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Report of doctoral thesis

The Concept of Variable in the Passage from the Arithmetical Language to the Algebraic Language in Different Semiotic Contexts

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The thesis is divided besides introduction into 5 chapters with 14 appendixes. It has respectable 167 pages. The aim of the thesis that is formulated in the introduction (page 7-8) is ambitious. One can characterize it by following hand-outs:

- the study of characteristics of passage from arithmetical to algebraic language;
- analysis of effect of various reception of variable (*unknown quantity* versus *functional representative*) when solving the given problems by students and consideration how the semiotic context influences on reception of variable in the viewpoint of the student;
- experimental verification whether the various reception of variable is epistemological obstacles or it has a didactical background;
- extension of the tools to creation of a-didactic situation and deeper understanding of communication processes.

The author could set this aim above all thanks to her own long terming research and educational practice and because she is very good acquainted with most important results in this field, as well as with historical evolution of concept of variable and algebraic language.

The structure is adapted to the aim of the thesis. It enables to follow transparently how is the author creating the suppositions to achieve successfully the stated aims and describes exactly the procedures that she has used.

The first chapter is actually historical study about introduction of algebraic language-overview of evolution of methods and strategies of equation solving in the periods that preceded formalization. Besides information values it is needed to highlight her motivational influence on further researches. Bibliography of the chapter consists of 37 sources, 4 of them are proper works of author, 3 works of supervisor and it documents the complexity of the approach.

In the second chapter the author studies some of the aspects of period of passage from arithmetical language (thus thinking as well) to algebraic. The key question identified by author on the

ground of analyze is the question of reception of variable. She describes proper didactic experiment carried out on sample of 11-12 and 14-15 years old pupils. This experiment with problem of *magic square* has verified hypotheses concerning the influences of insufficient acquisition of algebraic language. For each sample she describes phases of experiment: preparation of a-didactic situation and realization of a-priori analyze of problem; qualitative analyze of data; quantitative analyze of data by using software C.H.I.C. and S.P.S.S. Description is model application of approach of theory of didactic situation of G. Brousseau. She employs 25 sources in the chapter, 3 of them are her proper works.

The third chapter explores images of variable in different semiotic contexts. Point, it handles about questionnaire of 4 various questions (problems) in regard to algebra and analytic geometry for sample of 16-18 years old students. The tactics of the experiment and its description is similar as in the prevenient chapter. In like manner (in addition with complement of graphical information) she interprets also obtained data for ascertained strategies and for verification of hypotheses. She comes out of 25 sources again, 4 of them are her proper works.

The fourth chapter knots the research and conclusions of previous chapter. Its aim is analyze of using of two notions of variable by solving case situation. This time the sample is consists of 4 pairs of pupils. Output includes also pupils' protocols. As in the previous parts it is needed to highlight masterpiece analyze a-posteriori of pupils' outputs. 2 of sources are works of author.

The last fifth chapter is most brief. It is actually summary of realized experiments and findings. They are formulated in accordance with declaratived aims of thesis and despite of shortness they are significant conclusion of all done experiments. Bibliography of this chapter consists of 12 works, 2 of them are author's.

The subject of thesis is not only attractive, but markedly actual as well. Choices techniques of elaboration, used approaches and styles of analyze (quantitative analyze, implicative graphs, hierarchic diagrams and diagrams of similarity, factor analyze as well) speak about high level of theoretical and application readiness of author to scientific work. Above standard level of technical elaboration is increased by brilliant (facty, comprehensible, transparent) style of thesis, high language culture and really impeccable format. These all support its reading attraction and inspiration. In the viewpoint of conceptual approach of submitted thesis I do not have any restrictions and I state, that she has fulfill the specified aim.

I state as well that the work illustrates typical approaches of students of various age groups by creation strategies of solutions of mathematical problems. Formulated collusions are unique (new). They confirm the need and importance of didactic experiment to better knowing metacognitive processes, which are decisive for successful teaching of mathematics and so increasing the products of pupils. Mentioned I consider as best asset of dissertation for further development of theory of teaching mathematics.

Summary:

On the ground of arbitration I consider the submitted thesis as asset to the theory of teaching mathematics and at the same time as stimulus to further development. By its quality it highly satisfy

requirements that are set by regulation about doctor study number 131/1997 of Ministry of Education, Slovak Republic. Therefore I suggest to

admit the work of Elsa del Pilar Malisani and after its successful defense to grant her science-academic title PhD.

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