UNRAVELING STUDENTS' BELIEF SYSTEMS RELATING TO MATHEMATICS LEARNING AND PROBLEM SOLVING

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RESEARCH ON STUDENTS' BELIEFS AND MATHEMATICS LEARNING: AN INTRODUCTION

Inspired by Schoenfeld's work (Schoenfeld, 1985) and the initial findings that many students appear to hold a lot of naïve and incorrect beliefs about mathematics (e.g., Lampert, 1990), many researchers have been studying students' beliefs aiming, on the one hand, to identify the different kinds of students' beliefs that influence mathematical learning and, on the other hand, to understand the processes through which they develop and determine learning.

Almost two decades of research reveal how different categories of students' beliefs shape their cognitive as well as conative and affective processes in the classroom. First, several studies have demonstrated how *beliefs about the nature of mathematics and mathematical learning* and problem solving determine how one chooses to approach a problem and which techniques and cognitive strategies will be used (e.g., Lester, Garofalo, & Kroll, 1989). Research on the relevance of subject-specific manifestations of *epistemological beliefs* for mathematical learning and problem solving further supports these findings (e.g., Hofer, 1999). Apart from the research on these first two categories of beliefs that mainly dealt with the way students' cognitive processes are influenced by their beliefs, other scholars have investigated the motivational and volitional relevance of students' beliefs. More specifically, studies on students' *value and/or expectancy beliefs* in the context of mathematical learning and problem solving clearly show how these beliefs relate to students' motivation and the way they engage in mathematical learning and problem solving; these investigations also substantiate their influence on achievement (e.g., Kloosterman, 1996). Finally, *students' beliefs about teaching and the practices characterizing their specific classroom context* have been found important factors to be taken into account if we want to understand fully the academic behavior in the mathematics classroom. More than students' beliefs about the specific classroom context as such, it appears to be the closeness of fit between students' more general beliefs about mathematics teaching, learning, and the self, on the one hand, and the perceived practices typical for their classroom, on the other hand, that enables us to explain some of the motivational and emotional reactions of students. Until now, little is known about this relation between students' beliefs and their emotional processes in the classroom. Nevertheless, the few studies that investigated the relation between beliefs and emotions (e.g., Seegers & Boekaerts, 1993) indicated that indeed students' beliefs about mathematics education provide an important part of the context within which emotional responses to mathematics develop.

Notwithstanding the general agreement among researchers that students' beliefs have an important influence on mathematical learning and problem solving, from a conceptual as well as from an empirical viewpoint there is still a lack of clarity (see Op 't Eynde, De Corte, & Verschaffel, in press). Despite, or maybe just because of the attention paid to the multiple ways in which different student beliefs influence mathematical learning and problem solving, research on this topic has not yet resulted in a comprehensive model of, or theory on students' mathematics-related beliefs. As a matter of fact, most of the studies are situated in, respectively, cognitive, motivational or affective research traditions and in many cases operate in relative isolation from each other. The isolated study of specific categories of beliefs within these distinct research traditions in many ways has prevented the study of different students' beliefs in relation to each other, i.e. the analysis of students' belief *systems* related to mathematics learning and problem solving. Indeed, students' mathematics-related belief *systems* are rarely intensively studied, in spite of the fact that Schoenfeld even in his initial publication (1985) already pointed out that the systemic nature is one of the key features of the functioning of beliefs. He clarified that:

> *Belief systems* are one's mathematical world view, the perspective with which one approaches mathematics and mathematical tasks. One's beliefs about mathematics can determine how one chooses to approach a problem, which techniques will be used or avoided, how long and how hard one will work on it, and so on. Beliefs establish the context within which resources, heuristics, and control operate." (p.45, our Italics)

We are convinced that the study of students' mathematics-related belief systems, more than the study of isolated beliefs, can push the field forward. It might present a unifying framework for research on students' mathematics-related beliefs, resulting in more systematic research efforts and leading to a more comprehensive understanding of how beliefs influence mathematics learning and problem solving. In this contribution we will discuss an initial study based on a survey study of 365 Flemish junior high students, that analyzes the structure of students' mathematics-related belief systems. It tries to identify the different constituting components of students' belief systems in relation to each other.

