Humanitarian role of mathematics in training of a teacher

V.V. Afanasiev, E.I. Smirnov.

When a pupil of a primary school in France was asked "what is the sum of 2 and 3", he answered: "it is the sum of 3 and 2, because adding is a commutative. He didn't know what is the result of the addition, and even couldn't understand what he was asked about. (V.I. Arnold. On teaching of mathematics.)

The changes in the structure of a higher pedagogic education of Russia, emergence of secondary schools of various type: lycees, grammar schools, technical colleges and so on, democratisation of public life are based on a radical turn to humanitarian positions of the contemporary education. The main tendency in development of education is existence in the 21st century of such a teacher who is capable and willing to provide a an individual with a possibility to get the education of a required level and profundity at any period of the individual's life. The modern stage of development of the secondary education puts forward great demands to professional training of a teacher (in particular in respect of a specific subject), who is armed with the latest methods and technologies of teaching, being a creative driving force of an educational process.

One of the key aims of a pedagogical training of a mathematics teacher of a secondary school is transformation of the student and his personality into a professional teacher, capable of solving the diversity of the problems, connected with training and upbringing of schoolchildren. Improvement of professional training of a mathematics teacher requires not only more effective ways of organising the process of training and upbringing in the teacher's training college, but an overview of the structure and content of mathematics schooling of students in order to put it on a technological level of learning.

This tendency in a considerable degree influenced continuity of the content of mathematics education at the secondary and higher levels as well as a scientific approach to development of theory, concepts and methods of mathematics teaching. Individualisation of teaching, differential approach, utilisation of the latest research in psychology, and human physiology, pedagogics in order to improve the process of teaching, search for new optimal conditions, aimed at learning complicated mathematics content demand from a teacher in addition to competency in his subject an ability for self-education and creative activity. The identity of a teacher and requirement of his profession are a prerequisite for this.

Since nowadays mathematics methods are more widely used in natural science, engineering and interfacing sciences, which are reflected in the changing programs of school and university mathematics education, it has become rather important to use psycho-physiological mechanisms of integral perception of information by a pupil, development of his mathematics abilities, thinking and culture.

The society's requirements in mathematics training of its citizens has changed considerably within the last ten years. The theory of games and artificial intelligence, stochastics and theory of information are becoming more available for an average researcher due to logic of development of sciences which are more significant in practical application, but nevertheless are not represented in mathematics education of a schoolchild.

On the other hand, namely this new knowledge serves as a powerful motivation drive in learning mathematics disciplines, and as a result increase of interests to the teaching profession due to the fact that mathematics education suits best development of qualities of thinking, development of theoretical thinking (comparison, heuristics, analogy, intuition, analysis, synthesis, etc…). Mathematics thinking is characterised by a logic scheme of reasoning, argumentation, laconicism, clear distribution of the course of thinking, ability to pinpoint the essentials, analysis, synthesis. Not by accident a famous mathematician and teacher A.Y. Khinchin was of the opinion that high level of mathematics thinking is an indispensable element of the overall culture of a person [3].

Mathematics, as a branch of science, belongs to the natural-scientific cycle, its subject being methodology and language of other disciplines, which represents polysemically the existing reality. Thus mathematics occupies an unique place even among the disciplines of the natural-scientific cycle.

Mathematics not only facilitates appearance of new knowledge of nature, society and man, but discovers in related sciences real stimuli for its development. Thus, development of theory of the locally convex space in functional analysis was stimulated by physical problems of quantum electrodynamics and the tasks of finding generalised solution of equations of mathematical physics, the theory of unlimited operators in banach space – problems of quantum mechanics, tensor analysis – problems of mechanics of

elastic mediums, theory of function of many complex variables – problems of the quantum field theory, etc…

On the other hand in mathematics itself within the last decades came to existence some branches, which have relatively independent subject and specific methods of research: artificial intelligence theory of mass operation, theory of random processes and functional analysis, theory of games and mathematical programming, algebraic geometry and multiple-theoretic topology as well as others.

