Some experimental observations on common sense and fuzzy logic

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1.0 COMMON SENSE AND FUZZY LOGIC

Bart Kosko's works on fuzzy thinking have drawn great attention to some features concerning our everyday behaviour in relation to our decision making ability or our ability to be in control of a situation.

The author (of the well known book: "fuzzy thinking") has highlighted that when we are facing a decision making problem, we usually try to get good solutions on the basis of global evaluations regarding the different possible problem options.

When time is a critical factor, our way of thinking, analytical as it can be, is not able to help us taking a decision quickly unless weighted "fuzzy" averages of *pro* and *contro* are used (*Kosko, 1993*).

Keeping on talking about our behaviour, we can say that there is a "rule" whenever we associate ideas and we link a thing, an event or a process to another thing, event or another process. "If-then" clauses in natural languages, as well as in computer programming ones, help us managing our reasoning.

Kosko claims that rules on which we build our reasoning are "fuzzy" rules. This claim is widely accepted. In fact a fuzzy rule makes a connection between fuzzy sets and presents an "IF X is A then Y is B" structure; such is the case, for example, of the following phrase: "if the weather is very hot, then I will put on light summer clothes"; if X = {days of the year}, A = {days with very hot weather}, Y = {clothes}, B = {light summer clothes}, we definitely get an "IF X is A then Y is B" structure. ($A\subset X$, A fuzzy set, $B\subset Y$, B fuzzy set).

Thus, despite the fact that we can have different perceptions on feeling hot or finding clothes light, still the common sense gives a common meaning to the fuzzy rule "if the weather is very hot, then I will put on light summer clothes".

2.0 FUZZY LOGIC AND COMPLEXITY

Mathematics is "the language in which is written the Book of Nature" whose characters "are triangles, circles and other geometric figures" or at least this was the position of Galileo Galilei and of many other after him.

Today, if one wants to use *fuzzy* logic alongside with *bivalent* logic must take into account the possibility of using "qualitative relations" besides traditional mathematical formulas as well. A qualitative relation deals with linguistic variables, in fact its values are words rather than numbers.

According to this sense Lofti Zadeh says that fuzzy logic is a methodology which makes possible calculus using words. But Zadeh's thought points out wide-ranging issues which involve way of reasoning, of conjecturing, of learning and therefore of teaching.

All this leads to new inference processes where premises and conclusions have different meaning from their ordinary sense, and where new terms like "fuzzify" and "defuzzify" are used, these terms mean respectively passing from a mathematical formula to a fuzzy rule and vice versa.

New fuzzy scenarios appear, but new specifications for a fuzzy science are not fixed yet. Which attitude should take a teacher? And what can a researcher do?

They can certainly wait and see what happens, but the fuzzy logic is a challenging issue which walks with another compelling contemporary issue: the complexity. Once again Nature provides a common denominator. Investigating emergent behaviour of complex system, raging from anthill to human brain, requires much more flexible tools than classical mathematical formulas. And this means that we must go beyond the classical two-valued logic. Or, as it is stated by Zadeh: *"As the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and relevance become almost mutually exclusive characteristics."*

3.0 THE HYPOTHESIS

Fuzzy logic is paying off in the marketplace, where there is a growing interest for products which are based on fuzzy logic as their underlying logic (usually for control systems, which are called "fuzzy expert systems"), the Japanese manufacturers were the first to embrace this technology and have been building real products around it since the early nineties.

The fuzzy logic applications in reality are now innumerable and they range from control system for small domestic appliances or electronic devices (on washing machines or camcorders, for example Matsushita, Hitatchi products) to optimized planning of industrial control applications (Kawasaki Steel industry), from efficient and stable control of car-engines (Nissan) to controlling of subway systems (Sendai city subway system). These successful applications have convinced almost everyone, not only the Japanese engineers, that fuzzy systems actually "work".