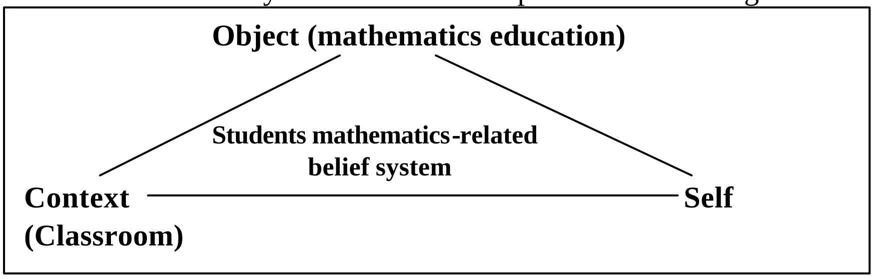
STUDENTS' MATHEMATICS-RELATED BELIEF SYSTEMS: THEORETICAL FRAMEWORK

An analysis of the nature and structure of beliefs indicates that students' beliefs are grounded in their social life and are as such fundamentally social. They are determined by the broad social-historical context in which students are situated. Finding themselves in a specific class context, students will interpret its rules and practices on the basis of their prior beliefs and knowledge and as such develop their own, to a large extent shared, conceptions about it (Cobb & Yackel, 1998).

Evidently, beliefs and knowledge operate in close interaction. Schemas or mental models are considered higher-order constructs that characterize on a conceptual level the integrated functioning of knowledge and beliefs. Although closely related in their functioning, there are, however, fundamental differences between the structure of belief and knowledge systems. One of the distinctive characteristics being that a belief system has a quasi-logical structure, whereas a knowledge system has a logical structure. Indeed, the equilibrium a belief system is trying to hold is psychological in nature. Snow, Corno, and Jackson (1996) rightfully acknowledge that

> Human beings in general show tendencies to form and hold beliefs that serve their own needs, desires and goals; these beliefs serve ego-enhancement, self-protective, and personal and social control purposes and cause biases in perception and judgment in social situations as a result. (p. 292)

In summary, the analysis of the nature and the structure of beliefs and belief systems points to the social context, the self and the object in the world that the beliefs relate to, as constitutive for the development and the functioning of these systems. The constitutive dimensions of students' mathematics-related belief systems can then be represented as a triangle:



Students' beliefs about mathematics education are situated in, and determined by, the context they participate in as well as by their individual psychological needs, desires, goals etc. Framed in another way, students' mathematics-related belief systems are constituted by their *beliefs about mathematics education, beliefs about the self, and beliefs about the class context.* The large amount of studies done on each of these categories separately learns that useful distinctions can be made between subcategories of beliefs within each of the three categories. The category beliefs about mathematics education contains: (1) students' beliefs about mathematics, (2) about mathematical learning and problem solving, and (3) their beliefs about mathematical teaching. Students' beliefs about the self refer to (1) their intrinsic goal orientation beliefs related to mathematics, (2) extrinsic goal orientation beliefs, (3) task value beliefs, (4) control beliefs, and (5) self-efficacy beliefs. Within students' beliefs about their specific class context one can differentiate between (1) beliefs about the role and the functioning of their teacher, (2) beliefs about the role and the functioning of the students in their own class, and (3) beliefs about the socio-mathematical norms and practices in their class.

Based on these insights on the key dimensions and the functioning of belief systems, and the broad categories of beliefs that turned out to be constitutive, students' mathematics-related belief systems can be defined as *the implicitly or explicitly held subjective conceptions students hold to be true about mathematics education, about themselves as mathematicians, and about the mathematics class context. These beliefs determine in close interaction with each other and with students' prior knowledge their mathematical learning and problem solvingactivities in class.*

The categories identified here are not really new and can be recognized in much of the research done in the past years summarized in the Introduction. However, the complementary and theory- based way in which the different categories and subcategories are defined, and the scope of the beliefs involved distinguishes the developed categorization from earlier work. Especially, the encompassing nature with its focus on the relations *between* the relevant categories (i.e. the systemic nature) and not only on the identification of each one of them, might turn this categorization into a valuable framework to understand, and further investigate the role of mathematics-related beliefs in students' learning and problem solving behavior.

