TEACHING ALGEBRA CONCEPTS IN THE EARLY GRADES

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Abstract: Building a foundation for understanding algebraic concepts is an important step in the teaching and learning process. Grounded in the work of Freudenthal, materials developed cooperatively by mathematics educators from the Freudenthal Institute and the University of Wisconsin-Madison provide an American version of "realistic mathematics." A progression of contextual problems can be used to introduce students from ages 9 to 12 to basic algebraic concepts. This development is one of progressive formalization, beginning with informal discoveries and gradually extending to a formal understanding as students mature both physically and mathematically. Generally algebra is taught as symbolic manipulation, equation solving and graphing algebraic equations. Through a contextual approach and relying on visualization as a critical element in developing understanding, young students can be introduced to concepts such as pattern recognition and investigation, symbolic representation leading to the concept of a variable, writing expressions and formulas, and making decisions based on generalized number concepts.

INTRODUCTION

Algebra can be thought of as the manipulation of symbols and is usually taught to students beginning in lower secondary school. In this paper I will give examples of problems presented through pictorial representations and in context making them approachable by young students and laying the foundation for more formal study of algebra. These problems and ideas stem from a joint project between the Freudenthal Institute in The Netherlands and the University of Wisconsin in Madison, Wisconsin, funded by the US National Science Foundation resulting in a middle grades mathematics program Mathematics in Context. The curriculum was developed in response to a call for mathematics programs that exemplified the National Council of Teachers of Mathematics vision of standards *Curriculum and Evaluations Standards (1989*). The philosophy of the program was guided by the following principles:

- Mathematics is a human activity for all.
- The real-world contexts support and motivate learning.
- Models help students learn mathematics at different levels of abstraction.
- Students reinvent significant mathematics.
- Interaction is essential for learning mathematics.
- Valuing multiple strategies is important.
- Teachers and students assume different roles.
- Students should not move quickly to the abstract.
- Mastery develops over the course of the curriculum.
- The mathematics is often new and different.

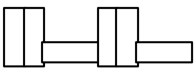
Algebra in *Mathematics in Context* emphasizes the study of the relationships between variables, the study of joint variation. Students learn how to describe relationships with a variety of representations and how to make connections among these representations. The goal is not to merely learn the structure and symbols of algebra but to also use algebra as a tool to solve problems that arise in the real world. To use algebra effectively students must be able to make reasonable choices about which algebraic representation, if any, to use in solving a problem. The curriculum—especially the algebra strand —is characterized by progressive formalization. Students initially rely heavily on their intuitive understanding of a concept and later work with it more abstractly. The realistic problem contexts support this progression from informal, intuitive understanding to a more formal, abstract understanding. Young students move back and forth between informal and formal strategies.

For example, to informally introduce students aged 11 or 12 to systems of equations, they are given a problem involving a two-pan balance (Kindt, Abels, Meyer, & Pligge, 1998). On one pan are ten bananas and on the other pan are two pineapples, and they are in balance. In a second picture the balance has one pan with 3 bananas and an apple in balance with one pineapple. A third picture shows an apple on one pan, and the second pan contains a question mark soliciting what would be necessary to make it balance.

Children ages 11 and 12 are able to answer these kind of problems intuitively by reasoning from the pictures. Some students will make a new balance by replacing the two pineapples in the first picture with six bananas and two apples. They subtract off the six bananas and have four bananas balancing two apples and conclude that one apple is the same as two bananas. Other students will make a new balance with three bananas and one apple balancing five bananas reasoning that one pineapple is equivalent to five bananas. They subtract off three bananas and have two bananas balancing one apple.

Students are solving a system of equations by substitution without being formally taught that process, manipulating pictures of real objects. The groundwork is being established for an abstract approach, but this takes place gradually over a period of time. Students can build confidence by working such problems, and later the abstraction becomes less difficult and is more apt to make sense to the learner. By the end of this set of lessons, students are working with combinations of variables. Students return to the solution of systems of conditions in three other units over the next two years: *Decision Making* (Roodhardt, Middleton, Burrill, & Simon, 1998); *Graphing Equations (*Kindt, Wijers, Spence, Brinker, Pligge, & Burrill, 1998); and *Growth* (Roodhardt, Spence, Burrill, & Christiansen, 1998). By age 13 or 14, they are involved in solving equations in the traditional manner.

Most algebraic concepts can be approached in similar ways. Another example is the concept of a variable, very important in understanding and success in algebra. The following illustration is taken from *Patterns and Symbols* (Roodhardt, Kindt, Burrill, & Spence, 1997), a unit for upper primary students. Again, students begin with a pictorial representation of a situation – building walls of different patterns using standing and lying bricks. Symbols become an efficient way to represent situations and to identify patterns. Students begin by drawing the pattern that is being repeated, for example two standing bricks followed by one lying brick, followed by two standing and one lying



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Students quickly find that drawing the pictures becomes very tedious and adopt or invent the use of symbols such as S and L to represent the attributes standing and lying. SSLSSLSSL. From a natural use of symbols, students expand their understanding of symbolic representation and order of operations using parentheses by describing a pattern of standing and lying bricks in a garden border using expressions such as 4(2 S+5 L) = 8 S + 20 L, with the symbols replacing the numerals-unknowns. (Wijiers, Roodhardt, van Reeuwijk, Burrill, Cole, & Pligge, 1998). Through a series of units students progress to using a symbol to represent any number in an infinite set-variable (Kindt, Wijers, Spence, Brinker, Pligge, & Burrill, 1998), (Kindt, Roodhardt, Spence, Simon, & Pligge, 1998), (Roodhardt, Spence, Burrill, & Christiansen,. 1998), (Roodhardt, Kindt, Pligge, & Simon, 1998).

Topics such as linear growth, recursion, arithmetic and geometric sequencing, slope and intercepts can be introduced with the idea of plant growth and using the NEXT/CURRENT reasoning along with the visual representation obtained by using graphs, (Roodhardt, Spence, Burrill, & Christiansen, 1998). The fact that a plant grows 6 inches every week is described by stating that the NEXT length will be the CURRENT length + 6 inches. From this the students can develop a direct formula, i.e. new length = current length + 6N where N represents the number of weeks of growth. Students collect the data for such linear growth and use graphs to describe and analyze this natural phenomena. These real data problems reinforce the concepts of slope and intercept.

The introduction of equation writing can be done informally with the notion of arrow language and can then be quite easily abstracted to the formal notation. For example, consider a problem of determining the price to ride in a taxi from *Expressions and Formulas* (Gravemeijer, Roodhardt, Wijers, Cole, & Burrill, 1998). A company charges a starting amount of $2.00 and an additional $1.50 per mile. Arrow language or arrow strings help students understand how the fare is determined.

number of miles —(x $1.50)→ —(+ $2.00)→ total price

With these tools, students create and use formulas that are precursors of algebraic equations.

Using a philosophy of progressive formalization, a solid foundation can be built to provide students with a strong and basic understanding of algebraic concepts. The key elements are to begin with realistic situations that can be represented pictorially in some way. Students gradually leave the pictures behind as they move to the use of symbols and finally come to understand that algebra is the representation of a situation in symbols in order to manipulate them absent of context to find a solution, once again in context, to a given problem.

Evidence from the use of this approach in the middle grades curriculum Mathematics in Context in the United States indicates that this development of concepts greatly increases the success of all students taking algebra

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