Misconceptions about triangle in Elementary school

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1. Introduction

The intent of studying the misconceptions on the triangle was suggested by the direct contact with the scholastic reality. During the training activity it was possible to experiment a teaching unit directed to pupils of the forth form of the primary school, having the objective of analysing with pupils the relations intervening when examining triangle sides.
In the course of observation it was possible to notice their erroneous interpretation on the concept of a triangle resulting in a miscomprehension of the unit we were considering. Hence the necessity of taking into consideration these misconceptions through an experimental work to be utilized in building up functional units aimed at preventing and correcting them.
Experimental data have been collected in an open question questionnaire elaborated on the basis of observation during the training activity and an epistemological analysis of the argument we were dealing with.
The study of experimental data was based on the a priori analysis of the pupils' behaviours, the descriptive analysis (trough the use of EXCEL software) and the implicative analysis of variables (trough the use of CHIC software).

2. Experimental work presentation

The research aim is that of discovering the pupils misconception on the triangle.
The observation has been applied to 77 student aged 11-12 years, in the first classe of "Scuola Media Vittorio Emanuele" of Palermo at the beginning of the 2001-2002 school year. The knowledge level of the group of student examined, is that they acquired in the enfant and primary school.
The "ipotesi alternativa" on which the questionnaire was built is the existence of erroneous conceptions on the triangle that can result in obstructing the pupils' comprehension; the "ipotesi nulla" is that there aren't erroneous concepts on the triangle that can result in obstructing the pupils' comprehension.
It isn't possible to falsify directly both the above hypothesis, however if we reject the "ipotesi nulla" the "ipotesi alternativa" result acceptable with a validity level equal to intensity of implication shown by the CHIC software.
The charts obtained through the software give us the possibility of controlling and choosing the level of acceptability of the implication established in line with the probable laws of inferential statistics.

3.0 Questionnaire on the triangle concept

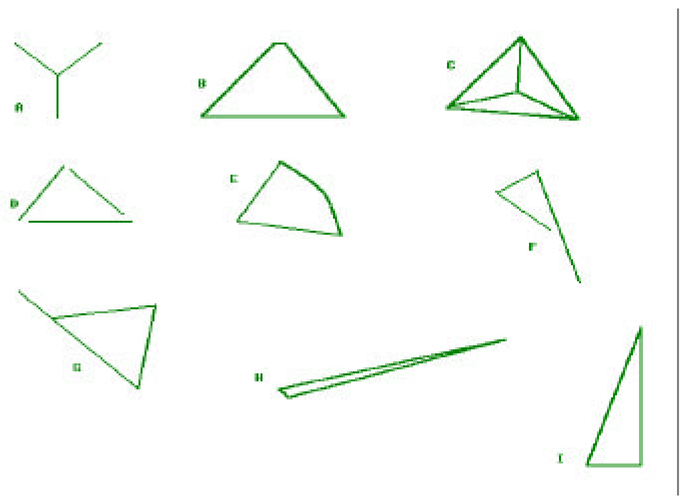
3.1 Preliminary remark

The observation has been made through an open question questionnaire having the following purpose:

- In the first question the student are invited to say if what they see is a triangle and why. This helps them to think about characteristic of a triangle and obtain an implicit model towards the triangle.
- The second question asks the pupil to draw a triangle and define it by words. The drawing they have made corresponds to their mental picture of the geometrical figure.
- The third and fourth questions are aimed at discovering if pupils understand the relation among sides and among angles of the figure and which strategies they adopt in choosing the measures.
- The last question is aimed at putting in evidence the concept of the height they have gained and misconceptions which hamper the comprehension of height in a given triangle.

3.2 Questionnaire

A. Look at these figures. Which is a triangle? Which isn't? Why?



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B. Draw a triangle and define it by word

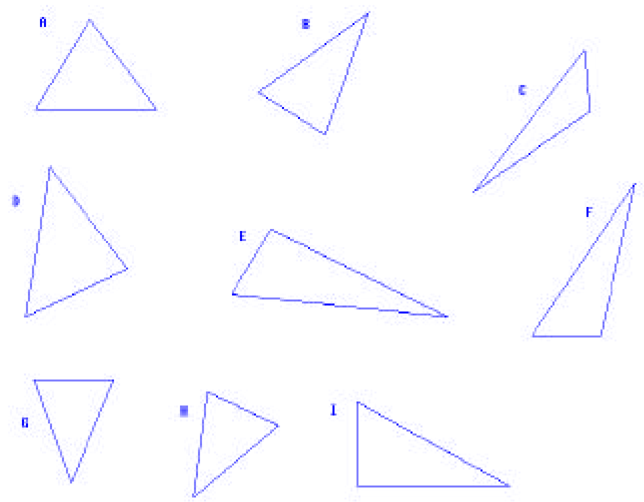
C. Complete the following table with sides' length.

	Triangle a	Triangle b	Triangle c	Triangle d	Triangle e
AB					
BC					
CD					

D. Complete the following table with angles' width.

	Triangle a	Triangle b	Triangle c	Triangle d	Triangle e
A	90°				
B	100°				
C					

E. Draw these triangles' height or heights.



4. A priori analysis of the students' behaviours

The following table shows the possible behaviours of the student, when they answer the questionnaire's questions.