Indeed, this hypothetical framework of students' mathematics-related belief systems grounded in what we know about the nature and the functioning of beliefs is in line with Schoenfeld's more general view on the different kinds of beliefs that determine a person's cognitive actions in research settings (Schoenfeld, 1983). He points out that cognitive actions are

> often the result of consciously or unconsciously held beliefs about (a) the task at hand, (b) the social environment within which the task takes place, and (c) the individual problem-solver's perception of self and his or her relation to the task and the environment. (p. 330)

THE STRUCTURE OF STUDENTS' MATHEMATICS-RELATED BELIEF SYSTEMS: AN EMPIRICAL ANALYSIS

Research question

Although the presented framework on mathematics-related belief systems is in accordance with much of the empirical evidence provided by the many "isolated" studies of students' beliefs, the lack of research that focuses on belief systems as a whole rather than each of its constituents separately, seriously questions the validity of the model. Therefore, our main research question for this study was: *Can we find empirical evidence supporting the validity of the structure of mathematics-related beliefs as presented in the theoretical framework?*

Method and instruments

In order to be able to test the validity of the presented framework we constructed a mathematics-related beliefs questionnaire consisting of several scales and subscales designed and intended as operationalizations of the different categories and subcategories constituting the model. Starting from existing questionnaires who usually measure only one kind of beliefs (e.g., or beliefs about math, or beliefs about the self), we developed a more integrated instrument that asked students about their beliefs on mathematics education, on the self in relation to mathematics, and on the social context in their specific class. Since beliefs about the social context have been very rarely studied using a questionnaire, we limited our operationalization of this concept to one component of it, although we are aware that others, as for example the role of fellow students, might be as important. More specifically, recognizing the important impact the teacher has on students' behavior in the class, we focused in this study on measuring students' beliefs about the cognitive, motivational and affective dimensions of their teachers' behavior. These dimensions refer respectively to students' beliefs about how their teachers organize instruction in class, how motivating they are, and how empathic and sensitive they are to students' needs. This resulted in the experimental version of the Mathematics-Related Beliefs Questionnaire (MRBQ) containing 58 items that are scored on a 6 point Likert-scale, from 0 (I completely disagree) to 5 (I totally agree).

Procedure and subjects

The data we will discuss are gathered from a sample of 365 Flemish junior high students (age 14). The experimental version of the MRBQ was administered once during the spring of 1999. Twenty-one classrooms were sampled spanning the different tracks students can follow in the second year of secondary education in Flanders. Although the core subjects-matter domains, including mathematics, are the same for everyone, students choose optional subjects that can be either vocational oriented (technical courses), humanities oriented (courses in humanities), and/or classical oriented (Latin/Greek courses). Generally speaking, the choice of optional subjects is not neutral, but related to the intellectual level of the students. Moreover, the optional subjects taken by the students are used in most schools as a grouping criterion for classes, resulting in relatively homogeneous class groups. In our sample 109 students were vocational oriented (low intellectual level), 119 students took humanities courses as optional subjects (moderate intellectual level) and 137 students were classical oriented (high intellectual level).

Data analysis

A principal component analysis was performed on all the items. The number and meaning of the principal components derived from this analysis could shed light and provide clarification concerning the question: Which beliefs categories and subcategories have empirical grounds? Our decision to choose this type of analysis, other than for instance a confirmatory factor analysis, was based on the exploratory nature of the study. After all, we did and do not know of any study so far that investigated the validity of the different categories of students' mathematics-related beliefs in relation to each other.

Results

An analysis of the scree plot reveals that not more than six factors should be extracted (they all have eigenvalues >1). A four-factor solution accounts for 38.3% of the variance and allows for the best interpretation of the major common factors.

Items with a high loading on Factor 1 (Beliefs about the role and the functioning of their own teacher) included the following: "Our teacher is friendly to us", "Our teacher appreciates it when we have tried hard", "Our teacher really wants us to enjoy learning new things".

Examples of items that are highly loading on Factor 2 (Beliefs about the significance of and competence in mathematics) are: "I like mathematics", "I can understand even the most difficult material presented in a mathematics course", "I'm interested in mathematics". These are all items relating to task-value beliefs and self-efficacy beliefs.

Items with a high loading on Factor 3 (Mathematics as a social activity) refer to the usefulness of mathematics in real life and, more generally, to the fact that mathematics is grounded in human practices and is perceived as a dynamic discipline. Items as, for example, "Mathematics enables men to better understand the world he lives in", and "Mathematics is continuously evolving, new things are still discovered" that refer to a socio-constructivist view of mathematics (Ernest, 1991) load significantly on this factor. This is also the case for items that refer to the related socio-constructivist perspective on (mathematics) learning and problem solving, as for example, "Anyone can learn mathematics" and "There are several ways to find the correct solution of a mathematics problem".

