

The Image of Rational Numbers in Students

N. Çetin*

Anadolu University, Department of Mathematics, 26470 Eskisehir-TURKEY

Abstract: The concept of rational number is taught at all educational levels from primary school to the university. Nevertheless, results of an exam on rational numbers administered to an experimental group of high school and university students revealed that both have difficulty comprehending these numbers. In this study the students' incorrect answers were analyzed in order to determine why they had reached such false conclusions.

The concept of number is something that we often face in our everyday lives and is also one of the basic concepts of mathematics. Early on, we are introduced to natural numbers such as 1,2,3 ... [1]. Moreover, rational numbers are quite common in our daily lives; i.e., half of an apple, a quarter of the cake or the percentage of the general population that is literate. Similar examples can be given. However, when people are asked to name a number spontaneously, almost all their answers consist of natural numbers. People are exposed to different numbers during their education as well as their daily lives. That people have difficulty in remembering rational and irrational numbers apart from natural numbers when they are asked to give a number may bear some relation to the inadequacies in teaching numbers at all levels of education or to the difficulty that students have in remembering these numbers [2], [3]. Based on this hypothesis, a group of high school and university students was given a series of numbers and asked to determine whether these numbers were rational numbers or not, explaining their choice. In the following section, the exam is explained in detail.

1. The Exam

The experimental group consisted of a total of 125 students 91 students of The Science and Education Faculties, and 34 high school students. This experimental group was given a seven question exam for which there was no time limit. Below are the seven questions asked of the experimental group.

1. Is “2” a rational number? Why?

2. Is “ $\frac{\sqrt{3}}{2}$ ” a rational number? Why?

3. Is “1,34” a rational number? Why?

4. Is “ $-\frac{\sqrt[3]{27}}{3}$ ” a rational number? Why?

5. Is “ $7,\overline{32}$ ” a rational number? Why?

6. Is “ \mathbf{p} ” a rational number? Why?

7. Is “x” a rational number? Why?

These questions were designed to determine where the students' deficiencies and misunderstandings in learning rational numbers lay. The aim of the first question was to see if they could acknowledge the fact that a natural number is actually at the same time, a rational number. On the second question, students were required to recognize that numbers in numerators and denominators are actually integer numbers. The third and the fourth questions required them to demonstrate that rational numbers can be symbolized in different ways. The fifth question obliged them to show that an infinite decimal number is a rational number. Whether frequently used irrational numbers should be considered as rational numbers or not was the aim of the sixth question. On the seventh question, students needed to realize that

variables used to symbolize numbers should be determined according to the numbers they represent.

2. Results and Discussions

The results of the experimental group's performance on the exam are shown in Table 1.

Question Number	The Number of Students of University, Who			The Number of Students of High School, Who		
	Gave Correct Answer	Gave Wrong Answer	Gave No Answer	Gave Correct Answer	Gave Wrong Answer	Gave No Answer
1	70	19	2	24	10	-
2	57	25	9	18	16	-
3	82	5	4	30	4	-
4	61	25	5	19	15	-
5	63	19	9	26	8	-
6	29	54	8	1	33	-
7	43	31	17	19	15	-

Table 1: Performance of The Students

21% of the students in the university and 30% of high school students gave the incorrect answer to the first question. The reason why approximately 25% of the students did not give the correct answer was because of their belief that rational numbers should be expressed as fraction. As for the second question, 34% of the students did not solve it correctly. When all their answers were analyzed, it was determined that they considered the number as rational because it has numerator and denominator and they ignored the square root sign. . Almost all the answers were correct on the third question which was in decimal form. When we examine the placement of decimal numbers in the syllabus, we found that decimal numbers were taught together with their fractional way of expression: i.e., teaching 0,5 with its fractional way of expression, $\frac{1}{2}$. Therefore, students automatically used the fractional number equivalent to the given decimal number. On the fourth question, nearly all the students made the necessary simplifications resulting with a negative integer number. Despite this, approximately 32% of the students could not solve the problem correctly. Analysis of their incorrect answers indicated that one of their two principal mistakes was identical to that made on the first question. Their second mistake was, on the other hand, quite striking. According to the students' explanations the number they got after simplification could not be a rational one because it was negative.

Students were led astray on question five was because the given number was an infinite decimal number. The students thought that as such, it was equal to an indefinite number and could not therefore be rational number. As seen in the Table 1, the question that almost all the students answered incorrectly was the sixth one. In primary school, " π " is generally taught using its approximate value, i.e. $\frac{22}{7}$ or 3,14 to make it easier for the students to grasp. Consequently, students falsely believe " π " to be a rational number. Analysis of the answers to the seventh question indicated that students considered " x " to be a integer number and thus interpreted it as a rational number without thinking that " x " could be any number.

According to the findings of this study, it may be claimed that the students do not fully comprehend the concept of rational numbers fully. As a follow-up to this study, a new method will be developed and its efficiency tested.

References

- [1] G. Ifrah, Histoire Universelle Des Chiffres, Editions Robert Laffont, S. A. , Paris, 1994.
- [2] N. Mahir, N. Çetin, S. Canbek, Comparison of Numbers, (2000), Mathematics for Living.
- [3] A. Baki, Okul Matematiğinde Ne Öğretim Nasıl Öğretim, 1996, Milli Eğitim, 130, Milli Eğitim Basımevi, Ankara.