General Education in Mathematics Lessons
An Introduction to Humanistic Holistic Education

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Abstract Mathematic subjects by themselves are not the goals of mathematics lessons for pupils in regular classes. Rather the general education children are learning by working with mathematics should be the aim of mathematics teaching. Therefore reflection on mathematics and mathematical thinking is necessary especially under the aspect of humanistic holistic education.
Remarkable points are: Mathematics as system of concepts which give us clarifications and orientations, mathematical thinking as special way to work on problems, mathematical knowledge and skills as tools, the method of simulation, limitations of mathematical modelling and the aesthetical effects of mathematical objects.
Mathematical teaching then can be reflected under these points in five dimensions: Pragmatic dimension, Enlightment dimension, Social dimension and dimension of Personal factors and Critical reflection.

Since the middle of the 80th in Germany general education is discussed again because the science-orientated teaching in the 70th gave no orientation and no midpoint to integrate the lot of learned details. The famous German pedagogue Wolfgang Klafki therefore stimulated the discussion on general education again whereat his revised version of the classical concept of general education focuses on the following three points:

- education for all (all classes of population and all nations)
- universal education (learning by head, heart and hand)
- education with respect to the so-called 'key-problems' of our world today
  (especially maintenance of peace, prevention of hunger, ecology and economy)

Also in didactics of mathematics the discussion about general education was forced again[1]. Since 1989 within the German Society of Didactics of Mathematics[2] we have a working group[3] whose aims is it to support general education in mathematics lessons.
Mathematics education has a long tradition and the subjects seem to be self-evident. But *what is the sense, the aim of mathematics education at school?* We know that a lot of people have problems with mathematics. This alone should push us to reflect on the relevance of mathematics education. The other way round I could ask you: What do you answer when you are asked why do you teach one subject rather another respectively why do you pick up one method rather than another one? Of course we have to differentiate whether we are asked by a pupil or a parent of a pupil or a colleague or whether we want to get clarification for ourselves. Also we can't deal with this question every time. However the question about the relevance of teaching within the scope of humanistic holistic education should be borne in mind at any time we are teaching.
This means that a teacher at least should know why he or she deals with a certain topic, why it is presented in a particular way and most of all to what aspects one should call special attention.

In the following I want to give you some assistance for this work. I begin with reflections on mathematics and mathematical thinking in everyday world that is I will refer to remarkable points of mathematics and mathematical thinking which are important in respect to general education.

1. First of all <u>mathematical knowledge and skills as widely known can be used as tools to solve problems of everyday</u>. We can find many situations in everyday life where mathematical tools can help a lot[4]. How broad normal people should learn this has to be discussed more precisely. At the moment I only want to point out that the modelling of world and the application of mathematics are things that have to be learned extra, in addition to mathematical knowledge and skills. Moreover in a certain

[1] See e.g. Graumann 1990, 1993-1998b and Heymann 1996 or especially Graumann 1992.

[2] Named „Gesellschaft für Didaktik der Mathematik“.

[3] Under the leadership of Karl Röttel and Günter Graumann.

[4] See e.g. the MISP-material of Alan Rogerson or Graumann 1987a/b.

degree it is also possible to train general abilities (like creativity and problem solving ability) which are helpful for solving problems of everyday by acting with pure mathematical problems.

2. A fundamental characteristic of human beings is that they organize life with help of concepts. We know of course that with concepts and modelling we never catch our empirical knowledge totally, but we have no other chance to get orientation for our life. In addition we get the possibility to reflect about experiences of the past and to make plans for the future. Mathematics therefore is to be seen as a specially elaborated system of concepts which put us clarifications and orientations at our disposal.
For example the concept of a straight line or parallel lines helps us to think about the shortest distance between two locations respectively the constant distance of the two rails of a railway or the same direction of sun-beams. Also the generalized use of parallelism like parallel swings by skiing or parallel classes in school we only can understand if we know the mathematical concept. In this way we can find a lot of mathematical concepts which have importance for everyday life. Also several terms, idioms and facts we use in everyday life have their roots in mathematics - especially the history of mathematics - (like the subdivision of hours in minutes and seconds or the idiomatic expression "squaring the circle") or in the mysticism of numbers (like the subdivion of the week in seven days or the trinity of god, godsun and holy spirit).
In this context of mathematical concepts we also can mention the so called operative formation of concepts where the function of a concept is well to be the fore. In a plane for example we have homogeneity of all points and we can move in all directions or if we want movement of water or sand or another similar material in one direction by rotating an axis of a machine we have to build something like a winding surface. Also the concepts of numbers can be seen in this functional way (e. g. natural numbers as abstraction of counting acts).

