



CAN TECHNOLOGY SAVE CLASSROOM MATHEMATICS?

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Introduction

Mathematics as a school subject is in crisis. All over the world, the shortage of mathematics teachers worsens, and pupils are treating the subject with less and less respect. Can technology help to reverse this downward spiral? Can technology add a sparkle to the mathematics classroom?

This presentation gives an emphatic yes, so long as governments do not think for a moment that technology can be used in any sense to replace teachers. Teachers are essential to the process, to lead and inspire the young. It is my view that the use of electronically based lesson plans and dynamic images as a routine part of delivery can make the subject more fun to learn, and more fun to teach. It is a long hope, but if the subject is made more visual, more exciting and the learning process made more effective in this way, the devastating processes described above could yet be reversed.

Hardware.

First, the right classroom hardware needs to be in place. We are about 3-4 years away from affordable large screens (using light-emitting polymers). I have linked to some research papers on this technology: www.argonet.co.uk/oundlesch/class.html and the companies involved will undoubtedly have a far reaching impact on education.

If they can get the price down far enough it will enable all classrooms to have a large display screen, on which (importantly) the teachers can write with a white-board marker, thus combining their traditional methods with the occasional computer generated image. It will be connected (by radio) to a small portable computer, the local intranet and the internet. It seems a long wait, but we must be ready. The presently available technology for large screens is generally not affordable. Battery technology is also set to improve dramatically, making portable devices last a full day – this will remove the need for the final wire, the electricity cable! .

The use of standard office tools.

Standard tools, such as MS Office, can be used to help add a sparkle to lesson plans and hand-outs. In particular, Word can be the common denominator of all electronically stored lesson plans and teaching resources for mathematics. First set up some extra tools on the toolbar:



● WORD documents can contain equations as text

The UNICODE font system enables each of the major fonts to have its own set of symbols. (though unfortunately a MAC or a PC system prior to W-98 will not display these characters).

Using “Insert” => “Symbol” => “Font” => “Normal Text” you can set up short-cut keys as required so that the current font can use its own symbols. The *suggested* ALT keys to set up are:

ALT	2	3	4	H	R	I	O	0	<	>	+	/	↑	↓
Symbol	²	³	¼	½	√	∫	°	∞	≤	≥	±	÷	↑	↓
ALT	A	B	D	E	F	L	M	N	P	S	T	X	←	→
Symbol	α	β	Δ	Σ	φ	λ	μ	ν	π	σ	θ	χ	←	→

Also available for occasional use (using “Insert” => “Symbol” => “Font” => “Normal Text”):

$\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$, γ ε ζ η κ ξ ρ ζ ω, ℓ, Γ Π Φ Ω, ≡ ≠ ≈ ∩ ∂

The proper MINUS sign (“-” or “en-dash”) is available on Ctrl – using the keypad ‘-’

Generally, it is worth creating equations and formulae as **text** if possible. These expressions can then be pasted anywhere, eg to an email or in a spreadsheet.

Examples:

$$y = x\sqrt{(1 - x^2)} \qquad y = \pi \pm \sin^{-1}(\frac{1}{2}x)$$

$$y = e^{\sin^2 x} \qquad \sigma^2 = (1/n)\Sigma(fx^2) - \mu^2$$

$$\chi^2 \sim N(\mu, \sigma^2) \qquad \int \sin^2 \theta \cos \theta d\theta = \frac{1}{3} \sin^3 \theta + c$$

$$\int \sin^2 \theta d\theta = \int \frac{1}{2}(1 - \cos 2\theta) d\theta = \frac{1}{2}\theta - \frac{1}{4} \sin 2\theta + c$$

● **WORD documents can contain equations as graphics**

These can be edited only using the Equation editor. Equations created this way are graphics and require format and layout control, Short-cuts can be placed in the “Auto-correct” list when selected in ‘move with text’ mode.

eg type “qf <ENTER>” for the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Other equation editor examples:

