, since activate students as both instructional resources for one another and owners of their own learning. Jenkins (2005) suggests looking at web technologies to promote the use of different assessment methods within a range of approaches, including peer-assessment, self-assessment.

In touch with these ideas, we refer to the model for the use of technology to support FA developed within the European project FaSMEd (Cusi et al.,2017). The model extends the bi-dimensional model of Wiliam and Thompson (2007), where one dimension is identified by the agents involved in FA processes (the teacher, the learner, the peers) and the second dimension by the key strategies for FA. This latter is represented by: (a) clarifying and sharing learning intentions and criteria for success; (b) engineering effective classroom discussions and other learning tasks that elicit evidence of students' understanding; (c) providing feedback that moves learners forward; (d) activating students as instructional resources for one another; (e) activating students as the owners of their own learning.

In the promotion of formative evaluation processes, digital technologies can contribute to the development of key functions of formative evaluation processes, such as the promotion of active student participation in discussions, monitoring of progress, or immediate feedback that can induce students to reflect. Based on these considerations on the use of technology to promote formative evaluation processes, an extension of the Wiliam and Thompson model (2007) was developed within the FaSMEd project, which also contains the functionalities of formative evaluation technology. These functionalities are:

- *sending and displaying*, when the technology is used as a support for communication and for activating discussions in class;
- *processing and analyzing*, when the technology is used to analyse data and information collected during the lessons;
- *providing an interactive environment*, when the technology is used to create a shared learning environment, where students work individually or collaboratively, or a learning environment in which to explore mathematical contents.

The additional dimension of the Wiliam and Thompson model is the one we have most stressed for our research study, in particular we refer to the *sending and displaying* and the *processing and analysing functionalities* within undergraduate level. That in order to investigate how an online platform may be exploited to promote mathematics FA processes involving the three agents—teacher, students and peers— and in particular peers in a blended modality.
Another element considered in our framework is the effective feedback from the different "actors" involved in the FA process, in order to put in practice the keys FA strategies. Hattie and Timperley's (2007) distin-

guish different levels of feedback. In particular, in our research, we focus on feedback about (i) the task and (ii) the processing of the task. Task feedback is focused upon the learning intent and the specific requirements of the task; the attention is on issues related to the interpretation of the problem text or the correctness of the answer given. Process feedback is aimed at the processes, skills, strategies and thinking required by the learner to complete the task.

3. Methodology

3.1. The didactical design

The setting, which refers to in this paper, is a University with a 3-year Bachelor of Science degree in Electronic Engineering and first year students taking part in a two trimester intensive modules in mathematics. Our research focuses on the second module, referring to the topics of the Geometry, Algebra and Logic's course. The module develops along 12 weeks, with 3 face-to-face classes (referred to lectures and exercises sessions) of 2 hours, per week. Moreover, in order to avoid that the student, especially in classrooms with a large number of students, such as the engineering classes, plays a passive role, additional learning resources and communication tools have been provided by an e-learning platform.

In the light of previous experiences on e-learning platforms to involve students as responsible for their own learning process and their peers (Albano&Pierri, 2014, Pierri, 2019), in this work we report the use of a specific advanced tool of the web-based Moodle platform (called "workshop") in order to support peer work in the perspective of online formative assessment (Aldon & Sabena, 2015).

The workshop tool allows students, on the one hand, to upload the resolution of specific problems/exercises and, on the other hand, the automatic and anonymous redistribution of work to be evaluated by other students and related to a specific theme of the course.
Students are invited to provide a feedback devoted to make evident the errors found and to give suggestions to the peer in order to improve her solution; in such a way, as pointed out by Black and William (2009), they are activated both as "instructional resources for each other and as owners of their own learning".

The evaluation is guided by specific criteria established by the teacher related to key aspects of the different arguments proposed, in particular with respect to *correctness, clarity* and *completeness*. The criterion of correctness refers to the lack of mathematical errors in the answer and in the justification given; the criterion of clarity refers to the communication plan and the comprehensibility of the answer by an interlocutor (peer, teacher); the criterion of completeness refers to the explanation of the various passages leading to the conclusion of the argument.