3. Traditionally mathematics concentrated on the aspects of quantification and spatial form. However in modern mathematics the abstract forms, structural aspects and functional connections are more important. With this we also touch a point of everyday life which can be named as general abilities developed while working with mathematics. Just in our world of today the *structuring of fields of experiences* is a necessary task to master life in an appropriate way. Therefore mathematics education can supply us a *special way of thinking for better understanding of our world and better ability to act.* Just mathematics built a wide field for experiences of connections and functional relationships. Moreover alive mathematics demand not only linear, algorithmic thinking (as – unfortunately – it is often to be seen in school) but also *creativity, problem-solving, more-dimensional thinking, looking out for connections and structures* as well as *finding and holding out till the end while solving problems*. I think this to learn especially today is a very important task for everybody because of the lot of things we get presented bit by bit.

4. Recently a new practice of solving problems in science and technology - the method of simulation - gains importance more and more. Already at ancient times it was possible to predict certain effects of the realisation of special plans with help of mathematics, but with the modern computers we can simulate complex events which can't be calculated. These simulations then give us ideas of the possible consequences. So we can think about consequences without real experiences which may have bad unrepairable effects.
In mathematics lessons we can learn such methods already on a small scale e.g. by discussing different systematic variations in a given word problem (even without using a computer) or by simulating lotteries (with a random generator). In upper grades we could simulate a more complicated situation with computer e.g. different predicted growths of mankind.

5. Not for all problems respectively all situations a solution with help of mathematics is appropriate. Therefore the reflection on senseful limitation of mathematical modelling is also a necessary topic of learning mathematics.

This begins in grade 1 with discussing that calculating money doesn't constitute the whole life or with clarifying that the problem "5 - 7" is unsolvable and can end with reflecting on chaos-theory.

6. Furthermore in mathematics especially you can make experiences that human mind starting with everyday problems can come to pure theoretical questions and that pure theoretical questions can fascinate men. This only speculative aspect of human mind is also one part of our life and should not be left behind in the education. In mathematics we find this aspect first and foremost with *looking out for rules, structures and mathematical aesthetics or making investigations and problem posings in special problem fields*.

7. Last but not least in respect to our reflections about mathematics and mathematical thinking it has to be mentioned that parts of mathematics, especially in geometry, have normal aesthetical effects. The feeling of aesthetics is also one aspect of life which should be developed in school.

I hope you got already an idea of the points at which we have to pay attention in respect to general education in mathematics education. But we also can bring closer the ideas to us if we look for the intention of learning at school in general.

8. The aims of school in general sorted into five dimensions which bring us also to aspects of humanistic general education in mathematics lessons can be described as:

a) Pragmatic dimension (that is: what we directly need or what can help us in everyday world)
b) Enlightment dimension (that is: for better understanding of our world)
c) Social dimension (that is: becoming an active member in groups and working with on social problems)
d) Dimension of personal factors (that is: all those abilities we need to become an integrated person and a person which can master life)
e) Dimension of critical reflection (that is: reflecting our acting and the limits of techniques we are using)

In the following for each dimension I will give a short explanation and some suitable examples of mathematics education.

Ad a (Pragmatic dimension): The first task of school is to equip every person with knowledge, skills and abilities so that he or she can master his or her present and predicted future life in an appropriate way. Here the terms "life" and "to master life" have to be seen in a wide meaning. With the term "life" you must think of all outward events but also the causal and social interlockings with nature and with communities like family, community of a house, municipality, nation or mankind. And with the term "to master life" you have to see the solving of problems not only in the definitive way - we often do in mathematics - but also by getting different ideas to work on the problem or by comming to the conclusion that the problem can *not* be solved with the presently given abilities. Also we have to concede solutions with the aid of special literature or with help of experts or by working in a team.

Thus the pragmatic dimension of general education doesn't include only solving problems of everyday life. Also special ways of thinking as already mentioned and abilities of communication have to be included. But for all of that we have to acquire clear and funded concepts.

For ***mathematics teaching*** therefore first of all it is important to *learn concepts not only by one definition but also by using them in connection with problem solving and underlining different characterisations as well as clearing their different use in everyday world*. In addition the pragmatic dimension includes *skills of calculation and drawing* as well as *abilities of modelling and of applying mathematics in problems of reality* and abilities which belong to the dimension of personal factors like the *abilities of classifying, ordering, formalizing, systematizing, generalizing, finding analogies and heuristic strategies and the ability of imagination in space*.

