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G.R.I.M. (Department of Mathematics, University of Palermo, Italy)

THE MATHEMATICS TEACHER IN THE MODERN SOCIETY

Michael Gr. Voskoglou
Professor of Mathematical Sciences
Higher Technological Educational Institute
Meg. Alexandrou 1 – 26334 Patras - Greece
e-mail: <voskoglou@teipat.gr>

Abstract

In the present paper we refer to the status of the mathematics teacher in the contemporary society. In particular we consider the role that the teacher plays for the learning of mathematics and the desirable characteristics of a competent mathematics teacher. The proper professional training, that helps the acquisition and development of these characteristics is also a parameter of our investigation. Examples from Greece are used to illustrate our remarks and conclusions

1. Introduction

The process of learning has been fundamental to the study of human cognitive action. But, although every one knows what learning is, and despite to the fact that there are very many theories and models developed from psychologists and education researchers to describe the mechanisms of learning, the understanding of the nature of this process has proved to be complicated. This basically happens because it is very difficult for someone to understand the way in which the human mind works, and therefore to describe the mechanisms of the acquisition of knowledge from the individual. The problem is getting even harder by taking into consideration the fact that these mechanisms, although they appear to have some common general characteristics, actually they differ in the details from person to person.

In earlier papers we have considered the use of mathematical modelling as a tool for learning mathematics (Voskoglou, 2006) and we have described the process of learning in terms of a fuzzy model (Voskoglou, 2008). In the present paper we focus on the role that the teacher plays for the learning of mathematics and we try to give a convinsing answer to the following question which is strictly related to this role: What does it mean to be a competent mathematics teacher?

2. The learning and teaching of mathematics

The ***constructivist view of learning*** and the ***socio-cultural theories*** have been highly influential in addressing mathematical knowledge and the learning of mathematics over the last four decades.

The constructivist view involves two principles: Knowledge is actively constructed by the learner, not passively received from the environment, and ***coming to know*** is a process of adaptation based on and constantly modified by a learner's experience of the world (Von Glasersfeld, 1987).

On the other hand, according to the socio-cultural approach, learning takes place within some socio-cultural setting. Shared meanings develop through negotiation in the learning environment, leading to the development of common knowledge.

Mathematics teaching is intended to promote the learning of mathematics. But, while theory provides us with lenses for analyzing learning, the position of mathematics teaching remains theoretically anomalous and underdeveloped (Jaworski, 2006). We might see one of the problems to lie in relationships between learning, teaching and the ***practice of teaching***.

Theories help us to analyze, or explain, but they do not provide recipes for action; rarely do they provide direct guidance for practice.
According to Jaworski (2006) one way to draw on theories of learning to explain or characterize teaching is to see teaching as a social practice, in which teachers are practitioners. For example, like the novice tailor being drawing in to the practice of tailoring practicing alongside old-stagers in the community, perfecting processes and skills, learning the trade etc, we might see the novice teacher being drawn similarly into the practice of teaching.
For this practice Jaworski suggests a form of ***critical alignment***, in which it is possible for participants to align with aspects of practice while critically questioning roles and purposes as a part of their participation for on going regeneration of the practice.
She also speaks about the importance of ***inquiry communities*** and their contribution to learning and development of teaching, addressing three forms of inquiry practice:

- ***Inquiry in mathematics***: Pupils in schools learning mathematics through exploration in tasks and problems in classrooms, teachers using inquiry as a tool to promote pupils learning in mathematics.
- ***Inquiry in teaching mathematics***: Teachers using inquiry to explore the design and implementation of tasks, problems and activity in classrooms, educators using inquiry as a tool to enable teachers to develop teaching.
- ***Inquiry in research which results in developing the teaching of mathematics:*** Teachers and educators researching the processes of using inquiry in mathematics and in the teaching of mathematics.

The above suggestions compose actually a socio-cultural transformation of the teaching of mathematics which results from the constructivist view of learning.