A) Look these figures. Which is a triangle? Which isn't? Why?	**frequency %**
A1: he recognises as a triangle a three sided figure	55
A2: he recognises as a triangle a three angled figure	39
A3: he recognises as a triangle a figure resembling a equilateral triangle	31
A4: he recognises as a triangle a closed figure	23
A5: he recognises as a triangle a plane figure	23
A6: he recognises as a triangle a polygon	0
A7: he recognises as a triangle a figure with three vertexes	6
A8: he recognises as a triangle a figure with consecutive sides	55
A9: he recognises the triangle but doesn't give explanations	0
A10: he says that C figure is a solid\pyramid.	26
A11: he says that a triangle mustn't have a round side	58
A12: he recognise as a triangle a isosceles triangle (it is a triangle because it is isosceles)	4
A13: he recognise as a triangle a scalene triangle	9
A14: he recognise as a triangle a rectangular triangle	5
A17: he sees the object as a frame referred to their own real experience and not as a geometrical object	45
A 18: he recognise as a triangle a geometrical figure.	14

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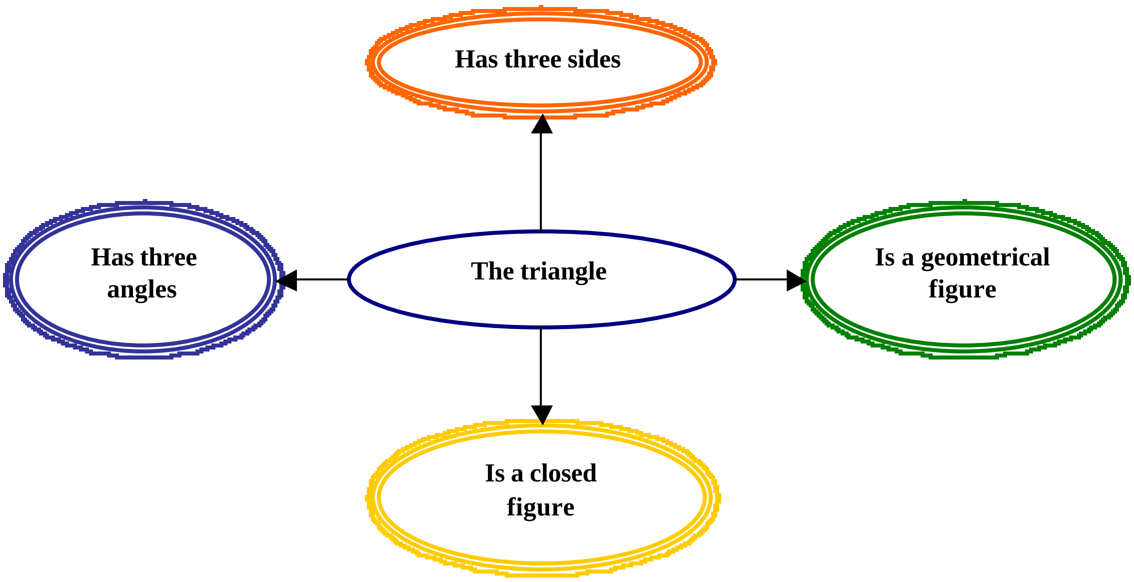
A 19: he sees: it is a triangle because it is a triangle	19
A 20: he sees that f e g aren't triangles because they have a too long side	48
A 21: he recognises as a triangle a "spezzata chiusa"	3
B) Draw a triangle and define it by word	
B1: it must be a polygon	4
B2: it must have three sides	55
B3: it must have three angles	48
B4: it must have three vertexes	8
B5: it must be a "spezzata chiusa"	5
B6: it must be a plane figure	3
B7: it must have three adjacent sides	6
B8: it must have three equal sides and three equal angles	22
B9: he draws a equilateral triangle intentionally	12
B10:he draws a scalene triangle intentionally	1,3
B11: he draws a isosceles triangle intentionally	1,3
B12: he draws a rectangular triangle intentionally	5
B15: he draws a equilateral triangle	47
B16: he draws a isosceles triangle	23
B17: he draws a scalene\rectangular triangle	5
B18: he draws the triangle with an horizontal base as it were a heavy body	91
B19: he sees the object as a frame referred to their own real experience and not as a geometrical object	3
B20: he recognises as a triangle a isosceles triangle	3
B21: he recognises as a triangle a scalene triangle	1,3
B25: he defines as a triangle a figure that can have three equal sides, three different sides or two equal and a different one.	8
B26: he recognises as a triangle a geometrical figure	23
B27: he recognises as a triangle a part of a plane	1,3
B28: he sees: to be a triangle it must have triangle's shape	3
B29: he says that a triangle mustn't have a round side	5
B30: the angles' sum must be 180°	3
B31: the base must be shorter than sides' sum	1,3
C) Complete the following table with sides' length.	
C1: he doesn't use any role to choose sides' length	19
C2: he always chooses equal sides' length	13
C4: he chooses almost equal sides' length	17
C6: he always chooses two equal sides' length and a different one	5
C7: he draws some triangles and measures each side	23
C8: he chooses each side shorter than other sides' sum	1,3
C9: he chooses three equal sides, three different sides or two equal and a different one.	45
C10: he writes right sides' length but doesn't use the formal rule	45
D) Complete the following table with angles' width.	
D1: he doesn't use any role to choose angles' width	36
D2: he always chooses equal angles' width	4
D6: he always chooses two equal angles' width and a different one	1,3
D7: he draws some triangles and measures each angles	5
D8: he chooses two angles wider than 90°, or equal to 90°.	56
D9: he notices that (a) can't be a triangle.	16
D10: he applies the role: the angles' sum must be 180°	9
D11: he chooses 3 acute angles, or 2 acute angles and a recto (or obtuse) one .	17
D12: he chooses three equal angles, three different angles or two equal and a different one.	23

