Opponent report

THE CONCEPT OF VARIABLE IN THE PASSAGE FROM THE ARITHMETICAL LANGUAGE TO THE ALGEBRAIC LANGUAGE IN DIFFERENT SEMIOTIC CONTEXTS

Applicant: ELSA DEL PILAR MALISANI

Supervisor: PROF. FILIPPO SPAGNOLO

Given work is as a doctoral thesis relatively comprehensive (167 pages). The dissertation is well structured and organized into the five chapters.

The first chapter (*Historical evolution of the algebraic language*) is about the history and introduces the construction of algebraic language and evolution of the methods and strategies of resolution of equations in the periods that preceded the formalization. The deep historical analysis on the construction of the algebraic language allows to highlight the principal conceptions, the precursory procedures, the passages from a concept to an other and, particularly, the passages through the linguistic levels of different phases: rhetoric, syncopated and symbolic. This approach gives possibility thoroughly acquaint with epistemological obstacles that the pupils meet in the situations of learning the algebraic language. Single stages of development are written critically with rich literal reference.

The next three chapters (2, 3 and 4) describe the core of the research work. The second chapter (*The magic square*. An experience on the transition between the arithmetical language and the algebraic language) explains the purpose to study some aspects of the period of transition from the arithmetical language to the algebraic language. There are analyzed different conceptions of variable in the resolution of problems.

The third chapter (*The notion of variable in different semiotic contexts*) deals with study of the relational-functional aspect of variable in the problem-solving, considering the semiotic contexts of algebra and analytical geometry. This chapter also deals in more details with investigation whether the notion of unknown interferes with the interpretation of the functional aspect.

The fourth chapter (*The variable between unknown and "thing that varies"*. Some aspects of the symbolic language) describes and analyzes how the conceptions of unknown and functional relation are activated and used in the process of resolution of a problematic situation. This chapter also describes the study of translation process from the algebraic language into the natural one and the representation of the syntax-semantics relation within the algebraic code. This part is very interesting. The fifth chapter summarizes final conclusions.

It can be remarked that this dissertation deals with concept of variable, an interesting topic, from the perspective of teachers' convictions. The researcher provides an analytical historical approach to the topic and covers the concept of variable by providing an accomplished theoretical framework. The research questions and methodology are well documented and cover the study's needs. The results are well analyzed and present and provide answers to the research questions. In conclusion it can be remarked, that work is written clearly and transparently. Due to a precise language style it is attractive for a reader. The work is conducted with a lot of scrupulousness and scientific correctness. Attention is given to the contents and also to the structure of work, which is comprehensive. The publication shows the high level of knowledge of the problem and the ability of own creative academic work, by the way, which is evidently the original contribution to the development of didactics of mathematics. Given work with its form highly exceeds requirements set to PhD's thesis. Therefore I recommend to accept the Thesis and after its successful defence to grant the applicant Elsa Del Pilar Malisani the academic degree PhD. (philosphiae doctor) in the specialization 11-17-9 Theory of teaching mathematics.