MATCHING UNUSUAL WORD PROBLEMS WITH GIVEN ANSWERS

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1.Introduction

A series of studies accomplished over the past few years have investigated students' reactions when confronted with non-reasonable or nonsensical problems. It has been observed that students solve these problems in a stereotyped, mechanical, thoughtless and unrealistic way (Verschaffel, Greer & De Corte, 2000).

In addition to pupils' responses to unreasonable or unsolvable problems, the present study examines their reactions when asked to choose only one among three problems matching with a given numerical answer in some groups of problems.

2.Theoretical Background

Based on the literature, it can be revealed that one of the most significant aims of the teaching of mathematics is the development of the mathematical skills of problem solving. Problem solving does not only need the use and performance of typical arithmetical strategies in order to arrive at a numerical result. Mostly, it requires the proper use of **real-world knowledge** and **sense making faculties**. The pupils' tendency to approach word problems without paying attention to the context and without any reference to their common sense was observed by Verschaffel et al.(1994) (in Verschaffel, Greer & De Corte, 2000). In fact, a test consisting of ten matched pairs of items (10 standard items and 10 problematic items) was given to pupils (10-11-year old) in Belgium by the researchers we mentioned above. It was found that pupils demonstrated a very strong tendency to exclude real-world knowledge when dealing with the problematic versions of the problems. Because of the replications of these findings in several countries (i.e. Germany, Japan, Switzerland), Verschaffel, Greer & De Corte (2000) support the universality and the consistency of these findings observed in that specific study.

A factor contributing to the pupils' mindless and unrealistic way of problem solution may be the stereotyped, artificial, [without variations]=unvaried, ordinary and **traditional nature of word problems** that are included in the mathematics textbooks and in the standard forms of assessment, or posed during the instructional practice of word problem solving (Verschaffel, Greer & De Corte, 2000). As a result, pupils resort to the generalization and the development of routine behaviors even in situations that it is not appropriate to do so. This statement can explain the same disappointing findings of studies similar to the study of Vershcaffel et al. (1994), which involved hints that some of the problems need more careful consideration, and gave pupils help to consider alternative responses taking into account realistic considerations (in Verschaffel, Greer & De Corte, 2000). An example of such a study was the one conducted by Yoshida, Verschaffel and De Corte (1997) with Japanese pupils, proving that the pupils' tendency to exclude realistic considerations from their interpretation of arithmetic word problems was "deeply entrenched and resistant to change" (in Verschaffel, Greer & De Corte, 2000, p.33).

Another factor, which can explain pupils' behavior toward the process of solution of word problems, is associated with school and class climate, as well as, teachers' norms and expectations, as far as problem solving is concerned, and generally with the classroom culture. The latter factor may be considered as a part of the concept of **"didactical contract"** which was first introduced by Brousseau (1983). According to him, it is a set of partly explicit and mainly implicit set of rules that defines the relationship between the teacher, the pupil and the mathematical knowledge. Learning cannot be obtained under the conditions of the "didactical contract", but under the [breaks]=breaches of it (in Gagatsis, 1992). The findings of the studies, presented above, support that most pupils' behavior, when solving word problems, is explained through this particular concept. For example, pupils are obliged to give an answer to every problem presented to them, which (according to them) is always correct and sensible, so they combine (all) the data of the problem to arrive at an answer.

An approach that Verschaffel, Greer & De Corte (2000) suggest, so that pupils can deal with word problems in a competent, effective, realistic and sensible way, is an instructional strategy that promotes the **mathematical modeling** presented in Figure 1. According to them, mathematical modeling is the link between the two aspects of mathematics: reality and abstract - formal model (mathematical structures).