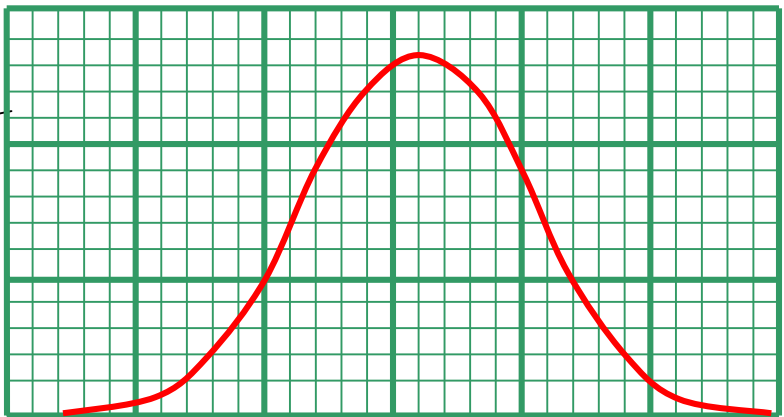
$\mathbf{a} = \overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ <p>Vectors and Matrices: $A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & 3 & 1 \\ 0 & 0 & 1 \end{pmatrix}$</p> $\det B = \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix}$	<p>Integrals: $\int_{\pi/3}^{\pi/2} (1 - \sin^2 2x) dx$</p> <p>Summations: $\bar{x} = \frac{1}{n} \sum fx$</p> $s^2 = \frac{1}{n} \sum fx^2 - \bar{x}^2$
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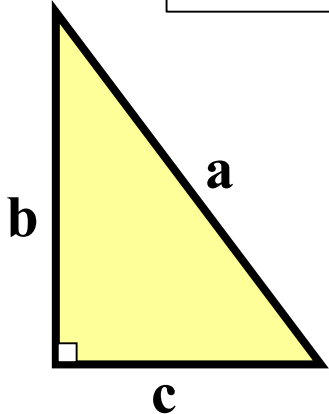
● **WORD documents can contain object-based diagrams**

The drawing toolbar has many more mathematical tools than many realize, though there are some disappointing omissions.

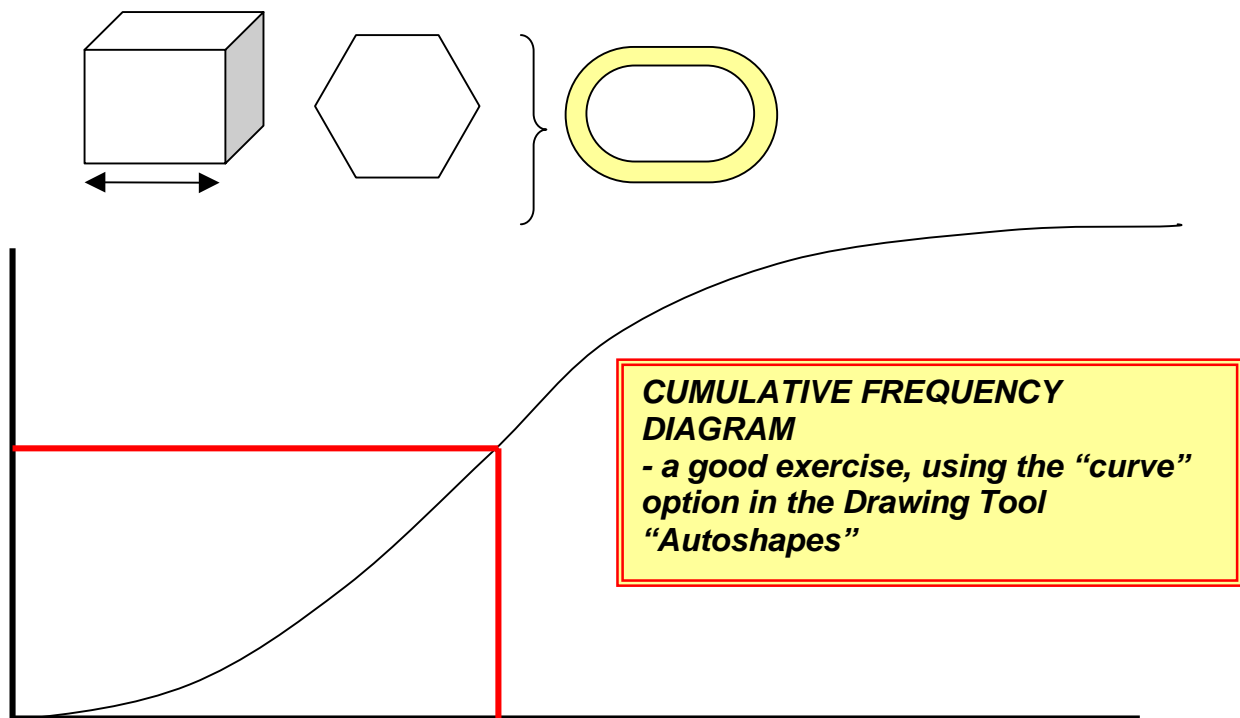
GRAPH PAPER

Use Ctrl-D to duplicate lines (it learns what you want)






To put these diagrams together, users need to appreciate the concept of a selectable OBJECT, and the multiple-selection and grouping of objects (using SHIFT) - the basis of all modern dynamic software.



Points to note: the use of SHIFT and CTRL is crucial when creating these objects (Shift: makes the object regular, Ctrl make it centred). Also ensure that “Snap to Grid” is ON, so that objects fit together neatly (use “ALT” to over-ride this).