E) Draw these triangles' height or heights.	
E1: he always marks a vertical line	39
E2: he marks the height on the sides	13
E3: he always marks the height inside the triangle	56
E4: he marks the height starting from the highest vertex	6
E6: he marks as height a line that divides in two parts the bases of the triangle	23
E7: he marks a vertical line starting from the highest vertex perpendicular to a plane, say the floor, on which the triangle has the base.	16
E8: he marks a not perpendicular line starting from a vertex to the opposite base.	31
E9: he marks one right height	5
E10: he marks three right heights	3
E11: he marks two or three wrong heights	13
E12: he marks one right height when the figure is like a equilateral triangle	21
E13: he marks a perpendicular line starting from a side to another.	9
E14: he marks a right height when the triangle has an horizontal base as it were a heavy body	18

5. Conclusions

Data analysis has allowed to get the following cognitive chart regarding the triangle concept in the tested group of children.



It is possible to think of a hierarchy in properties discovered by children that is "having three sides" first and then "having three angles".

The more diffused misconception are:

- Some confusion, also regarding the linguistic aspect, between the geometric contest and the daily contest . Notwithstanding children knew they were coping with a geometrical questionnaire, 45% of them has compared some figures to daily real object (such as needles, flags, alphabet letters, etc). This is proved by the fact that children using the expression "geometrical figure" has this misconception. This might means that the term has not a clear meaning for them. From this we derive the existence of a conflict between mathematic language and children daily language.
- The presence of a rigid mental scheme that brings student to generalize to all triangle the property: "having about the same length of sides and angles' width". This is supported by the descriptive analysis: 31% of children sees as triangles figures resembling the equilateral triangle, 59% draws an equilateral triangle, the remaining pupils draw a triangle with about the same length of sides and angles' width.
- A strongly stereotyped mental image, regarding the totality of children, is drawing a triangle with an horizontal base as a heavy body.

As it concerns the choice of measures of sides and angles, the percentage of children that remembers and applies the formal rule is negligible. The majority of student (45% for sides' measures and 23% for angles' measures) uses the classification of triangle with respect to congruence of the sides, choosing three equal measures, three different measures or two equal and a different one. For analogy they extend the same rule to angles. Part of the children (19%

for measures of sides, 36% for measure of angles) doesn't adopt any system in the choice of measures. This lack of strategy is more common in the choice of angle which seems to create major difficulties. Thanks to the implicative analysis we know that part of the sample doesn't make use any strategy neither in the choice of sides nor in the angles.

The sample of children presents three meaningful misconception as regard the concept of height:

- The height is a vertical line.
  This stereotype doesn't allow to mark the height if the triangle has not an horizontal base. Remember that 91% of children has drawn triangles with an horizontal base.
- Height must be drawn inside the triangle.
  This misconception might be the cause of the difficulty children have had in marking the height in a not rectangular and scalene triangle.
- The height must divide in two parts the base of the triangle.

These stereotypes might be due to the conflict between geometric meaning and the common meaning of the word "height". In this sense it is worth mentioning the E7 strategy: "make a vertical line starting from the highest vertex perpendicular to a plane, say the floor, on which the triangle has the base".
To draw the height of a triangle we can suppose students use the same strategy they use to measure their own tallness, by marking on the wall the distance from the floor to the point up to their heads.
Data implicative analysis suggests us that a consistent number of children has contemporary more than one misconception about the height.
In particular when children use E7 strategy, said above, and E14 strategy "he marks a right height when the triangle has an horizontal base as it were a heavy body" , they are convinced that height must necessarily be vertical.
Both the strategies might be referred to the E1 misconception "he always marks vertical line", that would be of great importance.
The descriptive analysis has also shown:

- A considerable percentage of student doesn't remember formal rules and definitions
- An inaccuracy in terms they make use.

In fact only a few student remember the rule to define the relation among the three sides and the three angles of triangles or make use of proper terms such as "polygon".

1.5 Open problem

The result achieved allow as to discern other points of reflection that could be the starting point of other researches:

- Which are the genetic factors or environmental factors that contribute to create the supposed misconceptions?
- How do they affect learning?
- Which is their evolution during the school years?
- In getting information pupils rely too much to the drawing. Could the stereotyped drawing be the source of misconceptions and erroneous intuitions?

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