3. The status of mathematics in our modern society and the role of the teacher

Mathematical activity is an original and natural element of human cognition. The status of mathematics in the contemporary society is described in an elegant way by Wittman (1995, pages 359-360). In our modern society we must conceive of mathematics as a broad social phenomenon whose diversity of uses and modes of expression is only in part reflected by specialized mathematics, as typically found in university departments of mathematics. This diversity includes mathematics developed and used in science, engineering, economics, computer science, statistics, industry, commerce, art, daily life, and so forth according to the customs and requirements specific to these contents.
Mathematics educators and teachers need a lively interaction with mathematics and they must devote an essential part of their professional life to stimulating, observing and analyzing genius mathematical activities of children and students.
As part of mathematics, specialized mathematics must be taken into consideration as one point of view that, however, has to be balanced with other points of view. The history of mathematical education clearly demonstrates the risks of following specialized mathematics too closely; perhaps the best example of this mistake is the "new mathematics" movement. A second mistake is that educationally important fields of mathematics, which are no longer alive in specialized research and teaching, lose the proper attention in our schools today; a characteristic example is the elementary geometry (e.g. see Voskoglou, 2007a, section 4). Imagine therefore the position of a university teacher of mathematics in a department of Architecture (where I am currently lecturing in Patras), or of Civil Engineering, who wants to present, some, even elementary, topics from the theory of surfaces - that constitutes the necessary mathematical background for the design of roofs in buildings with wide openings - when students have difficulty even to realise that a straight line in space can be defined as the section of two levels!
Mathematics educators and teachers must see the school mathematics as an extension of pre – mathematical human capabilities, which develop within the broader societal context provided by

mathematics (Schweiger, 1994 and Dörfler, 1994). It is only from this perspective that the unity of mathematics teaching from the primary through the upper secondary level can be established. It would also be beneficial if the mathematical specialists perceive themselves as partners in the larger mathematical system described above. Without some change of awareness on their part, any attempt to change the public image for mathematics is very difficult to succeed.

The teachers must take part in the design of the teaching units, but this can be no excuse for mathematics educators to retrain from this task. On the contrary: The design of substantial teaching units, and particularly of substantial curricula, is a most difficult task that must be carried out by the experts in the field. By no means can it be left to the teachers, though teachers can certainly make important contributions within the framework of design provided by experts, particularly when they are members of, or in close connection with a research team. Also the adaptation of teaching units to the conditions of a special classroom requires design of a minor scale, which must be carried out by the teachers. Nevertheless, a teacher can be compared more to a conductor than to a composer, or perhaps better to a director than to a writer of a play, with the second roles of the above two parallelisms corresponding to the researcher of mathematics education .

It is finally important to underline the remarkable influence that the existing philosophical views about the nature of mathematics have on the development of the mathematics education, a fact that causes recently successive substantial changes in the field (Voskoglou, 2007a, section 5). This means that, regardless of personal beliefs and options, mathematical educators and teachers of mathematics ought to prepare the conditions under which the mathematics education will receive and assimilate gently and creatively the advancing changes, getting the maximum possible profit from them.

4. The characteristics of a competent mathematics teacher

Let us now turn our attention to the question that we have asked in our introduction about the characteristics of a competent mathematics teacher.

The common experience teaches that a brilliant mathematician need not be a good teacher of mathematics. However the converse of this argument is not true, because a good teacher of mathematics must be first of all a good mathematician. In fact, in order to be able to help efficiently students to understand mathematics and to use it properly in their everyday lives, the teacher must be able first to do so for him (or her) self! Further a wide and deep knowledge of mathematical topics relevant to his (her) teaching subject enables him (her) to acquire a spherical view on it , and therefore is a very important factor that helps to have a good teaching performance.

In Greece the teachers of primary education are teaching mathematics together with language, history, physical sciences etc, i.e. they are no teachers specialized in each one of the above subjects (unless if they have undertaken some postgraduate studies). Concerning the teaching of mathematics at least, this fact causes many problems, particularly in the last three years of primary school (10 – 12 years old). In fact, many of these teachers, although they undertake one or two courses of mathematics - usually not of the required depth and width - in parallel with their pedagogical studies in the university, they have not the proper mathematical background needed for this purpose. The problem is getting more serious due to the well known fact that a student, who does not acquire the correct mathematical background in primary school, it is very difficult to do so later in high school.