- **WORD documents can contain HYPERLINKS**, using  linking to bookmarks in the same document, eg [back to the start](#) linking to files on the hard drive (eg an [Excel](#) file)

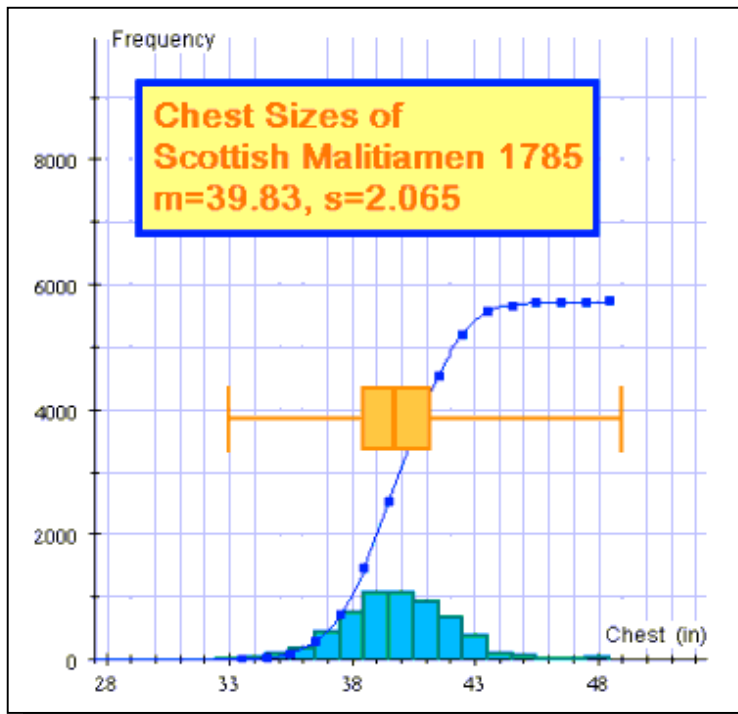
linking to web pages (URLs), eg www.argonet.co.uk/oundlesch

eg, some useful web links from the Oundle site with dynamic images for the classroom:

MATHEMATICAL RESOURCES	
UK	PLUS - Maths magazine - NRICH - Online Maths Club Counton (ex Maths Year 2000)
AUSTRALIA	Maths300 (Lessons on the Web)
JAPAN	Mathematical JAVA applets
SWEDEN	TIPTOP Virtual Laboratory - JAVA applets
MATHEMATICAL ENTERTAINMENT	
	Ron Knott's Fibonacci pages Integer Lists to full accuracy History of Mathematics (St Andrews)
STATISTICAL RESOURCES	
DATA SETS	[there are further data resources on the Autograph site]
	DASL: USA Data and Story Library The UK Lottery Data from UN, by country Statistical web resources from Sidney Tyrrell , Coventry University, UK

● **WORD documents can contain items pasted from other applications**

eg a graph or text from Autograph – best placed in a text box for greater control:



Scottish Militiamen (1785):

x	f	cf.
33	3	3
34	18	21
35	81	102
36	185	287
37	420	707
38	749	1456
39	1073	2529
40	1079	3608
41	934	4542
42	658	5200
43	370	5570
44	92	5662
45	50	5712
46	2	5714
47	3	5717
48	18	5735

$\Sigma f = 5735$ $\Sigma fx = 228000$
 $\Sigma fx^2 = 9125000$ Mean = 39.83
 SD = 2.065 Var. = 4.265

● **WORD documents can contain items pasted off a web page**

text (which can include their own hyperlinks)

graphics (images pasted straight off a web page)

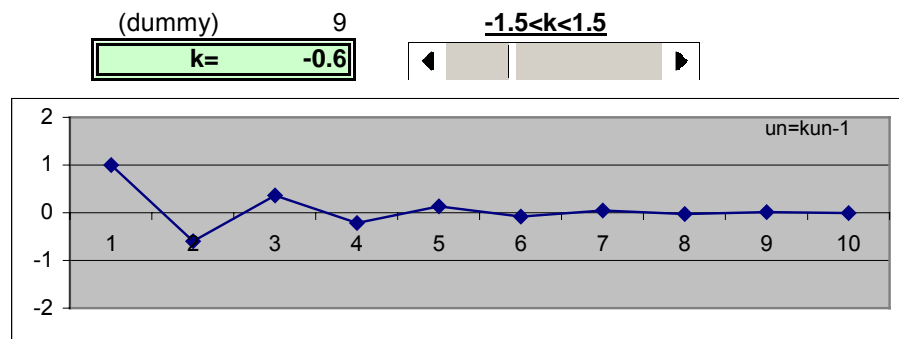
data (best taken via Excel to sort out any problems, though this is not without difficulties)

● **WORD documents can contain embedded spreadsheets, using **

This way the spreadsheet remains ‘live’ and can be activated by double-clicking

ILLUSTRATING THE GEOMETRIC PROGRESSION

n	$u_n = ku_{n-1}$
1	1.0000
2	-0.6000
3	0.3600
4	-0.2160
5	0.1296
6	-0.0778
7	0.0467
8	-0.0280
9	0.0168
10	-0.0101