To date, basically the engineering research in this field is limited to fuzzy expert systems in which the main problem is to find good fuzzy rules. The FAT theorem demonstrates that it is possible to model any system using fuzzy rules; from a mathematical point of view, this means that a fuzzy system based on a finite number of rules can approximate, in an uniform way, any continuous system.

The point is that, in most cases, finding good rules in order to model a real system effectively is not that easy. To solve this problem new adaptive fuzzy systems have been recently developed: the basic idea is to build fuzzy systems that are able to model their own rules learning how to do it directly from data of the problem they must solve.

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Lots of techniques have being developed to carry out such a task, and they usually involve back-propagation neural networks for determining the most appropriate rules to use, and genetic algorithms for defining the type and the shape of the membership functions. Both techniques are based on adaptive principles and are able to deal with raw data as a starting point to build their own "knowledge" of the problem.

Such an adaptive fuzzy system acts like a human expert: it is able to learn from experience and to use new data to perfect its own knowledge and ability domain. Adaptive fuzzy system developers and researchers have taken their inspiration form neural network of our brains in order to build neuron-fuzzy systems that are able to learn fuzzy rules, thus adapting their own dynamical structures to the problem.

All these things concern learning, and therefore they could be of great interest for a teacher dealing with scholastic learning issues. **Thus, one wonder whether spontaneous inferences used by an individual who argues his own first approach to learning something by actually "doing" something are fuzzy rules**.

Control problems provide useful and simple examples of fuzzy logic applications in reality. Their main characteristic is readily recognized: from the input values it is necessary to calculate the output values that can assure a satisfactory behaviour of the system. That translated in fuzzy term sounds like: IF present conditions, THEN do a particular action.

The proposed activity has been introduced to a 28 pupil class of the second year of a "liceo scientifico" as a computer science activity, and it deals with a classical control problem better kwon as the "inverted pendulum". Of course pupils whose age ranges from 14 to 16 have not got yet the proper tools to model the problem by means of mathematical formulas in order to solve it. What has been investigated, however, is their first approach to the problem by using the natural language with nobody there to guide them, but they are only given the assignment instructions.

Anyway, the class have already tackled, in the previous scholastic year, an algorithm unit and the first elements of Pascal programming language. Such activities are part of the planning that proposes to open again the computer science education issue that was interrupted.

4.0 THE ASSIGNEMENT A computer science activity (time: 3 hours)

Phase I.

The class pupils are divided up into groups, each containing just two pupils; besides each group is provided with a pole, a sheet of paper and a pen. (time: one hour)

1. ***A*** tries to balance the pole on his hand while ***B*** observes and writes down on the paper the actions made by ***A*** relating them to the actual positions of the pole.

2. the couple of pupils exchange their role and repeat the previous operation.

3. ***A*** and ***B*** compare what they have written and they make a list of "rules for balancing a pole on a hand palm".

Phase II The class in now divided up into groups, each containing two of the previous groups, they have the written rules, a sheet of paper and a pen. (time: two hours)

1. each group discuss the possibility to use a properly programmed robot to carry out this balancing task jotting down any draws, symbols, variables, constraints and whatever they think it is necessary in order to do this correctly.

2. after they have discussed, accepted and jotted down the hypothetical conditions under which the robot should be able to execute the instructions, every member of the group tries and express all the ones he thinks right using the natural language (should he search for an algorithm?).

3. the group, once they have examined all the material that have been produced, express only a program for the robot.

Note: the group must give the teacher all the material produced, even if it was considered not useful.

4.1 A PRIORI ANALYSIS

The considered activity is clearly an "open test". To undertake an implicative analysis of the variables, assumable behaviours must be taken into consideration as well as appropriate and significant variables that must be consistent with the assumed hypothesis. As far some possible behaviours are concerned, we thought it right to consider expressions such as "the pupil does…", whereas for other cases it was thought that expressions such as "the pupil does not do a certain thing" would be more revealing.