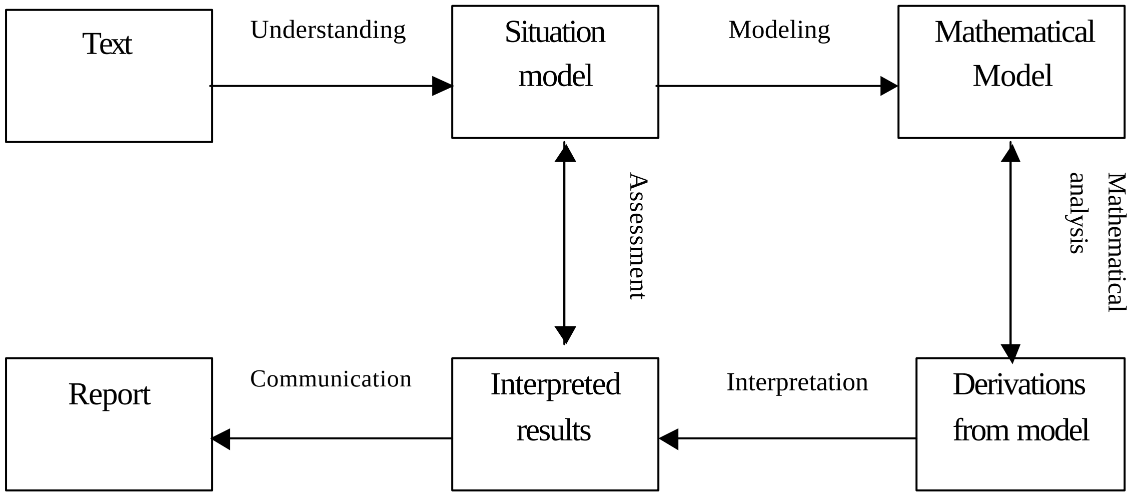


Figure 1

3.The study

Purpose The purpose of this study was to investigate whether the need to select one problem among three word problems (a standard, a problematic and a "parallel" one) so that it matches with a given numerical answer (which is the result of the standard item and the non-realistic answer to the problematic item) could help pupils include and use realistic considerations in their interpretations for the solution of word problems; in other words could lead pupils to a breach of the didactical contract.

Methodology *Data Sources*

In order to collect the data needed for this study, a questionnaire was constructed. It consisted of seven groups of problems. Each group consisted of:

(a) a standard item (S-item) that could be solved unproblematically by applying the most obvious arithmetic operations with the given numbers, and was the one that matched with the given numerical answer

(b) a problematic item (P-item) for which the appropriate mathematical model was less obvious if realities of the context were taken into account, but if not paying attention to these realities, the derived answer matched with the given numerical one.

(c) a "parallel" item (O-item) which had nothing to do with the given numerical answer but in most cases its context was similar to the other two items.

For each group of problems a numerical answer, which was the correct result of the S-item and the unrealistic result of the P-item, was given. The example below presents one (the fifth) of the groups of problems: **6**

A. A boat travels with a speed of 45 km per hour. How long would it take it to travel 270 km? (S-item)
B. A car drives with a speed of 120 km per hour. How far can it go in 20 minutes? (O-item)
C. Tom' s best time to run 1000 meters is 3 minutes. How long would it take him to run 2000 meters? (P-item)

Five of the S-items and P-items were based on the item pairs that were involved in Verschaffel et al. (1994) (in Verschaffel, Greer & De Corte, 2000). Actually, in the P-items (a) of the 1st group, an exact numerical answer couldn't be found because it was somewhere between two numbers, (b) of the 2nd group, there was an intersection between the two sets that was not defined, so there was not only one answer, (c) of the 3rd group, the interpretation of the remainder in the division was needed, (d) of the 5th group, analog couldn't be used as a strategy for solving the problem, (e) of the 7th group the connection of two pieces of planks was not possible. The other groups contained P-items with contradictory (P4) and superfluous data (P6).

Pupils were asked to solve all the items explaining their solution strategies and circle the item that they singled out as the only answer matching the number they were given above? every group of problems.

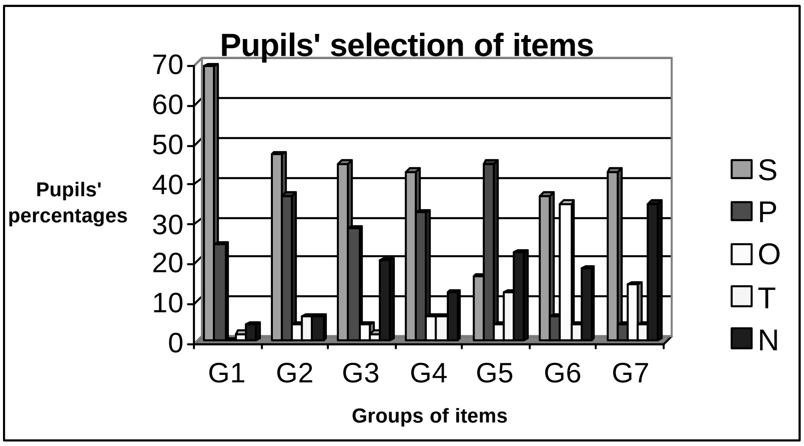
Participants The sample of the research consisted of elementary school children and more specifically 22 fifth grade children and 27 sixth grade children. A total of 49 children participated in the study.
Data Categories For all the pupils and the groups of problems, pupils' choices were collected and grouped in five categories based on the problem, which was considered as the one that matched the given answer:
S if the S-item was selected, P if the P-item was selected, O if the O-item was selected, T if two items were selected
K if no item was selected

The ways they used to solve the P-items were grouped into separate categories too:
Realistic Answer (RA), Non-realistic Answer (NR), A technical mistake in the execution of arithmetic operations (TE), No Answer (NA), Other Answer (OA)