THE USE OF DYNAMIC IMAGES IN MATHEMATICS TEACHING

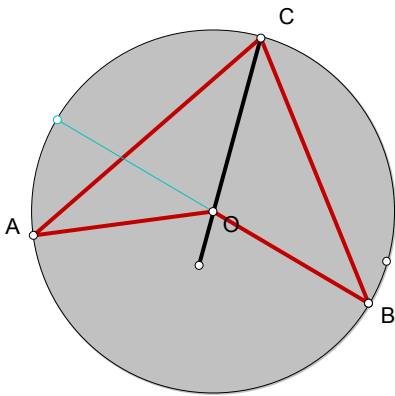
There are countless aspects of the secondary and college level mathematics curriculum for which this approach is effective. But the teachers need to practice making it effective, and training issues are a serious consideration. If the images proceed too quickly the learning can be lost. It is necessary to involve the class at each stage in the process, to get the students to anticipate what is about to happen, maybe to invite a student to draw on the screen first before the computer does it. This is a tried and tested technique, allowing the teacher to determine the pace, and if necessary to force errors from the class to help make good teaching points.

● The Circle Theorems illustrated (dynamically) in Geometer's Sketchpad

$$m\angle AOB = 141.6^\circ$$

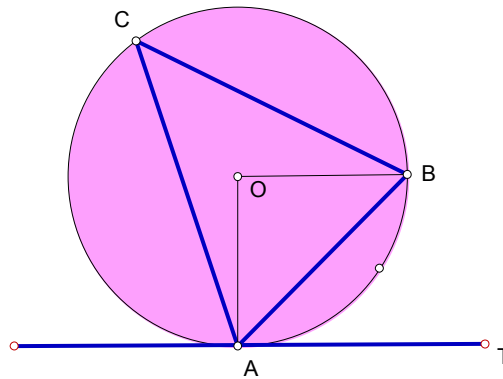
$$m\angle ACB = 70.8^\circ$$

$$\frac{m\angle AOB}{m\angle ACB} = 2.0$$



$$m\angle TAB = 45.3^\circ$$

$$m\angle ACB = 45.3^\circ$$

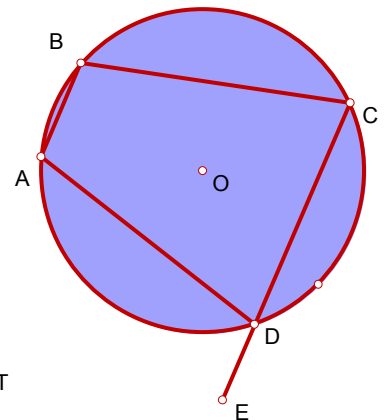


$$m\angle EDA = 104.9^\circ$$

$$m\angle ABC = 104.9^\circ$$

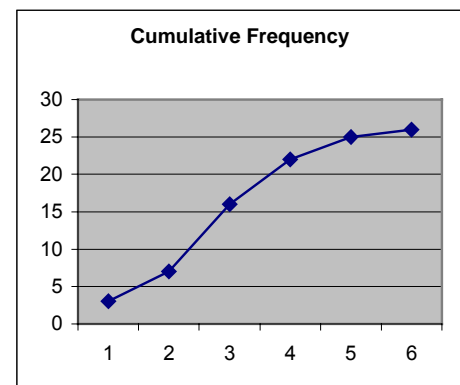
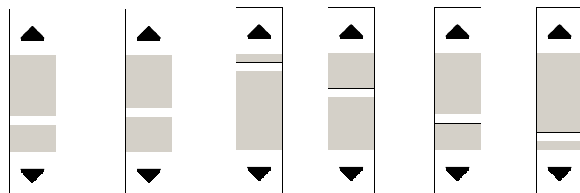
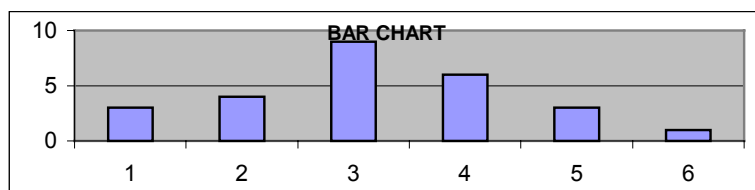
$$m\angle CDA = 75.1^\circ$$

