

## **Pre-service Mathematics Teachers Conducting Research to Enhance Their Knowledge of Children's Thinking**

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**Abstract:** This paper examined the effect of having direct experience with children through researching their thinking on pre-service mathematics teachers' knowledge and beliefs about children's thinking. Also examined were the ways pre-service teachers think about teaching children mathematics. The sample of this study consisted of 21 pre-service teachers enrolled in a mathematics methods course. Subjects conducted research on children's mathematical thinking and reflected on their experience through reports and interviews. The results revealed that the research projects deepened students' understanding of children's thinking, motivated them to do more research on children's thinking, and created a need for them to know more about children's thinking through reading. Also, students' ways of thinking about teaching children mathematics were changed.

### **Introduction**

It is essential for mathematics teachers to have strong mastery of mathematics content, mathematics pedagogy, and knowledge of children's mathematical thinking. Methods courses usually focus on pedagogy leaving the issue of children's thinking to educational psychology courses. In this study, students were given the opportunity – through a methods course- to examine children's thinking by conducting research on children.

Many researchers argue that there is a significant relationship between teachers' knowledge of mathematics, their conceptions of teaching, and their understandings of students' learning of mathematics (Fennema & Franke, 1992; Knapp & Peterson, 1995). Reforming math (teaching) ... at its heart is a problem of (teachers') learning. (And one of the critical things they must learn is) knowledge of children and their mathematics (which) is crucial to teaching for understanding. (Ball, 1994, p. 1).

Mathematics instruction should be changed so that students will be able to learn mathematics with understanding by actively participating in tasks that incorporate important mathematics (National Council of Teachers of Mathematics (NCTM), 1989, 1991, 2000). Teaching for understanding requires that teachers learn about children and their mathematics. This learning is critical if teaching mathematics is to be reformed (Ball, 1994, p. 1).

Researchers assert that deep change in teaching practice may be initiated by changes in teachers' knowledge (Clark & Peterson, 1986; Fennema & Franke, 1992; Putnam, Lampert, & Peterson, 1990). And there is growing evidence that teachers' knowledge of children's thinking has a powerful influence on them as they consider instructional change.

Children's knowledge and the teacher's understanding of that knowledge are central to instructional decision making. Teachers plan instruction using research-based knowledge about children's mathematical thinking (Carpenter & Fennema, 1991; Carpenter & Moser, 1983). Teachers seek specific information about individual students' thinking and understanding and then adjust the level of content to match individual students' performance levels.

Several studies were reported from the Purdue Problem-Centered Mathematics Project in which instructional activities were designed to offer children opportunities to construct their own mathematical knowledge (e.g. Cobb, Wood, & Yackel, 1990; Cobb et al., 1991). Teachers were given the opportunity to participate in workshop activities that guided their reflection about their instruction and students' thinking. Teachers were encouraged to ask their students to describe their thought processes as they engaged in the instructional activities provided. The researchers related the changes in instructional practices to what teachers learned about students' thinking.

Fosnot, Schifter, and Simon (Simon & Schifter, 1991; Schifter & Fosnot, 1993; Simon, 1995) involved teachers in workshops that reflected the constructivist pedagogy. The purpose of these workshops was to help teachers develop mathematical knowledge and pedagogy. As the teachers learned new mathematical ideas, their understanding of children's thinking increased.

In this study, pre-service teachers were given the opportunity to have direct experience with children through conducting research on them to analyze their thinking. The study aimed at answering two research questions: first, does conducting research on children's mathematical thinking affect pre-service teachers' knowledge and beliefs about children's thinking ?, and second, does conducting research on children's mathematical thinking affect pre-service teachers' beliefs about teaching children?

## **Method**

The sample for this study consisted of 21 students enrolled in a methods course. They were encouraged to conduct research by giving them extra credit on doing it. The instructor prepared a list of issues to be studied, and students chose among them according to their interests. Five groups of students were formed to work on five different research projects. Students were guided to certain readings related to their projects, and asked to design studies that address their issues. The instructor met with each group to discuss their plans and refine it. Groups constructed tests, or conducted interviews with children.

Students were given about 80 days to complete their projects. This included writing reports about what they did, the results they obtained, and a separate reflective journal about what they benefited from their projects. In the reflective journal they were asked to reflect about what they learned and to compare that with their knowledge before doing the project. Also they were asked to write about any change in their beliefs about teaching children mathematics.

The instructor interviewed the groups and discussed with them their reports and reflective journals to obtain a clearer picture about students' gain out of the research projects. Groups presented their projects to the whole class and gave copies of their projects to other students.

## **The Projects**

### **Counting Principles:**

This project aimed at exploring children's understanding of "how to count" principles (Gilman and Gallistel, 1978), namely:

- a. The stable order principle
- b. The one-to-one principle
- c. The cardinality principle

In this project, kindergarten children were asked to count objects and asked about the number of objects they counted. Researchers videotaped children and analyzed their counting and answers. The child met the stable order principle if he/she orally counted correctly (in a stable order). The child met the one-to-one principle if he/she assigned one number to each object. The child met the cardinality principle if he/she knew that the number of objects was the last number he/she mentioned in the counting sequence.

### **Counting errors of kindergarten children:**

This project aimed at exploring error pattern of children's counting in Arabic. Children were asked to count up to as high as possible. When they made an error, children were corrected and continued to count until help was not useful.

### **Multi-digit Multiplication Informal Strategies:**

This project investigated children's informal strategies in solving Multi-digit multiplication. Children –who have not learned the formal procedure–were given multi-digit multiplication problems and asked to solve them the way they wanted. The problems were given in word form. Children were encouraged to ask for explanation if they did not understand the problem.