4.Results

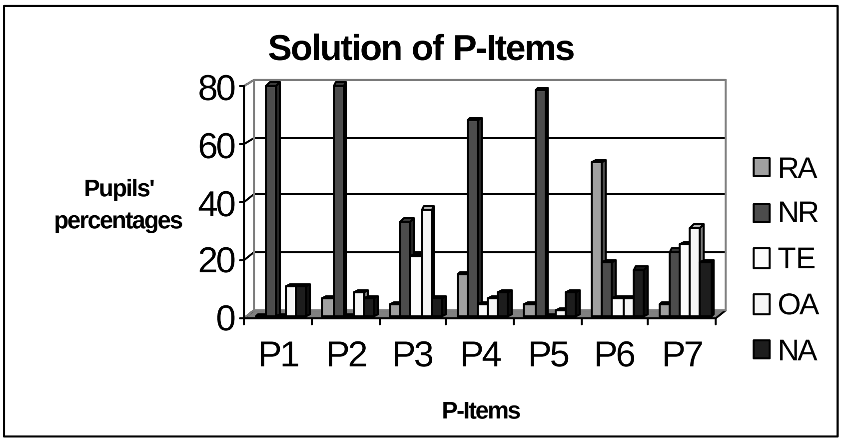
The results presented below are based on the comparison of percentages of the pupils' outcomes in the test. They are also based on the statistical analysis of the collected data performed according to the Gras's Implicative Statistical Model.

Graph 1 presents the percentages of pupils, based on their choice of the item they believed that matched with the given answer for every group of problems.



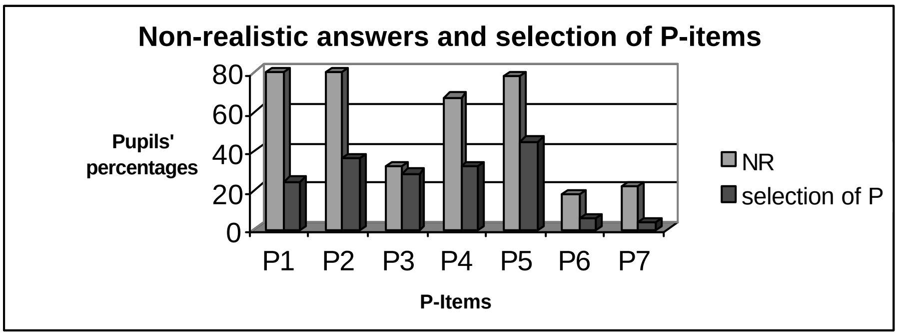
Graph 1

For all the groups of problems (except the 5th one), the majority of the pupils' selected the S-item as the one that matched with the given numerical answer. The percentages of pupils who chose the P-items were 4,1-44,9%. The highest percentage of the pupils who selected the P-item, was presented in the 5th group of problems (S-item with the boat and P-item with the runner, where analog could not be used as a solution strategy). Graph 2 gives the percentages of pupils based on the way they [used]=employed to solve the P-item in each group of problems. Graph 2:



The percentages of pupils who gave a realistic answer were relatively low (0-14,3%) in all the P-items, apart from P6 (problem with superfluous data), where the relevant percentage reached 53,1%. The largest percentages of non-realistic answers appeared to the P-items of the 1st, the 2nd and the 5th groups of problems.

Graph 3 gives the percentages of pupils who answered in a non-realistic way to P-problems and the percentages of pupils who eventually selected the P-item in each problem group.

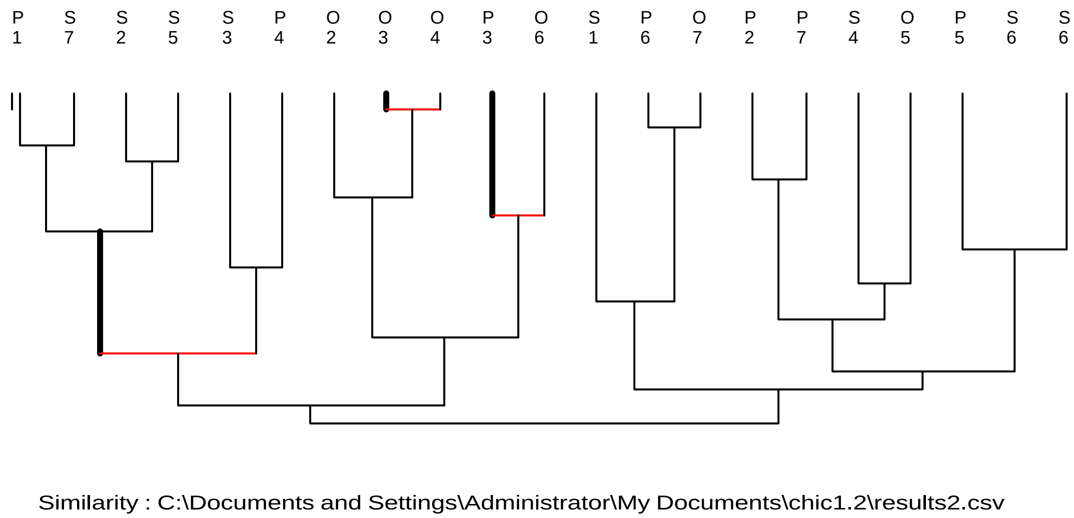


Graph 3

It can be observed that the percentages of pupils who answered in a non-realistic way to P-items are very high, compared to the percentages of pupils who decided to select the same problems, as those matching the given answer. The reason for this result is that the majority of the pupils chose the S-items instead (even though they ended in the same answer for P-items), as mentioned above.