$$m\angle ABC + m\angle CDA = 180.0^\circ$$



● Dynamic Histograms, in Excel

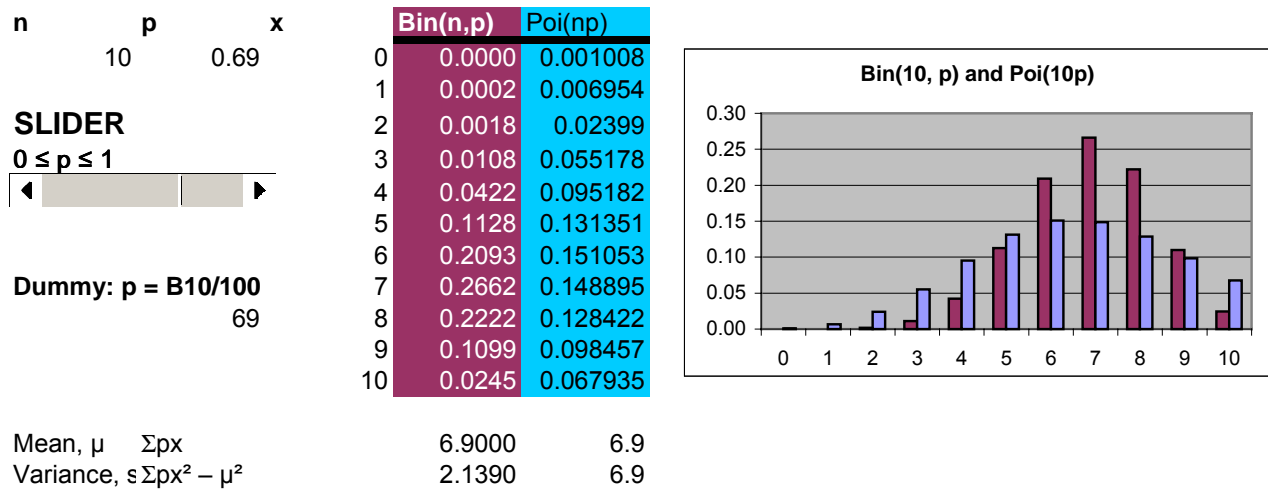
(from Peter Mulkerin)



	Σ						
Slider	7	6	1	4	7	9	34
f	3	4	9	6	3	1	26
cf	3	7	16	22	25	26	

Whilst Sketchpad and Cabri are naturally dynamic environments, Excel is now, and needs the use of extra tools, such as the slider bar, to make it come alive. The sliders only work for positive integers, so it is necessary to insert linear transformations with dummy cells (one of very many disappointments about Excel from a mathematical point of view)

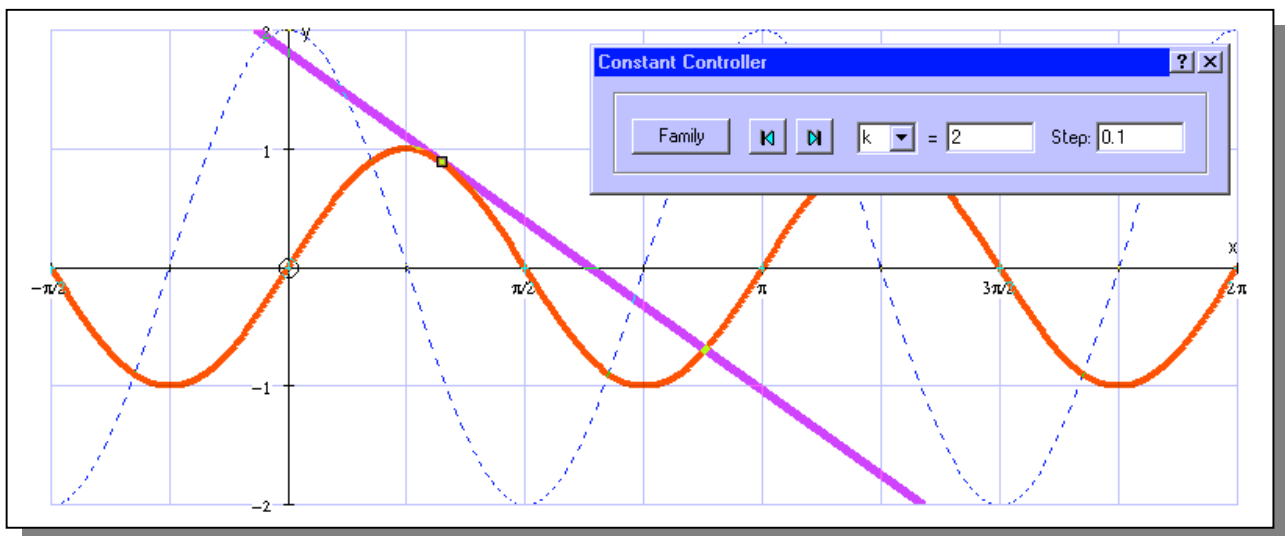
● Comparing probability distributions in Excel