Besides it was considered to be necessary to determine the reference words of the expected answers for the two phases, in more precise terms.

Phase I Point 3: ***making a list (one to one activity) of rules for balancing a pole***

Variable	**Description**
ra1	The pupil uses a table as a representation tool
ra2	He only tells between equilibrium and non equilibrium positions
ra3	He does not use more than four instructions
ra4	He does not consider numerical variables

Phase II Point 1 and 2: ***expressing the robot potentialities and the initial conditions***

Variable	**Description**
rb1	The pupil draws a robot that looks like a human being
rb2	He thinks that the robot is able to recognize the equilibrium state
rb3	He thinks that the robot is able to move
rb4	He thinks that seeing is one of the robot capabilities
rb5	He thinks that the robot is able to feel (tactile perception)
rb6	He specifies the way the robot receives instructions

rb7	He thinks that the pole length is a possible variable that influences balance
rb8	He does not specify whether the robot is able to make numerical calculations

Phase II Point 3: ***expressing the robot program***

Variable	**Descripion**
rc1	The pupil does not use a mathematical model, but he uses natural language instead
rc2	He does not assign numerical values to the state variables he is considering
rc3	He uses "IF-THEN" implications
rc4	He does not use instructions in sequence
rc5	He does not use instructions that require numerical calculations
rc6	He uses fuzzy sets in the instructions
rc7	He takes into consideration the angle between the pendulum and its equilibrium position
rc8	He takes into consideration the movement speed in order to balance the pole
rc9	He doe not consider the angle and/or the movement speed

How the variables are chosen The basic issue we are investigating is whether the pupils, under the proposed situation, are using fuzzy rules. So, the expected results concern the implications between the following variables: rc1, rc2, rc3, rc4, rc5 and rc6. The occurrences of such variables will be substantially weighted, and so will be the case with the ones of the first group ra1, ra2, ra3 and ra4. Any possible "strong implication" between the three groups must be interpreted in order to establish whether it is revealing about the way of reasoning.

4.2 IMPLICATIVE ANALYSIS USING THE CHIC PROGRAM

The implicative graph containing all the items shows that the strongest implications (98) are **rc4 ® rb5** and **rb6 ® rb5**, thus those who do not use instructions in sequence and those who do not specify the way the robot receives instructions consider the robot capable of tactile perception. On the other hand the implication hierarchical tree shows very clearly the high level of implication between rc4 and rb5, but the strong implication (coloured in red) goes from rb6 to rc4 →rb5 and keeping on analysing the graph: all this block, still red coloured, implies rc5 (those who do not use mathematical models). **It seems that giving up the "in sequence" structure of instructions one also gives up any mathematical model related to strictly numerical values, and is willing to take into consideration other qualitative aspects of the problem (such as the case of tactile perception capability of the robot)**.

Another strong (95) and extremely relevant implication is **rc3 ® rc6** i.e.: **those who use "if-then" instructions in association with fuzzy sets, formulating, in this way, actual fuzzy rules**.

Besides, let us look at the occurrence values of **rc3** and **rc6** variables, **70%** and **85%** respectively, this means that we are referring to a vast majority of individuals involved in our analysis. In the similarity tree, if we look at the block in which there are rc3 and rc6 (related by means of similarity to rc9, ra4, rc5, 5b8, rc1, ra3) we can deduce **that it is possible to put together those who use fuzzy rules with those who do not bind themselves to standard measures of numerical kind, or numerical calculations, or standardized mathematical models, but use only a limited number of instructions on the whole**.

The implications **rb5 ® rb8**, **rc2 ® rb8**, **rc5 ® rb8** indicate that those who use numerical variables and numerical calculations did not specified whether the robot was able to do numerical calculations, neither did those who consider the robot capable of tactile perception. **Considering the robot able of tactile perception replaces its ability of doing numerical calculations**.