On the contrary the teachers of secondary education undertake their undergraduate studies at the mathematics departments of the universities, all of which include during the last 10 -20 years a sector of didactics, history, and philosophy of mathematics. However many of them prefer to follow other directions during their undergraduate studies (pure or applied mathematics, statistics and operational research, etc), but they finally decide to become high school teachers, because they cannot find an alternative proper job. Notice that a graduate of mathematics, in order to become a teacher, must pass an extra general examination including topics of

mathematics taught in secondary education and of the didactics of mathematics of practical nature, but this is not enough to insure that he (she) has the proper pedagogical background needed in order to be a competent mathematics teacher..
My personal feeling however - from the time that I was a student many years ago and through my over of 35 years teaching experience of mathematics in the secondary and tertiary education - is that many mathematicians became good teachers despite to the fact that they had never attended courses on the didactics of mathematics and (or) pedagogical studies. Therefore the following question arises: To be a competent teacher of mathematics is actually a talent, or it is something that must be acquired through the proper professional training?
I believe that the truth lies somewhere in the middle, i.e. to be a good teacher is a talent in fact, but this talent must be cultivated through the proper teacher's training. In other words I believe that, under the pressing demands of the modern society, the teaching experience is not enough to mark out this talent, as it happened in the older times of a much simpler and less demanding society. On the contrary, I believe that less talented students could become nowadays competent mathematics teachers through the proper professional training
But which is actually the proper training for someone to become a competent mathematics teacher?
In other words is a competent mathematics teacher someone who has been trained in mathematics and receives some additional pedagogical or didactical training of theoretical and of practical nature, or someone who has received pedagogical training and moves on to teaching mathemaics after having had some amount of mathematical training? Further on, should the mathematics teachers learn be mathematics as such without particular regard to their future teaching profession, or should they learn mathematics in ways that are specifically focused on teaching at certain levels, i.e focus on what Schulman (1986) calls "pedagogogical content knoeledge" ? Yet another issue is the balance between theoretical study in a teacher training institution and practical work in school.
Thus delicate balances have to be struck and genuine dilemmas have to be resolved for the design, structuring and organization of pre – service education and in – service professional development of compentent mathematics teachers. But are the mathematics teacher training institutions today in position to offer the proper training to their students and future teachers? The answer to this question seems to be rather unsteady and complicated, because there exists a big problem towards this direction. In fact, according to Simon (1970), who speaks in general about the design sciences [1] – and mathematics education could considered to be a design science (cf. Wittman, 1995) – as proffessional schools are more and more absorbed into the general culture of the univesity, they hanker after academic respectability. In terms of the prevailing norms, academic respectability calls for subject matter that is intellectually tough, analytic and formalizable. Thus quite a number of mathematical educators and researchers of mathematics education instead of working into the core of mathematical education, are taking the approach to adopt methods and standards from the hard sciences and humanities, where the scientific background and their natural interests might be as influential as the wish to be recognized and supported by scientists in the related disciplines. Consequently, a great deal of didactic research adheres to mathematics, history and philosophy of mathematics, psychology, pedagogy, sociology and so forth. Thus the holistic origin of didactic thinking, namely mathematical activity in social contexts, is disolved into single stands. Very often also the adoption of frameworks and standards from related disciplines is linked to the dogmatic claim that these frameworks and standards were the only ones possible for didactics
In the views of many researchers this is a big problem that presently inhibits major progress in mathematical education. Heinz v. Forester states characteristically: "The hard sciences are successful, as they deal with soft problems. The soft sciences are badly off, as they are confronted with hard problems".
Niss (2006) suggests a new way to educate competent mathematics teachers, which has been developed as part of a larger project in Denmark, the so - called KOM project. According to the

results of this project a competent mathematics teacher is someone who in an effective and efficient way is

[1] The mission of the *design sciences*, or otherwise called *the sciences of the artificial*, is the design and manufacture of artificial objects having certain desirable properties, in contrast to the natural sciences, that describe and interpret the structure and operation of natural objects (Simon, 1970).

able to help students to build and develop the following eight mathematical competences, which are divided in two groups:

a) ***Mathematical thinking*** (mastering mathematical modes of thought), ***problem handling*** (formulating and solving mathematical problems),

modelling **(**being able to analyze and build mathematical models concerning other subjects or practice areas) and ***reasoning*** (being able to reason mathematically), that characterize the ability to ask and answer questions in and with mathematics, and

b) ***Representation*** (being able to handle different representations of mathematical entities), ***symbol and formalism*** (being able to handle symbol language and formal mathematical systems), ***communication*** (being able to communicate in, with, and about mathematics) and ***aids and tools*** (being able to make use of and relate to the aids and tools of mathematics), that characterize the ability to deal with mathematical language and tools.