Gras's implicative Model

Similarity Diagram



The similarity diagram shows how problems are grouped, based on the similarity of pupils' selections. It seems that no groups consisting of S-items or P-items are clearly distinguished. The "mixing" of the S and P problems indicates that the possible finding, which supports the breaks of the didactical contract by the general pupils' behavior (since the majority of the pupils selected S-items), is questionable. That is because the P-items and the S-items were approached and solved by the pupils in similar (mechanical and artificial) ways.

5.Conclusions According to the results, in almost all the groups of problems, the majority of pupils chose the S-item as the correct match of the given numerical answer. Contrary to the above finding, it has been found that, relatively high percentages of pupils (reaching 44,9% in some groups of problems) selected the P-items. This finding may be explained by the fact that most pupils solved the P-items in a non-realistic way, which led them to the answer that matched the given one. However, the percentages of pupils who selected the P-item in each group of problems were much lower than the percentages of pupils who gave a non-realistic answer to the P-item. For example, as far as P1 is concerned, the percentage of pupils who solved it in a non-realistic way was 79,6%, whereas the percentage of pupils who selected P1 was just 24,5%. The certain finding indicates that the majority of pupils who solved the P-items in each group of problems in a realistic way were not sure about the correctness of the solution they proposed and considered (maybe intuitively) the answers of the S-items more suitable. That was perhaps the reason for choosing the S-item instead of P. But, that kind of reflection did not seem to play a dynamic or meaningful role, so as to make pupils reconsider the P-item and their unrealistic-mechanical solution strategy they used to solve it and, therefore, activate realistic considerations related to it. This conclusion concurs with the similarity diagram of pupils' choices, produced by Gras's statistical analysis, where the non-existence of grouping between the P or between the S-items proved the similarity between pupils' reactions and processes toward P-items and their reactions and processes toward S-items. In other words, they confronted the S-items in the same way they dealt with the P-items.

As for the ways pupils followed to solve the P-items, the percentages of pupils who gave realistic answers were relatively low at almost all the P-items. This finding indicates the pupils' strong tendency not to use or include realistic considerations in their interpretations for word problems. It also concurs with the findings of many researchers (in Verschaffel, Greer & De Corte, 2000), who investigated the outcomes of students in several countries toward the same or similar kind of problems. The pupils' behavior presented above may be a result of the influence of the "didactical contract", which characterizes the process of problem solving in class. Actually, it makes pupils assume that they have to approach unusual and unrealistic problems in the usual way, even though they might have effectively thought about the problematic or unsolvable nature of the P-items (Verschaffel, Greer & De Corte, 2000). Contrary to this finding, it is worth saying that for one P-item, more specifically P6 (problem with superfluous data), the breach of the "didactical contract" was a fact, since the majority of pupils (55,1%) answered in a sensible way, while only 6,1% of them selected it as the one that matched with the given answer. This might be explained by the fact that the solution of a problem with superfluous data does not necessarily need to take into account the realities of its context or to activate and use realistic considerations. But, it certainly requires the careful and meaningful reading and understanding of the problem, something that the majority of pupils in this study obviously did.

The findings of the present study indicate the need for dealing effectively with the situation, which influences pupils to confront word problems in a superficial and uncritical way. Furthermore, it stresses the important role of effective reforms concerning the framework within which to teach word problems. In fact, according to Verschaffel, Greer & De Corte (2000), emphasis must be given to (a) the use of greater variations of problems, which should be complex, interesting, with multiple steps, superfluous or missing data, (b) the practice in realistic mathematical modeling and, (c) the motivation of pupils to plan and construct their own word problems. Moreover, pupils should be given the chance to confront not only ordinary and traditional problems, but unusual and non-realistic problems as well, in order to become competent to compare the different strategies they use for solving them, to be involved in decision-making processes and to develop metacognitive and self-assessment skills. A positive contribution to this effort of reform may be offered by the more systematic use of unusual tests or written tasks, having perhaps the form of the questionnaire used in the present study. Although, its form did not lead to a drastic breach of the "didactical contract", it seems that it directed pupils to deeper considerations, made them doubt their own solutions to some P-items and even question the logic and data of these problems.

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