Items that are highly loading on Factor 4 (Mathematics as a domain of excellence) refer to students' extrinsic goal orientation beliefs, on the one hand, and reflect an absolutist view of mathematical learning and problem solving, on the other hand. The following items are good examples: "By doing the best I can in mathematics I want to show the teacher that I'm better than most of the other students" (extrinsic goal orientation) and "There is only one way to find the correct answer on a mathematics problem" (absolutist view on mathematical problem solving). Overall, they deal with the importance to excel in mathematics and specific characteristics of the (problem-solving) process related to it.

The correlations between the different factors indicate that students holding a more social, dynamic view of mathematics (Factor 3) attach more value to mathematics and have more confidence in their mathematical capacities (Factor 2) (r = .48). Moreover, they also tend to have more positive beliefs about the teacher and his functioning in class (Factor 1) (r = .41). The correlation of .38 between Factor 1 and Factor 2 indicates that students holding positive beliefs about their teacher also consider mathematics more valuable and feel more confident about it. Rather surprising, a low positive correlation (r = .21) was found between Factors 3 and 4 which implies that both views of mathematics certainly should not be treated as the opposite poles of one dimension.

Following the exploratory analysis the internal consistency estimates of reliability (Cronbach's alpha coefficiënt) were computed for the scales representing the four factors. The scale on students' beliefs about the role and the functioning of their own teacher had a very high alpha (.92), as did the scale on the beliefs about the significance of and competence in mathematics (.89). There was a higher variability in students' responses on the Factors "Mathematics as a social activity" (alpha = .65) and "Mathematics as a domain of excellence" (alpha = .69) scales. Taken together, however, the principal component analysis and the alphas suggest that the four-factor model is a reasonable representation of the data and that an adjusted version of the MRBQ[1] might provide us with an instrument to validly and reliably measure students' belief systems.

Conclusions and discussion

The four-factor model resulting from a principal component analysis shows that there is some empirical ground for the proposed structure of students' mathematics-related beliefs. The three main categories differentiated in the hypothetical framework, can be identified in the four-factor model. Factor 1 refers to beliefs about the social context, Factor 2 to certain beliefs about the self, Factors 3 and 4 to beliefs about mathematics. Clearly, these empirical factors are not entirely constituted as theoretically expected. Many of the hypothesized subcategories are not validated or do not relate to each other in the expected ways.

There is, however, clear evidence for the relevance of students' beliefs about the role and the functioning of their own teacher. Indeed, they appear to have clear views on the cognitive, motivational and affective dimension of their teachers functioning that can be expected to influence their behavior in class. Moreover, the way students feel accepted by the teacher and find him sensitive to their needs, seems to be related to how motivating they perceive their teacher to be and how he organizes instruction, since items referring to these subcategories are significantly loading on the same factor.

Further, the results point to the relevance of students' beliefs about the self in relation to mathematics, and more specifically of the conceptions of their competence in mathematics and their views on the personal relevance of mathematics. The clustering of these two subcategories in one factor indicates that students who are confident about their mathematical ability are mostly also the ones who are convinced about the relevance of mathematics; this can create a solid motivational basis. Those with low self-confidence, on the other hand, are also not convinced of the importance of mathematics, which implies that there is a group of students that will be very difficult to motivate for mathematics.

Students' beliefs about mathematics appear to split up in two mildly related dimensions (r =.21). On the one hand, students perceive mathematics as a social activity, or not (Factor 3). On the other hand, they view it as a domain of excellence, or not (Factor 4). The socio-constructivist view of mathematics (Ernest, 1991) is clearly present in Factor 3. Items related to an absolutist view of mathematics, however, do not load on this factor at all (a negative loading was expected). Some, however, that refer to an absolutist view on mathematical learning and problem solving load on Factor 4. This might indicate that these two theoretically "opposite" positions towards mathematics and mathematics learning are not that contradictory in the classroom context as could be expected. A possible explanation could be that the orientation toward achievement and grading that up to a certain point always characterizes a mathematical school context, might account for the necessary presence and acceptance of certain absolutist characteristics. For example, on most items of a traditional mathematics test there usually is only one correct answer. These grading related aspects of

[1] Adjusting the scales in accordance with the components found and leaving out all items that load on more that one factor or have a loading of less than .40.

mathematics in school can be perceived by students quite independently of what one thinks mathematics and mathematical learning and problem solving should really be about, accounting for the presence of two different factors. Still, the results of the analyses have to be treated carefully since the internal consistency of Factors 3 and 4 are not very satisfactory. However, the data give good initial support to the hypothetical theoretical framework. Further analysis and development of the scales of the MRBQ, should allow us to upgrade the instrument, which will then make it possible to come to more reliable and detailed conclusions.

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