In the following, the criteria's details:

- *Criterion 1 (correctness)*: "*For each exercise, assess whether there are any errors in the solutions or solving process and whether all the answers are given. Are the theoretical recalls correct, if any? Are mathematical symbols used correctly?"*
- *Criterion 2 (clearness): "For each exercise, assess whether the solution is expressed clearly and unambiguously and whether the solving process is shown and intelligible. In other words, evaluate if the solving process express clearly, precisely and unambiguously what it means to say"*.
- *Criterion 3 (completeness)*: *"Evaluate whether all the solutions are given. Where required by the assignment, assess whether the processes are complete or whether there are lacking parts or gaps in reasoning, or unjustified conclusions*."

After the students have completed their task, the teacher finalizes the workshop by providing all the problems addressed in the current workshop together with a corresponding solution model. The solution models are extracted from the students' product, appropriately reviewed, integrated and commented on by the teacher. So each student receives two kinds of feedback: the assessment from three peers, that consist both in a punctual comment/feedback to each criterion and an additional general feedback; a less direct assessment that corresponds to the optimal solution for each problem posted by the teacher; this latter is a more general and indirect feedback and can be used by the student as a form of self-assessment.

All three criteria go in the direction of the two types of feedback considered in the previous section; in the specific, the first criterion seems to be more oriented to provide a type of feedback on the task while the others more stress the process feedback.

In this work, we mean to show how, starting with some samples of feedbacks received from the students' assessors, the FA strategy could enhance the students' argumentation competence according to the peers' assessment. Finally, in order to include the students' point of view, we submitted a questionnaire concerning the students' feelings about their participation to the activity and on their perceived consequences on the learning process.

3.2. The technological implementation

From the technological point of view, the activity has been implemented exploiting the Workshop module of the learning platform Moodle.
This module essentially covers two phases of the learning activity:

- Phase 1**- Solve**: "*Solve the problem you received. For each question of the problem, give suitable justifications, i.e. explain what you are doing, in a concise and clear way as to allow the reader to follow the procedure you have applied; refer to appropriate theoretical results (theorems, definitions, properties) underpinning the procedures you use. Then upload your product, being careful not to put your name inside it.*"
- Phase 2 **- Assess**: "*Correct and assess the received products, as if you were a teacher who should verify student's knowledge concerning the proposed topics, based on the products she has submitted."*

The starter of the activity is a software tool external to the Moodle platform, used by the teacher for delivering, through to the messaging services of the software itself, different problems to various students. The problems are chosen by the teacher in order to cover all topics of the course.

The facilities of the software tool allow to assign, by email, the same problem to a small group (4 or 5) of students randomly chosen. This ensure on the hand that each student doesn't know the classmate that has had the same problem to solve; on the other hand, that, when the student plays the role of assessor (Phase 2), he has a good chance of not receiving the same problem that he solved in the Phase1. In any case, the problems selected by the teacher foresee more than one solving paths; so that, even if a student must assess the same problem that he has received in the Phase 1, there is a good chance that her peer has solved the problem in a different way.

The sending of the problem is accomplished by specific requests:

- *"explain what you are doing as you go along, synthetically but clearly enough so that the reader can follow the reasoning you have done and the proof you have applied*";
- "*recalls appropriate theoretical results (theorems, definitions, properties) that are the basis of the procedures you use*".

The workshop module foresees a setting phase, within the teacher predisposes the environment: he fixes a time within students should accomplish the two phase (from 1 to 4 days for each phase); he defines the assessment elements associated to the three criteria (correctness, completeness, clearness); distributes a certain number of products for each student, excluding self-assessment (we chose 3 reviewers for each product); makes evident to the students the scores obtained on the basis of the scores received from their peers (processing and analysing functionality of technology).

4. Experimentation

We experimented online FA workshops in undergraduate mandatory course of Geometry, Algebra and Logic which are enrolled 175 freshmen students in Computer Engineering. FA online workshops constitute a support to a traditional face-to-face course, whose content mainly concerns linear algebra. The course foresees two written examination tests, one mid-term and one final, that are prerequisite to access the oral exams. For the experimentation five online workshops have been submitted, two before each of the written tests, and are referred to summary exercises for the written test. For each session, about 40 exercises have been prepared, and each of them has been submitted to groups of 3 or 4 students. The experiment has been performed with voluntary students, who were liked to be involved in a massive and more interactive use of the e-learning platform.

