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Opponent report

THE CONCEPT OF VARIABLLE IN THE PASSAGE FROM THE ARITHMETICAL LANGUAGE TO THE ALGEBRAIC LANGUAGE IN DIFFERENT SEMIOTIC CONTEXT

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Presented doctoral thesis goes into the perception of concept of variable as the point of critical passage between the arithmetical and algebraic thought in students´ minds. Work has 167 pages including appendixes and bibliography and it is logical divided into the introduction and 5 chapters.

The subject of doctoral thesis resulted from previous author's long-time researches in witch she compared the cognitive effort of the students between the ages of 14-15 in the determination of resolution of algebraic and geometric problems and diagnosed and classified errors in their resolution (Malisani 1990, 1992, 1993). This is an actual subject in term of Theory of didactic situations for the realisation the current pedagogical practice related with teaching of mathematics at primary and secondary schools (creation of a-didactic situation, levels of didactic situations and the environments appertain to them, didactic contract in the teaching...). These researches suggest to the necessary conceptual knowledge, especially the concept of variable, to understand and to describe the problem. Following presented researches as well as the researches realised by the others authors and following analyse of problems resulted from

historical background of evolution of concept "variable", (Malisani 1996, 1999) lead author to formulate the aims of thesis (p. 7).

The study of construction of the algebraic language and the evolution of the methods and strategies of resolution of equations in the periods that preceded the formalization (chap. 1) author creates the conditions for analyse the possible epistemological obstacles of perception of concept the variable in students´ minds. In conclusion of chapter focused on historical evolution of the algebraic language (chap. 1) author proposes the survey of 10 important facts from history those analogy we can find in the learning of algebraic language.

The second chapter deals with some aspects of the period of transition from the arithmetical language to the algebraic language. With the well-chosen experiment (p.48) author determines perception of concept of the variable and compares using of natural, arithmetic and algebraic language by students. In realization of experiment author goes out from *Theory of didactic situations* (Brousseau), in compliance with this theory she realizes qualified analysis a-priory of planned didactical situation. This analysis took place at two groups of pupils different in level of their knowledge (middle school and high school). Validity of hypotheses formulated in the introduction of experiment author verifies in two levels – qualitative and quantitative. Author in details analyses work of pupils in groups and completes it with statistical analysis (analysis with using of software C.H.I.C. and by the help of S.P.S.S.), what is a great achievement from view of actuality of qualitative evaluation of didactical experiments. At the end of chapter important results coming out from experiment are transparently summed.

The next chapter joins achievements coming from realized experiment and at the same time she deals with concept of variable in different semiotic contexts, in concrete regarding algebra and analytic geometry. For the verification of validity of four hypotheses formulated at the beginning of this chapter, the crucial meaning has the choosing of experimental group, as well as proper choosing and formulation of given four questionnaires´ tasks. During characterization of single level of research author througfully uses methodology and terminology of Theory of didactic situations, realizes

in details analysis a-priory, at the evaluation of research she uses as the base statistical software (already mentioned C.H.I.C. and S.P.S.S) and classifies pupils by fore defined profiles (p.94). Detailed analysis of research is worked out very transparently, at the high professional level and it is completed also with graphical documentation.

Conclusions of the third chapter are the base of the next chapter, which is dedicated for some aspects of symbolic language, in concrete author deals with comparison of perception of concept of variable by pupils namely as *unknown* or as "*thing that varies*" (functional relation). For fulfilment of goals of the fourth chapter ("to analyze how the conceptions of unknown and of functional relation are activated and used in the process of resolution of a problematic situation") author realizes analogical research as in the third chapter, however stated questionnaire she proposes to four pairs of pupils, their solutions are registered on the audiocassette and for detailed analysis documented also by pupils´ protocols. Choosing of methods and arrangement of research are supposition for precise analysis a-posteriori of pupils´ protocols.

The fifth chapter include summary of author's observations and researches´ conclusions confronted with aims of thesis, at the same place author proposes also questions that could be subject of the next research.

Thesis is characteristic with precise formal and graphical form, it is written in very transparent and comprehensive style. The end of each chapter is completed by bibliography and appendix documented text part of given chapter.

Subject of presented thesis is very actual and inspiriting for the scientific discipline Didactics of mathematics and at whole also for Slovak school of Didactic of Mathematics as well as for Slovak Educational System. Whole conception of the work, choosing of methods, processing of research, formulation of conclusion, logical structure of the work and way of subject processing – historical and epistemological development as analogy with pupils' ontogenesis (chap. 1), analysis of actual state of perception of concept variable by pupils (chap. 2), comparing preference of natural language before arithmetic and algebraic language and comparing variable as *unknown* or as "*thing that varies*" in pupils minds at problem solving (chap. 3,4) – fulfils needs given on doctoral thesis.

Submitted doctoral thesis with title: The concept of variable in the passage from the arithmetical language to the algebraic language in different semiotic contexts, by author ELSA DEL PILAR MALISANI is quality contribution for development of Didactics of Mathematics. It is based on study of scientific literature, realization of research; it includes new issues for discipline of Didactics of Mathematics and numbers of questions and proposals for the next development of this discipline.

Given work with its form highly exceed requirements set to PhD thesis. Therefore I recommend accepting the thesis and after its successful defence to grant the applicant Elsa del Pilar Malisani the academic title **philosphiae doctor** in the specialization 11-17-9 Theory of Teaching Mathematics.

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