● The use of Autograph

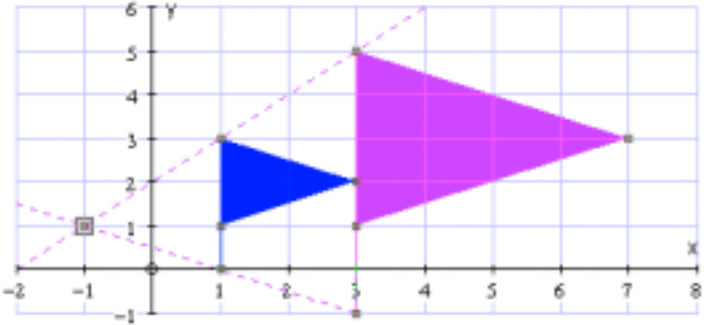
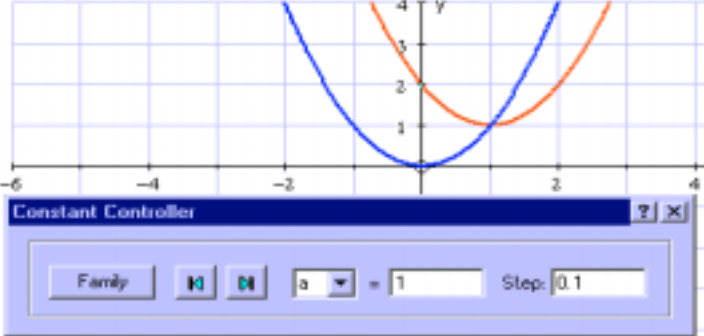
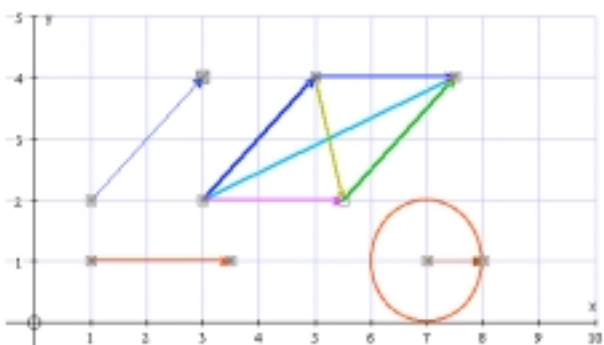
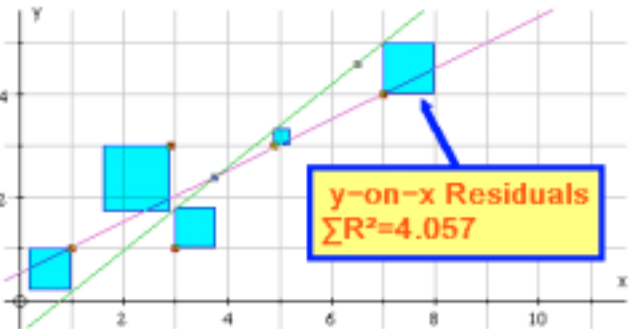
Autograph is a relatively new medium, bringing the world of dynamic and dependent objects to coordinate geometry and statistics. See: www.autograph-math.com for related resources. Some lesson plans follow, showing how dynamic software can add a new dimension to whole-class teaching of secondary and college mathematics.

By way of introduction, to demonstrate the Chain Rule using dynamically linked objects:



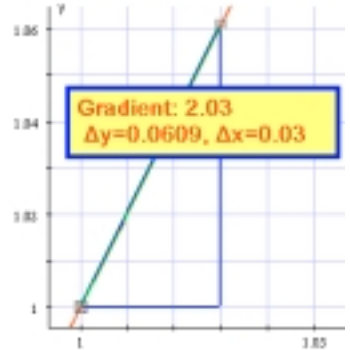

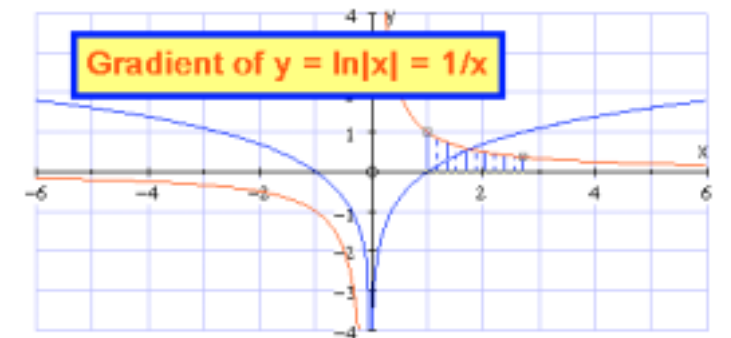
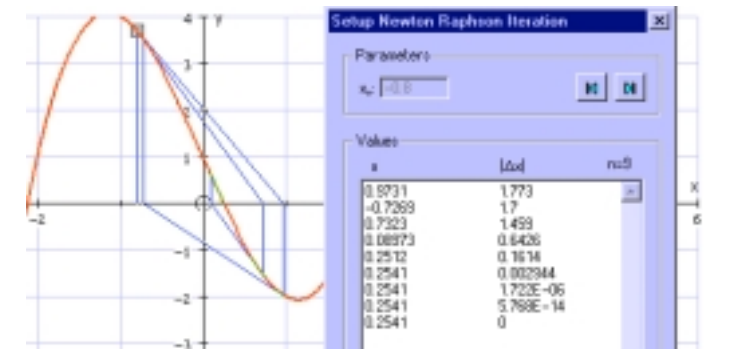
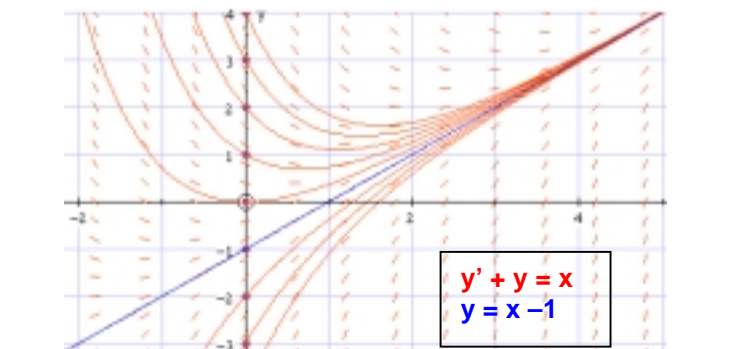
SAMPLE LESSONS USING AUTOGRAPH (1)