This includes for example a good comprehension of numbers (natural numbers, integers, rational numbers and decimal numbers) and quantities as well as techniques to compute with them or knowledge about instruments for measuring time or calendars and knowledge about computers. Also the concepts of a triangle, different types of quadrilaterals, the cube, pyramid, cylinder and sphere as well as the fundamental idea of probability is helpful in everyday life. Moreover the way of acting and behaviour in complex situations should be learnt in mathematics lessons while discussing simulations or chaos theory and working in complex problem fields. Even abilities of rational communication we have to integrate into this dimension.

Ad b (Enlightment dimension): If we want to use mathematical concepts, structures and techniques properly we have to know them in a good way. But to open the world for somebody also knowledge of objects and connections which can not be used in everyday world directly are necessary.

Some headwords of ***mathematics*** which may explain this dimension are: *Symmetry, infinety, different growth of special functions, behaviour of dynamic systems, principles of computers, basic ideas of probability and the role of real analysis in physics and technology.*

The phenomenon that the human mind by special situations of life can be stimulated to pure theoretical questions also belongs to this dimension. This can happen for example with *analysing parquets and ornaments or computing the probability of games of chance*. Other pure theoretical mathematical topics which should be heard of by everyone are *the distribution of prime numbers, the construction of Pythagorean triples and the figurate numbers, the proofs of irrationality of special roots, the mightiness and commensurability of real numbers and the extension of Euclidian geometry to projective geometry.*

Furthermore the so-called "cultural coherence" belongs to the dimension of enlightment. For ***mathematics education*** this means that we also have to teach e.g. about the *genesis of our calendar, numerals and ornaments in prehistoric times, the origin of proofs and its connection with religion or philosophy in the ancient world, the way of positional notations of numbers from Babylon to India to Arabia and finally to Europe, the development of algebra during the renaissance and its influence for music*, and last but not least *the discovery of non-Euclidian geometry and the arise of modern structural thinking in mathematics.*

Ad c (Social dimension): First of all social objectives concern the way of company with each other and paying respect to any other person (children included). These objectives are not directly related to mathematics learning but to the ***atmosphere in mathematics lessons***. Secondly in everyday world more and more problems are solved in a team. Therefore in school we have to develop abilities of communication and cooperation and to train teamwork also during solving problems of mathematics and especially problems of everyday world as mentioned above.

Ad d (Dimension of personal factors): A fundamental task of school is to develop the personality of each child/teenager. This includes the *development of perception, assimilation, receptivness, visualisation, intution, creativity, ability of abstracting and structuring as well as psychomotoric copetences, attitudes and self-confidence*. Also the *development of different individual talents and interests* should be added to this dimension of general aims of school.

Especially in ***mathematics lessons*** one should pay attention to the following points: *Spatial imagination, visualisation and intuition, ability of abstracting and structuring as well as finding out connections, logical thinking respectively ability of organisating abstract things, finding structures and holding out systems till the end, creativity and ability of discovery, playful handlings with abstract things, ability to find appropriate actings for complex situations.*

Mathematics education of course can have effects in respect to this dimension only in connection with other subjects, enviroment and family.

Ad. e (Dimension of critical reflection): First of all we have to be aware of the limitation of human beings and their thinking. Especially we have to reflect on the danger of the extensive use of special tools, techniques and methods.

In respect to ***mathematics education*** therefore the reflection on the *limitation of mathematical modellings and tools* are important. This can start with discussing the solutions of word problems in respect to reality or the not-existance of a natural number that solves the equation $3 - 7$ in primary school and end with the treatment of mathematical expansion during history in upper secondary school. In upper grades also the way of working on word problems and the discussion of their solutions arranges already a special attitude of mind. Or the dicussion of prime numbers and prime-twins in grade 4 or 5 as well as the dicussion of equations like $2x+3 = 2(x+3)$ and $x^2+1 = 0$ in grade 8 or 9 can show the limitations within mathematics.

Discussing the four-colour-problem can lead us to limitations mathematicians have had. Moreover *the treatment of mathematical expansion in history* can lead to a reflected view of mathematics too.

Also refletions on chaos-theory can show the limitations of our thinking. Hereby it is not meant to choose the chaos-theory as a subject for mathematics lessons (even though it is a good thing that pupils can make experiences with chaos-phanomenons like "game of life" in cellular automata). Rather in this connection I mean the reflection on the positivistic-mechanical thinking which still is commonly held in mind though most of the complex problems of today can't be solved step by step with lots of reductions. A wholistic thinkinking is necessary for that. Moreover in special situations which with chaos-theory can be characterized as "bi-furcation-points" the given causality doesn't work anymore. Summerising this we can say: *On one hand scientifical analysis of complex problems is necessary and on the other hand we can't repose on once done analysis, rather from time to time we have to make a new analysis and new decisions in respect to the given problem.*

In any case we have to keep in mind the limitation of human thinking in general and mathematical modelling in special.