In more specific terms a competent mathematics teacher must first be mathematically competent him – or herself in the sense outlined above. It follows that as regards the mathematical pre-service education of the teacher it is not sufficient to provide it in such a way that the study of mathematics is restricted to the study of mathematics in its didactical and pedagogical enactmment as an educational subject

Secondly a competent mathematics teacher must posses the following six didactical and pedagogical competencies:

Curriculum competency (to analyze, assess and implement existing mathematics curricula and to consruct new ones),

Teaching competency (to plan, organize and carry out mathematics teaching, including creation of a rich spectrum of teaching/learning situations),

Uncovering of learning competency (to answer, interpret and analyze students' learning of mathematics, as well as their beliefs and attitubes towards mathematics)

Assessment competency (to assess, characterize and communicate students' learning outcomes and competencies, so as to inform and assist the inndividual student and other relevant parties)

Collaboration competency (to collaborate with different sorts of colleagues in and outside mathematics, as well as others - parents, authorities- concerning mathematics teaching and its conditions) , and

Professional development competency (to participate in and relate to acttivities of professional development, such us in-service courses, projects, conferences, and to keep oneself up-dated about new developments and trends in research and practice).

As Niss concludes, the adoption from a country of this way of looking at mathematics teachers'compentencies and their development during pre-service education and in-service professional development activities most probably poses substantial challenges to its educational system. For the case of Greece for example, taking into account the existing situation with the teachers of mathematics described above, the adoption of the conlusions of the Danish KOM progect could suggest the establisment of new university departments for training mathematics teachers, either inside the mathematics or the pedagogical departments as an undergraduate specialization, or indipendantly outside of them. Alternatively it could suggest that the graduates of mathematics or pedagogical departments must earn an obligatory special postgrauate degree before moving on to teaching mathematics.

All the above were thoroughly discussed during a workshop on the role of the teacher for the learning of mathematics, that I organized within the 59th CIEAEM Congress, which took place on July 2007 in Dobogoko – Hungary (Voskoglou, 2007b). The participants of this workshop (researchers and mathematical educators from all over the world) underlined the fact that the undergraduate and the professional (during the service) training of the teacher of mathematics, even if they are of very good quality, they are not alone enough to create the competent mathematics teacher. For this a continuous personal effort of the teacher, accompanied with the critical alignment that Jaworski suggests (see section 2), is also necessary for his (her) improvement through the experience that he (she) gradually gains from practicing with the teaching of mathematics. Finally, but not less important, a continuous self-training of the teacher (by attending seminars and conferences, by reading books and articles, by using the new technologies, etc) is needed on the recent developments of mathematics and (mainly) of the didactics of mathematics.

5. Final conclusions

- The constructivist view of learning and the socio-cultural theories have been highly influential in addressing mathematical knowledge and the learning of mathematics over the last four decades. Jaworski's proposal about the inquiry communities composes actually a socio-cultural transformation of the teaching of mathematics which results from the constructivist view of learning.
- In our modern society we must conceive of mathematics as a broad social phenomenon, whose diversity of uses and modes of extension in most sectors of human activity is only in part reflected by specialized mathematics, as typically found in university departments of mathematics.
- The role of the mathematics teacher in the modern society can be compared more to the role of a conductor than to a composer, or perhaps better to the role of a director than to a writer of a play, with the second roles of the above two parallelisms corresponding to the researcher of mathematics education.
- The property of being a competent mathematics teacher is a talent, which has to be cultivated through the proper professional training. Under the demands of the modern society the teaching experience alone is not enough to mark out this talent, as it frequently happened in older times.
- The neglect of the core of mathematical education by a quite number of mathematics educators and researchers, who are taking the approach to adopt methods and standards from the related 'hard' sciences and humanities, is a big problem, which presently inhibits major progress in mathematical education and affects in a negative way the professional training of the future teachers in the universities.
- According to results of the KOM – project, developed recently in Denmark, a competent mathematics teacher is someone who is able to help efficiently students to develop eight concrete mathematical competencies and who is characterized by six concrete didactical and pedagogical competencies with particular regard to mathematics. The adoption of these results from a country most probably poses substantial challenges to its educational system.

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