### **Multi-digit addition Informal Strategies:**

This project investigated children's informal strategies in solving Multi-digit addition. Children –who have not learned the formal procedure–were given multi-digit addition problems and asked to solve them the way they wanted. The problems were given in word form. Children were encouraged to ask for explanation if they did not understand the problem.

### **The effect of language on children's problem solving ability:**

This project aimed at exploring the effect of Arabic language on children's mathematics problem solving ability. In this project, 40 fifth graders were randomly divided into two groups. Both groups were tested on a word problem test. One group did not receive any help in explaining the problems. The other group was encouraged to ask for help if a problem was not understood because of language. Only language related questions were accepted.

## Results and Discussion

Of the 21 participants, 3 did not submit reports and did not attend the interviews. So, 18 reports and 5 interviews (with 18 participants) were analyzed. In this section I present the results as answers to the research questions.

**First:** Does conducting research on children's mathematical thinking affect pre-service teachers' knowledge and beliefs about children's thinking ?

The analyses of the reports and interviews answered this question positively. All 18 participants reported/said that their knowledge about children thinking was enhanced, and their beliefs were changed. Following are some examples of participants' responses in the reports and interviews: Fatima from the "counting principles" project said<sup>1</sup>:

*Before this project, I did not even know if there were counting principles. But even if I did, I would not know that a child who could count orally could not apply the one-to-one principle. Also, the cardinality principle, it was so strange to me. It was surprising that some children could not determine the number of objects after a second from counting them.*

Samia, from the "Multi-digit Multiplication Informal Strategies" project reported:

*I never imagined that a child could invent his own strategy to solve a multi-digit multiplication problem. I always thought that solving such problems can be done only after the formal procedure is taught. Most of the children I worked with did very good jobs. Their ways of solving the problems were interesting. If I did not see this [children's work] myself, I would not believe that children can do this. As a matter of fact, I used to believe that children only memorize mathematics.*

Moza, from the effect of language on children's problem solving ability project, stated:

*Now I know why children hate word problems. It is not that they don't know mathematics. Actually most of them are smart. The language is a big obstacle. Now, I really think about it ... if I don't know what the question is about, how can I answer it? Some students answered the questions without help with language. These seem to be good on language. But in the second group, the one that received help in language, much more students solved the problems. Again this is not because mathematics itself is complicated, but not understanding the language of the questions is a problem.*

**Second:** does conducting research on children's mathematical thinking affect pre-service teachers' beliefs about teaching children mathematics?

The analyses of the reports and interviews answered this question positively. All 18 participants reported/said that their beliefs about teaching children mathematics were changed. Following are some examples of participants' responses in the reports and interviews:

Samar, from the Multi-digit addition Informal Strategies project, reported:

*Before conducting this research project, I was thinking of teaching my students the best way to do mathematics problems. Yes, I was thinking of using the best instructional aids, and use good explanation methods. All of this we learned in the college. But now, I learned new things. My students are not empty. They have their own ways of doing mathematics. I have to take this into consideration when I become a teacher of mathematics. Actually this might make my job easier. If students are given the opportunity to use their own strategies in solving mathematics problems, learning the formal ways –which is my responsibility- will be easier and faster to them.*

In the interview, this same student said:

*In my experience, teachers usually are not so open to their students. I mean, because of the time constraints and may be other reasons, I don't know, do not consider knowing what students might be able to do without instruction. That is not part of teachers' thinking. The only thing teachers care about is finishing the textbook. They want the students to follow the book procedures and get good grades on the test. I think it is the responsibility of the teacher to enable her students get good grades .... As for me, after this experience, I think I will be different. I discovered that students are so smart. They are not "stupid" in math as I used to think. When I teach, I mean when I become a teacher, I will give my students the opportunity to discover mathematics by themselves. I will allow them to use their own ways and strategies even if these are different from my ways. I learned from this project to respect students' ways of thinking and their ways of solving problems.*

Jamila, from the counting errors of kindergarten children project, asserted that:

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<sup>1</sup> a) names are not real, and b) quotes are translations from Arabic.

*Teaching counting should be different. The results of our project proved to me that we waste a lot of time teaching our children how to count. We should not concentrate on all numbers. Some numbers are easy to learn as we noticed from the project. Other numbers need more concentration. The children we interviewed did not make a lot of errors in 13 to 19, 21 to 29 and so on. Most of the errors were on: 1 to 9, 10, 11, 12, 20, 30, 40, and other multiples of ten. We learned that students can follow patterns in counting. The concentration should be on few numbers. And for the other numbers, we should focus on helping children discover and follow patterns.*

The results revealed that the projects deepened students' understanding of children's thinking, motivated them to do more research on children's thinking, created a need for them to know more about children's thinking through reading. Also, students' ways of thinking about teaching children mathematics were changed.

The results of this study suggest that preservice mathematics teachers should have the opportunity discover the ways of children's thinking. Telling them about these ways is not enough. It is the responsibility of the teacher education program to provide them with such opportunity. This can be done either by conducting research as it is the case in this study or by other means. The point is that those preservice teachers need to know and get convinced about what children can do and how they do it.

However, the extent to which the new knowledge and beliefs will be implemented in the real settings is another issue. This adds another responsibility to the teacher education programs. This responsibility is summarized by making good use of the student teaching experience provided to the preservice teachers. Cooperative teachers, and supervisors should be chosen such that they can enrich and assess preservice teachers' implementation of their knowledge and beliefs about students' mathematical thinking.

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