4.3 FROM THE PUPIL RECORDS

1. Some instructions given in the first or in the second phase :

- The pole is in motion, swinging for a few centimetres only; in this case you have to compensate the pole's movement by only moving the hand in the same pole's direction. But if the pole is swinging more rapidly, then you have to compensate the pole's movement by moving your arm, and if necessary, even your body (in the first phase).
- If the pole is in motion at low velocity, then compensate its movement by moving your hand at low velocity (in the first phase
- If you want to get more movement agility of your arm, then put the pole at a medium height (in the first phase).
- If the pole end leans forward, then increase the angle formed between your arm and forearm and at the same time increase the inclination of your wrist forward (in the second phase).
- To balance the pole do very slight, medium or fast movements, but make sure that they are always coordinated (in the second phase).

2. Some of the pupil observations on the robot potentialities:

- The robot activates the given instructions as soon its tactile sensor, placed on the platform where the pole stands, detects that the pole is loosing its equilibrium position;
- The robot has the ability to recognize the equilibrium position by means of its visual and tactile perceptions;
- Thanks to its visual sensors the robot is able to recognize when the pole is standing in its upright position, perpendicular to its hand palm (or should we say platform?).

4.4 A POSSIBLE INTERPRETATION KEY

The previous results can be more revealing if compared with the Edward De Bono's work on relations between creativity and lateral thinking.

The following table shows synthetically the analogies and the differences between the two different way of thinking that are usually indicated as *linear* and *parallel*.

PENSIERO VERTICALE **LINEAR THINKING**	**PENSIERO LATERALE** **PARALLEL THINKING**
• It is an intentional process • It is a mental attitude • It can be learned, put in practice, used	
• It is selective	• It is productive
• It chooses a path excluding the others	• It does not select a path but tries to open new ones instead
• It selects (or it searches for?) the best point of view	• It produces alternative approaches within the feasible solution space.
• It activates only if there is a clear direction indicating where to go	• It activates in order to produce a direction where to go
• It is analytical	• It is stimulating
• It is consequential	• It can jump from a place to another
• When it moves step by step, each step must be clearly justified	• It can move freely without taking into consideration any contradictions, provided that the final conclusions are right
• It uses negation in order to prevent from following some paths	• It does not use negation, thus it is possible to follow a wrong path
• It explore the most probable paths	• It explores the least probable paths
• It is expected to find an answer	• There could be an answer, but there could be many others too, besides there could be partial solutions.

The pupils that have carried out the activity have clearly used both thinking methods, but usally preferring the second one. The records, in which it is possible to see the abandoned solutions as well as the ironic comments on the robot look and all the attempts to give a solution at any cost, even using absurd instructions, present different characteristics that are readily traced back to one of the two afore mentioned way of thinking.

But it must be highlighted that the activity **requires pupil's absolute freedom of using non conventional schemes, which are usually used during the normal scholastic activities instead**.

Usually a pupil, during any scholastic activity, tends to follow the linear way of thinking rather than the lateral one, and that is due also to an "expected" logic capability that it is recognized and appreciated only if its way of reasoning was based on the linear thinking requirements, while it is usually misunderstood and not enough appreciated if it was based on the lateral thinking ones instead.

We can assume that besides the well known relation: **Linear thinking « ordinary logic (the bivalent one)**

there is also another one of the following kind: **Parallel thinking « fuzzy logic and fuzzy rules**

5.0 CONCLUSIONS

Pupils, during the proposed activity, did not use their mathematical and computer science knowledge and freely used natural language to express their models, in this way they ended up using actual fuzzy rules to describe the control instructions for some expected actions. The lack of sequential schemes has also encouraged fuzzy thinking and creativity. This, in a nutshell, was the main result of the experience. But it is necessary to comment further this experience in order to reveal its own specific characteristics:

- The way the activity was carried out can be easily lead back to the so-called "a-didactic" Brousoau's situations rather than to any conventional didactical approach (drills and class tests and so on). The pupil can talk with the class mates and can express his own opinion as well as face the proposed problem using the approach he thinks it is the best. He knows that will not be evaluated the process and the method he uses to get the solution, but what really matters is a convenient solution to the problem is found.
- There are no objectives referring to "logic capabilities", and everyone knows the programmatic choices that were made. It was created an atmosphere in which everyone had the feeling of being free to use whatever kind of thinking he liked, linear or parallel.
- In this work, speaking with the class about the fundamental choices, it was preferred to use terms such as "different mathematics" rather than simply "mathematics" also it was abandoned the idea of knowing the "reality" in favour of knowing the "realities".
- And finally, uncertainty, indetermination, incompleteness are terms that get to be familiar to pupils since their first year of school, facing, for example, games and problems which have no solutions or more than one, or situations that require a probabilistic approach, or open tests and so on, all situations that can easily happen during laboratory activities that are integrated in the ordinary scholastic work.

5.1 OPEN PROBLEMS

- Investigate the implications that reasoning using fuzzy rules can have on the possibility of **formulating conjectures (is it possible to speak of fuzzy conjecturing?)**
- Investigate the pupil knowledge of “**precision** and **significance**” used in their reasoning processes
- Investigate how the students use **quantitative and qualitative relations** in their formulating conjecture processes
- Investigate the possible implications that the habit of using “**fuzzy enquires**”, in natural language, can have on their learning approach.

APPENDIX

Bivalent logic and fuzzy logic:

The **bivalent logic** is what in the Western thinking is called *logic* par excellence, this kind of logic goes back to Aristotelian times and it is based upon the concept of true (value 1) or false (value 0) clauses. Within this logic is valid the Non Contradiction Principle (it is not possible that the same clause is true and false at the same time) and the Law of the Excluded Middle (a clause can only be true or false, there is not a third possibility). Upon this logic it is based the formal demonstration process.

The term **fuzzy logic** has two meanings. The first: it indicates a multi-valent logic in which the concept of truth of a certain clause can vary in a continuous way from values between completely false and completely true [0,1], introducing in this way the concept of “partial” truth. In this context the Law of the Excluded Middle is not valid at 100% and so is case for the Non Contradiction Principle, (there is no contradiction between **A** and **not A**, but only *degrees* of contradiction). This logic goes back to the beginning of the last century.

The second meaning was created by Lofti Zadeh in the sixties, and it **indicates the reasoning (and the calculus) using fuzzy sets and fuzzy rules**.

Fuzzy rules:

Conditional relations of the following kind: **IF X is A, THEN Y is B, where A and B are fuzzy sets** are called fuzzy rules. *"if there is poor visibility, then car headlights must illuminate farther".*

Every rule determines a “fuzzy patch” (AXB product) within the *state space* – the set of all possible input and output combinations - .

The larger the A and B sets, the more uncertain the fuzzy patch; on the contrary, more certain knowledge implies smaller fuzzy patches, i.e. more precise rules. From a mathematical point of view every fuzzy rule just operates as an associative memory that links the fuzzy B output to the input A, besides inputs similar to A activate outputs similar to B.

Fuzzy system:

A fuzzy rule based system that associates input to outputs is called fuzzy system. Every input activates all the rules of the system with different degrees of intensity, just like in an associative memory. The more precise the correspondence to the “if part” of the fuzzy rule, the more activated will be the related “then part” of it. The fuzzy system sums all these “then parts” up determining a weighted average (usually called “centroid”, also you have to considered that several method as to determine the output of the system exist), such a value is the output of the system.

The adaptive systems are able to learn their own rules directly from the problem data, such an adaptive fuzzy system acts like a human expert: it is able to learn from experience and to use new data to perfect its own knowledge and ability domain.

FAT theorem:

It is the **Fuzzy Approximation Theorem** that demonstrates that a fuzzy system based on a finite number of rules can approximate any continuous system in an uniform way (Borel – measurable)

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