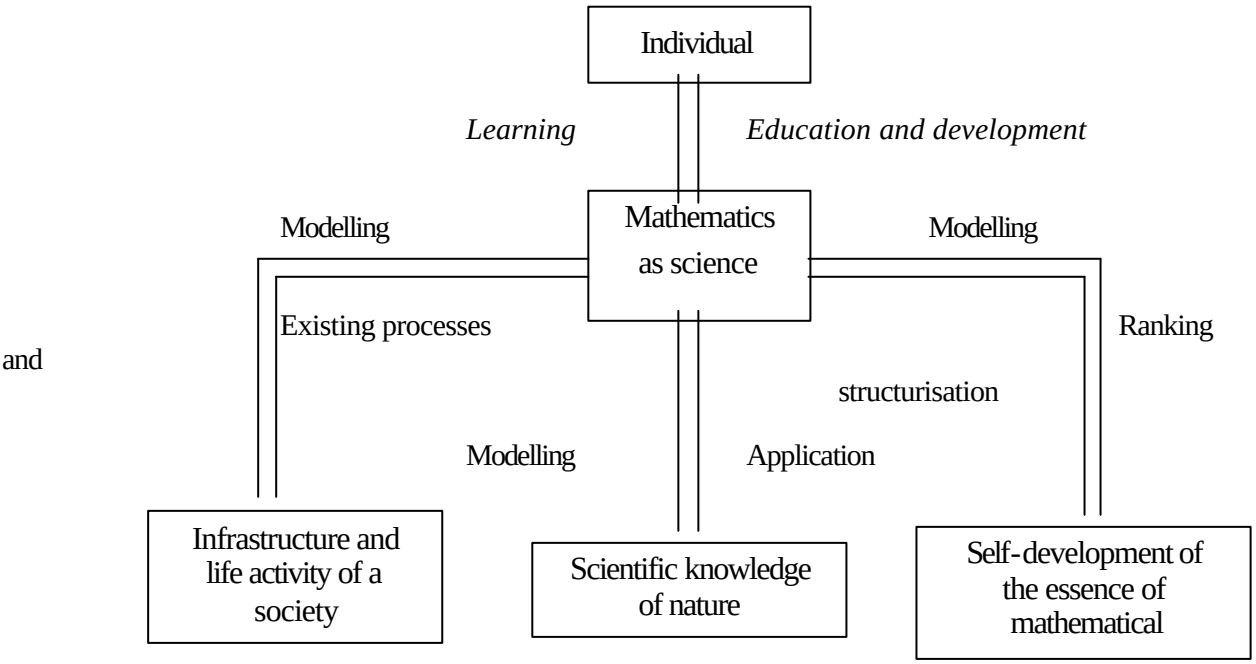
That is why due to a more pronounced tendency in fundamentally of mathematics, knowledge is connected with wide utilisation of mathematics methods in other sciences (inclusive of the humanities), part of which directly influence vital functions and socialisation of an individual in the modern world.

The body of mathematics is intended, in addition, to description of integral systems, functioning in the real world; it describes their structure, dynamics, statistics and integral characteristics. Profound correlations, expressed in a mathematical model of an integer, are described by the functional analysis and the theory of automations, algebra and the theory of random processes, statistical and probability methods. At the same time mathematical concepts, theorems, algorithms, demonstrations and so on being mathematical objects of teaching of mathematics, must acquire properties and characteristics of integrity as foundations for storage, processing and transfer of information to the new generation.

Within the last decade mathematics as a pedagogic objective is subjected to an unprecedented pressure caused by the public in connection with content of teaching as well as methods of teaching. The fact is that the depth of its formalisation even in its standard applications and mechanism of inner structure of mathematics conflict with ontogeny of development and with socialisation of an individual as well as requirements of a society in provision of its vital function. That is why teaching of mathematics both in secondary school and the Higher School must be revised in direction of a wider visualisation, visual modelling and explanation of a social status of mathematics.

It is worth noticing that modelling as the highest form of alphanumeric and symbolic activity that leads to discovery of new knowledge about nature and technological processes in engineering, law of social development and mechanism of thinking, perception and a man's memory is the key mean that facilitates development of new formations.

Recently, the role of mathematics as a mean or humanisation and socialisation of an individual's education in the modern world, as an indispensable attribute of the education paradigm of an individual of the 21st century is becoming more important.



Drawing 1. An individual and mathematics

And what is more important, mathematics as an educational subject is treated more and more as a humanitarian (of general culture) subject rather than part of a natural science. Productivity of thinking and perception, development of objective speech, logical value of argumentation, development of mental capabilities can tangible results of mathematical education on condition that it is sensibly organised.

Thus social and cultural role of mathematics can be represented in the following way (see drawing 1).

Within the last decades the whole world took great delight in bathing in the sea of informational technologies in education: multimedia, remote education, telecommunication, graphic calculators and so on. At the representative international forum on problems of mathematical education in Greece (Samos, 1998), the majority of reports, announcements and "round tables", in one way or another touched upon questions of introducing informational technologies into the process of learning. Hundreds of universities all over the world (for instance American Distance Education Consortium, comprising 55 universities) carry on informational exchange of educational programmes vie Internet, teaching various specialists on the basis of remote education): according to the latest data the number of such students reached hundred thousands. But in connection with this it is necessary to underline possibilities of professional training of a teacher: informational technologies as a mean of training – yes, informational technologies as cross-linking factor of the pedagogical system – yes, informational technologies as paradigm in training of a teacher, an alternative to the personality of a teacher – no (at least on the current stage of development of means of communication and exchange of information).