After having received by email the problem/exercise collected by the teacher, students have been asked to solve it and to upload their solutions within a given deadline. Then, thanks to the automatic redistribution functionality of the workshop module, all problems and their solutions have been assigned to the students, so that each student receives three of them. Within a second deadline, fixed by the teacher during the preparation of the activity, students have been expected to assess the received products according to the criteria of correctness, clearness and completeness. In the final phase of the activity, each student receives detailed feedbacks from three reviewers. When the activity has been concluded, the teacher gives a general more indirect feedback by uploading on the platform a folder including optimal models of solutions. Each student can access to it for a self-assessment activity.

Following we report some sample of feedback provided by the students, with the role of the reviewers (indicated with R# in the Tables). The feedback is referred to the three evaluation criteria.

Table 1. Feedback about the correctness' criterion.

	Comments
R1	*At the end of the computations, we have three non-zero rows, so the given vector cannot belong to the space V and, accordingly, it cannot be expressed as linear combination of the basis*
R 2	*There are mistakes in using mathematical symbols and some formal shortcomings (see attached file)*
R3	*You cannot perform the inner product among three vectors*

Table 2. Feedback about the clearness and completeness' criteria.

	Comments
R4	*Specify, when [...], that you have replaced the value of h with 1*
R 5	*Theoretical references fairly present but lacking in formalism and deep knowledge*

In the Table 2, R4's feedback is referred to clearness criterion, while R5 to the completeness's one.

The questionnaire submitted to the students at the end of the activity has meant to explore the students' feeling when they played the role of the assessor as well as of the assesse. Moreover, we have investigated the impact that their participation to this activity has had on the learning of the subject, the method of study and the success of the examination. In the specific, the questions submitted to them are:

- How did you feel when you were assessing your peers' work?
- How did your participation in the workshops affect your learning of the course content?
- How did your participation in the workshops affect your to learning approach for being successful in exams?

5. Discussion and conclusion

In this section we provide a qualitative analysis of the data that we can extrapolate both from the feedbacks provided by the students, with the role of the reviewers, and from the answers to the questionnaire.

We can observe as R1 argues on why there is an error. He starts from the computation [*…we have three non-zero rows…*] and interpret them in terms of theoretical meaning [*…so the given vector cannot belong to the space V…*] and finally he gets a further theoretical result as a consequence *[…accordingly, it cannot be expressed as linear combination of the basis…*]. R3 focuses its feedback on cognitive/language aspect, by underlining that different objects should have different names [*…in order to avoid confusion…, name them differently…*]. Concerning the clearness's criterion R4 stresses the importance to made explicit some implicit facts; such as regarding to the completeness R5 feels to point out a lacking deep learning.

The qualitative data obtained from the answers to the questionnaire show that the feelings of the students in their role of assessor are different: responsible, that it is evidence from the sentence [*…Responsible and it allowed me to understand many mistakes that I also made…*], helpful, as another student affirms [*…In some tasks, I have tried to steer the student examined towards the correct execution of the exercise, motivating with the right theoretical references where necessary. In those situations, I felt comfortable with the idea of being able to help…*]. So, we can observe, in this sentence, as this goes alongside feeling responsible, not only for her own learning, but above all for peers' one. Sometimes the sense of responsibility was to feel not up to the task, so a sense of frustration has emerged, as some students reported [*…Sometimes in trouble for fear of misjudgement…*]

Respect to the impact on content learning, among the benefits derived by the participation in the workshops activities, the main ones consist in acquiring regular rhythm of studying, the self-assessment of what understood during the lectures [*…Participation in the workshops allowed me to understand where there were more gaps, so that I could focus more on them…*], as well as the identification and recovering of gaps [*…It has allowed me to understand where it is most possible to make mistakes in an exercise and what are the errors or inconsistencies I was making…*].

For consequence it has been a positive impact also on the students' learning approach (the third aspect of the questionnaire), offering continuity in studying [*…She "forced" me to study gradually and not only at the end of the course and near to the exam, which otherwise I would surely have done…*], focusing on theoretical and argumentation aspect [*… It positively impacted leading me to a more in-depth theoretical study…*], [*…It helped to fix the theoretical concepts…*].

As conclusion, the qualitative analysis suggests that formative peer assessment practice in blended environment can be of great help in overcoming one of the main difficulties in the transition from high school to university. The outcomes provide an encouraging answer to our research question: peer to peer online activities can be an effective strategy for the promotion of mathematical thinking, both at cognitive and metacognitive levels.

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