Concluding remark:

I hope I could stimulate you for more discussion about general education in mathematics lessons and fasten the opinion that the sense of mathematics teaching must be seen in the points of general education. Of course this has to be discussed more detailed especially in connection with concrete topics[5].

References

Graumann, G. 1986. Computers and Geometry Teaching. In: Kupari, P. (Ed.) 1986. Mathematics education research in Finland. Yearbook 1985. Jyväskylä, 61 – 79.

Graumann, G. 1987a. Geometry in Everyday Life. In: Research Report 55. University of Helsinki. Department of Education, 11 – 23.

Graumann, G. 1987b. Geometry in Everyday Life. In: Blum, W. et al. (Ed.), Applications and modelling in learning and teaching mathematics, Chichester 1989, 153 - 158.

Graumann, G. 1989. Problem-Orientated Geometry Teaching with Consideration of Computers.In: Pehkonen, P. (Ed.) Research Report 74. University of Helsinki. Department of Education, 141 – 150.

Graumann, G. 1990. "Allgemeinbildung durch Mathematik" als Aufgabe der Lehrerbildung. In: Beiträge zum Mathematikunterricht 1990, 103 – 107.

Graumann, G. 1991. Wie können wir aus der Chaostheorie ein neues Denken für die Behandlung komplexer Probleme ableiten? In: Köhnlein u.a. (Ed.) Wissenschaft und Verantwortung. Münster: LIT-Verlag.

Graumann, G. 1992. The German school system and mathematical education. In: Morris/Arora (Ed.), Studies in mathematics education - Moving into the twenty-first century. Unesco/Paris. 45 - 54.

Graumann, G. 1993. Die Rolle des Mathematikunterrichts im Bildungsauftrag der Schule. In: Pädagogische Welt 5/93, 194 - 199 + 204.

Graumann, G. 1994. Mathematikunterricht und Allgemeinbildung - mit Blick auf Schlüsselprobleme unserer Welt. In: Mathematik in der Schule 1/94, 1 – 7.

Graumann, G. 1994. Wodurch wirkt der Mathematikunterricht allgemeinbildend - Vier Beispiele aus dem Geometrieunterricht. In: Arbeitskreis Mathematik und Bildung. Mehr Allgemeinbildung im Mathematikunterricht. Buxheim-Eichstätt: Polygon Verlag, 55 – 68.

Graumann, G. 1998a. Beweise - ihre historischen Wurzeln und ihr Sinn im Unterricht. In: Graumann, G. & Röttel, K. (Ed.), Mathe, ja bitte-Wege zu einem anderen Unterricht.Buxheim-Eichstätt:PolygonVerlag,39 - 46.

Graumann, G. 1998b. Ordnen, Strukturieren, Sytematisieren - Gewinn jenseits des Rechnens. In: Graumann, G. & Röttel, K. (Ed.), Mathe, ja bitte - Wege zu einem anderen Unterricht. Buxheim-Eichstätt: Polygon Verlag, 47 - 53.

Graumann, G. 2001. Mathematical Views of Pupils and First-Year-Students. In: Cohors-Fresenborg, E. & Maier, H. & Reiss, K. & Törner, G. & Weigang H.-G. (Eds.), Developments in Mathematics Education in German Speaking Countries, Band 2: Selected Papers from Annual Conference of Didactics of Mathematics, Leipzig 1997, Hildesheim 2001, 62 –74.

Graumann, G. 2002. Übersehene und versteckte Geometrie. In: Graumann, G & Röttel, K. (Ed.), Mathematik: Unsichtbar und doch allgegenwärtig und überall wirksam. Eichstätt: Polygon Verlag (in press).

Hershkowitz, R. 1994. The Role of Geometry in General Education. In: Gaulin et al. (Ed.) Proceedings of the 7th International Congress on Mathematical Education Québec 1992. Sainte-Foy: Les Presse De L'Université Laval, 160 – 167.

Heymann, H.-W. 1996. Allgemeinbildung und Mathematikunterricht.Weinheim: Beltz.

Klafki, W. 1985. Neue Studien zur Bildungstheorie und Didaktik. Weinheim: Beltz.

[5] See e. g. my paper „General Aims of Mathematics Lessons" in this book.