The following argument will support the last thesis:

-"uncontrolled formation of methods of thought activity". Exactly this factor led to unsatisfactory results of realisation of the ideas of programme education (E. Thorndike, B. Skinner, N. Crowder and others) in the 60-70th of the 20th century. The failure was due to a groundless transfer of principles used in training animals on the process of teaching of a human being, which possesses specific particularities. N.F. Talizina explains the reasons for failure of Skinner's approach by a lack of correspondence of the psychological theory, confirming utilisation of functional theory of learning with receiving qualitatively new results, which prove that it is possible to form rational methods of man's thinking. But in this case the principles of programming of a learning process with a real interaction of a teacher and a pupil are meant [4];

-"lack of real (non interactive) interaction of a teacher with his pupils, between the pupils", which has possibilities for activation of directed and interdependent polyfunctional factors of adequate perception of new information: perceptional, mnemonical, emotional, volitional, and so on.

-"irregularity of integrity of interiorization" of the visual-logical row of perceptional images of new information due to artificial limitations of perception field and the dynamics of handling the repertory of short-term and long-term memory.

The above mentioned deals with questions of effectiveness of remote and conventional education, and what more in the field of professional and subject training of a mathematics teacher; naturally, longer terms for remote education as well as creation of specific didactical methods and improvement of communication means are capable to make up for the shortcomings enumerated above.

At the same time, treating mathematics as a pedagogical task, one has to face the problems of an adequate presentation, differentiation, formation, stability of perception and reproduction of mathematics knowledge and revelation of the specific particularities of the phenomenon of mathematical thought in all three hypostasises of mathematics (see drawing). The new situation in education, that came to life within the last decade, being in favour of real steps aimed at increase of interest to mathematics, inclusive of the pedagogical task and an effective way of development of a pupil's intellect, makes it the more important. All this was facilitated by the following factors:

- deep concern of pupils, parents, teachers with content of mathematics education and its influence on development of personality;
- democratisation and humanisation of educational processes at school, university, bringing to the forefront problems of personal development of schoolchildren, especially in the period of ontogenetic new formation in thinking;
- involvement wider range of information facilities for provision of educational process: computer classrooms, Internet, services software, multimedia, remote education, etc…

- intensive development of methodological foundation for provision of pedagogical processes: psychology and physiology of a human being, artificial intelligence, engineering psychology and psychology of individual and joint activity, theory of management, theory of educational systems, and so on.

How can one tell a pupil that Fermat's last theorem (the problem over which the best scientists had been racking their brains for 300 year) was proven by A. Wilse in 1995, and trisection of an angle and squaring the circle can be done with the help of a ruler and a pair of compasses? How can thought processes of a pupil (logic, analysis, synthesis, generalisation, concretisation, analogies and others) be most efficiently developed in the process of teaching mathematics, which objectively must be the most powerful developing facility (and is not at present)? How to reflect in the process of teaching mathematics its role in the life of a society and development of other sciences, including such activity as space flights and safety of air transportation? How to bring it to a pupil that physics – a powerful component of life and outlook, without knowledge and utilisation of mathematics is nothing but observation and experiment, and psychology without statistics methods of processing and analysis of experimental data and modelling of psychological processes is a tendency to external phenomenology and empiricism, without revealing the internal substantive mechanisms of psychological processes?

All these questions are only part of the crucial but far from being solved state of separate problems of the school mathematics, that exist in Russian as well as foreign systems of education.

Analysis of the educational systems of the school mathematics makes it possible to refer to the following most acute contradiction:

- between integrity of mathematics as a system of scientific knowledge and the way it is represented in school curriculum and programmes on separate subjects: algebra, geometry, elements of analysis, stochastics, and others;
- between significance and role of mathematics in the life of the society, development of science and engineering and the way this function of mathematics is represented in the process of formation of motivational and emotional-volitional sphere of learning;
- between the essence of the knowledge formed in the process of teaching mathematics, skills and abilities, mathematical methods and procedures and their real formalised display in the pedagogical process.
- between objective and intensive development of psychological processes in transitional age (12 – 16 years) and methods (facilities and technologies) of the external influence on the personality of a pupil during the educational process.

Diversity of pedagogical systems and theories of teaching mathematics creates a wide choice of world experience, which creates complicated vital problems of understanding and universilisation of advanced ideas in methods and conceptions. Interpenetration of methodologies and effective monitoring of educational systems at present doesn't meet the demands of an adequate reflection of essence and integrity of mathematical knowledge.