The most effective way to use Autograph in the classroom is with a screen that the pupils can all see, and one which you can write on with a white-board marker (eg a TV or, ideally, a projector displaying on a white board). All these examples form part of the “45 Exercises”, all of which have been recorded as ScreenCam demonstration, available for download on the Autograph web site.

<p>TRANSFORMATIONS A perfect dynamic situation: the original objects can be moved around, and the parameter that created the second object can be animated. Write on top of the images on the while board to get the students to anticipate the results before the computer.</p>	
<p>TRANSFORMING CURVES Several approaches are possible here. Illustrated here is: $y = x^2$, and $y - b = (x - b)^2$ and then vary a and b Using function definitions works well too, eg plot $f(x) = x^2$, then $y = f(ax + b) + c$; vary a, b, c Then redefine $f(x)$ as $\sin x$.</p>	
<p>FUN WITH VECTORS The principle of copying, adding and subtracting vectors can be explored dynamically. Also scalar multiples (vector equation of a line). The difficult concept of a unit vector can be explored by constructing a unit circle over it and varying the parent vector.</p>	
<p>ILLUSTRATING RESIDUALS Here a variable line is constructed through the centroid and another random point. This line and the data set (which is an object) are selected and the y-on-x residuals illustrated as square. Vary the second point to watch the squares minimize.</p>	

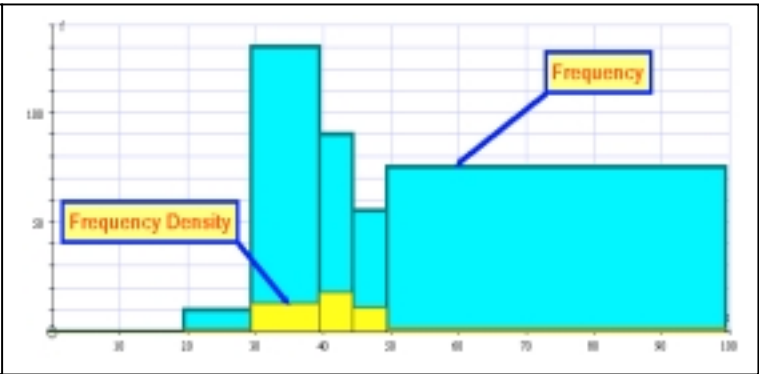
SAMPLE LESSONS USING AUTOGRAPH (2) – the principles of calculus

There are many aspects of the calculus that can be taught very much more effectively with dynamic images. The old thought that a picture is worth a thousand words is never more true, and students gain new insights by observing movement through animation, driven either by the teacher in ‘whole-class’ presentation, or by the students themselves working through instructions in a lab.