Average growth of indicators based on three tests: numbers, algebra, geometry (on the basis of 50 points for each test)

Country	Numbers			Algebra			Geometry			Σ		
Age	13+	14+	Growth	13+	14+	Growth	13+	14+	Growth	13+	14+	Growth
Russia	**26,5**	**31,2**	**4,7**	**19,5**	**29,7**	**10,2**	**17**	**24,5**	**7,5**	**63**	**85,4**	**22,4**
Poland	24	29,2	5,2	16,6	24,9	8,3	13,6	22,4	8,8	54,2	76,5	22,3
Singapore	33,4	34,6	1,2	23,9	30,7	6,8	18,1	26,9	8,8	75,4	92,2	16,8
GB	17,6	20,2	2,6	11,3	14,4	3,1	15,4	19,9	4,5	44,3	54,5	16,2
Germany	23,5	26,9	3,4	12,5	17,6	5,1	11,3	17,3	6	47,3	61,8	14,5
Scotland	18,2	22,1	3,9	8,8	12,7	3,9	14	18,6	4,6	41	53,4	12,4

Nevertheless, it is possible it make certain conclusions in connection with achievements of schoolchildren in different counties of the world. For example, at schools in Scotland the whole series of subject is divided into 2000 modules of three types: general, special, integrative. It should be noted that the research conducted by the authors within the framework of the Kassel-project under guidance of professor D. Bergerse (Great Britain) on the problems of schoolchildren achievements in mathematics in

various countries (including Russia), gave the following results at representative sample tests and identical tests carried out with a one year interval (the same schoolchildren).

Six school from the city of Yaroslavl took part in the research with a representative sampling of 425 schoolchildren of 6 – 8 grades. They were given a test of potential and twice (within a year) three mathematics tests (number, algebra, geometry) based on the volume of knowledge of 5^{th} – 8^{th} grade of a comprehensive school.

The results show substantial progress of Russian schoolchildren, though mainly traditional methods of teaching had been utilised. The table support the fact that Russian schoolchildren are ahead of schoolchildren of all European countries as far as an absolute indicator is concerned as well as the dynamics of growth of mathematics knowledge, although the average age of our schoolchildren was below the age of those in Europe. In fact it didn't come as a great surprise: our traditions in defining the level and volume of mathematical information for the subject as well as high level of professional skills of our teachers are well known in the world.

But these soothing figures speaking in favour of our success there are alarming tendencies, hidden from the sight of even a careful and experienced analyst. Firstly, the necessity of restructuring the content of teaching of mathematics (as well as other subjects) has become acute, in all grades from 1^{st} to 11^{th} (or 12^{th}). Till the 5^{th} grade mathematics must as much as possible facilitate socialisation and development of a personality, creating the necessary basis for the comprehensive school. In this school mathematics must be universal and unified, demonstrating its role and place in the life of society and unilisation in other sciences. It should be noted that special attention must be devoted to forming in schoolchildren of standards of calculation and algorithm, relying on mathematics. Universities maintaining their educational role must have their own specialisation, being capable of providing extensive training in various directions: humanitarians, engineering, mathematics, economy, etc…

Apparently, it will be followed by reduction of the general volume of mathematics knowledge and restructuring of the primary and secondary schools, but at the same time maintaining and enhancing their developing effect. It can be achieved only if modern theories and technologies of teaching are used and skills and level of knowledge of mathematics teachers at universities are improved.

Secondly, schoolchildren suffer from too much learning activity outside school (home assignments, extracurricular activities) – activity in class can be controlled. Some pupils spend more time over their home assignment than at his activity in the classroom. Of course, at the end it produces educational effect, but at the expense of a pupil's spare time, which could be devoted to health improvement, enjoying sociable activity, improvement of general culture.

The data as well prove that it is necessary to organise intensive exchange of advanced experience of work of various educational systems in the 21^{st} century with the aim of defining efficient methods, forms and technologies of teaching mathematics, determining optimal content of teaching and forming of mathematical culture of competent members of the world community.

At the same time our scientists and teachers in the last quarter of the 20^{th} century started to get worried with a certain drop in mathematical education in the teacher's training universities of Russia. The situation, which was pointed to by a famous German mathematician F. Klein in 1924, namely "double gap" between mathematics at school and university, has aggravate. The scientist stressed that the elementary mathematics should be taught from the point of view of the higher mathematics. The point is not only in decrease of the number of hours for teaching mathematics and in the objective situation when teacher's training universities receive students with average abilities (students with average and low IQ can in future become successful and creative teachers of mathematics), but in the quality and effectiveness of the mathematics knowledge, formation of experience of creating activity combined with outlook and life values. Unfortunately, fundamental nature of mathematics education is poorly connected with the prospective professional activity of the students.

In connection with this in order to improve professional training of a mathematics teacher it is necessary not only to introduce new, more effective ways of organising the process of teaching and upbringing in a teacher's training university, but to revise the structure and content of schooling in mathematics, optimise its fundamental and humanitarian components, raise theoretical grounding of didactic processes on a technological level.

Thus, the mathematics education being in effect at the teacher's training universities requires serious qualitative changes, which can determine a stage of its development under condition in modern Russia, entering the 21st century.

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