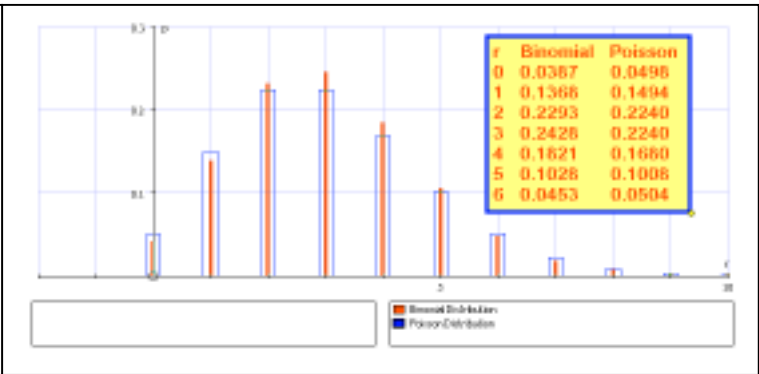
<p>THE CONCEPT OF GRADIENT</p> <p>(a) zooming in on the gradient of a chord as $x_2 \Rightarrow x_1$, observing $\Delta y = 0$ and $\Delta x = 0$ yet $\Delta y/\Delta x \Rightarrow 2$</p> <p>(b) Autograph can draw the gradient function slowly, and users can dynamically move a tangent along the parent curve</p>																																												
<p>$y = 1/x$ and e and $y = \ln x$</p> <p>This lesson first shows, by varying the right limit of the area under $y = 1/x$ to find e. Then plot $y = \ln x$, and show that its gradient is the right branch of $y = 1/x$. Then develop an argument for $y = \ln(-x)$ and finally $y = \ln x$</p>																																												
<p>NUMERICAL METHODS</p> <p>Solving equations by Newton-Raphson, and by the $x = g(x)$ iterative method both lend themselves perfectly to a dynamic approach. The start point can be dragged around, and the equations can have variable parameters.</p>	 <table border="1" data-bbox="981 1265 1348 1612"> <thead> <tr> <th colspan="3">Setup Newton Raphson Iteration</th> </tr> <tr> <th colspan="3">Parameters</th> </tr> <tr> <td>x_0</td> <td></td> <td></td> </tr> <tr> <td colspan="3">Values</td> </tr> <tr> <th>x</th> <th>$f(x)$</th> <th>n</th> </tr> </thead> <tbody> <tr> <td>0.9731</td> <td>1.773</td> <td></td> </tr> <tr> <td>-0.7269</td> <td>1.7</td> <td></td> </tr> <tr> <td>0.7223</td> <td>1.499</td> <td></td> </tr> <tr> <td>0.08973</td> <td>0.6426</td> <td></td> </tr> <tr> <td>0.2512</td> <td>0.1614</td> <td></td> </tr> <tr> <td>0.2541</td> <td>0.002944</td> <td></td> </tr> <tr> <td>0.2541</td> <td>1.722E-06</td> <td></td> </tr> <tr> <td>0.2541</td> <td>3.769E-14</td> <td></td> </tr> <tr> <td>0.2541</td> <td>0</td> <td></td> </tr> </tbody> </table>		Setup Newton Raphson Iteration			Parameters			x_0			Values			x	$ f(x) $	n	0.9731	1.773		-0.7269	1.7		0.7223	1.499		0.08973	0.6426		0.2512	0.1614		0.2541	0.002944		0.2541	1.722E-06		0.2541	3.769E-14		0.2541	0	
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<p>DIFFERENTIAL EQUATIONS</p> <p>Here is a lesson showing the principle of the complementary function and particular integral. The implicit form of the entry allows the RHS to be changed, say, to $\sin x$. Also, the original equation can have a variable parameter, eg $y' + ky = x$.</p>																																												

SAMPLE LESSONS USING AUTOGRAPH (3): Probability and Statistics

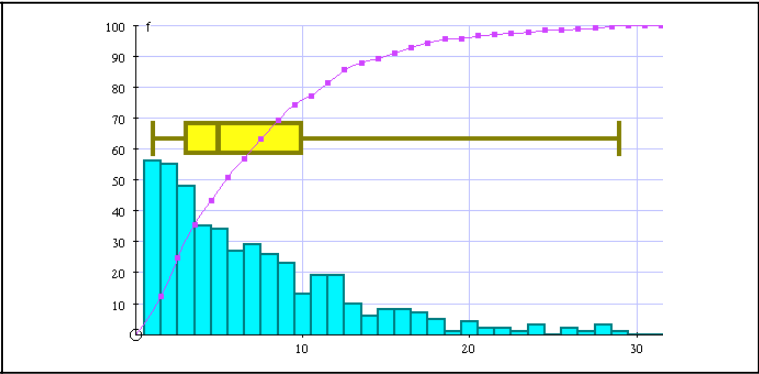
FREQUENCY DENSITY
 It is useful to be able to plot a set of grouped data so that students can see the difference between representing the data as **discrete** (displace to the left) or **continuous**, and by **frequency** or **frequency density**.



POISSON APPROXIMATION
 Students can get a good insight into the relationship between a binomial distribution and its poisson approximation by varying the binomial's 'n' and 'p'. Hypothesis tests on the binomial can also be illustrated dynamically.



DATA OFF THE NET
 The UK Lottery data (6 balls from 49) is collected from all previous results and downloaded to Excel. The distribution of the first ball (in order) copied to Autograph offers an unusual bar chart, box and whisker and cumulative frequency diagram.



Conclusions:

Mathematicians have been teaching the subject with nothing more than a writing surface and a piece of chalk for centuries, and to good effect, certainly with the more able – these are pupils who can, by and large, visualise the subject for themselves. The real beneficiaries in the new, visual approach are the next ability level down. The rewards to them can be considerable, and there is every chance that this approach can help numbers who want to take the subject further to rise.

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