Three roles for technology:
Towards a humanistic renaissance in mathematics education

Barry Kissane

Murdoch University, Australia, http://wwwstaff.murdoch.edu.au/~kissane, kissane@murdoch.edu.au

Abstract: Although modern technology and humanistic perspectives on mathematics education seem unrelated, it is suggested that important and positive connections can be made between the two. Three roles for technology in mathematics education are described, exemplified and defended, and some implications of these for a humanistic view of mathematics education are drawn. A *computational* role is concerned with humans using technology to complete otherwise tedious or difficult mathematical tasks. An *influential* role suggests that the availability of technology needs to be considered in deciding what mathematics is most important for the mathematics curriculum. An *experiential* role emphasizes the new possibilities for teaching and learning mathematics afforded by technology. It is suggested that these roles are only important when technologies are genuinely available to students, so that attention is focused on the graphics calculator, which is interpreted as a humanising device, the best available example of a personal technology for secondary school mathematics.

Introduction

Twenty years ago, David Wheeler (1982) suggested prophetically that mathematics teachers were in the midst of three major educational upheavals: mass secondary education, the rise of new and available technologies and the revolution of humanizing mathematics. Regarding technology, he described the upheaval in terms of "… the technological revolution of cheap electronic calculators and micro-computers. We are in the midst of this and had better get involved in it, if we aren't already, in order to give it a direction consonant with our best hopes for the education of our students." (1982: p. 23) Curiously, Wheeler said little more about the significance of technologies in this mixture, so that the present paper is a brief attempt to make some of the connections from the vantage point of a generation later.

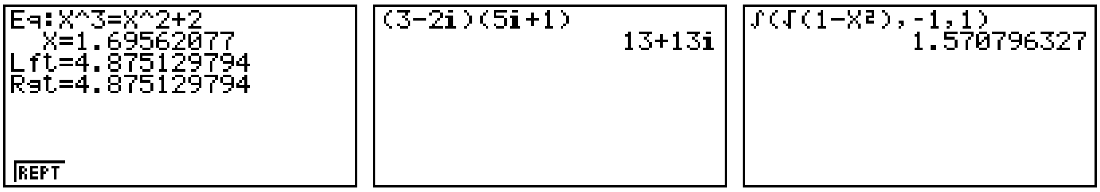
At first glance, it may seem odd to associate technology with humanism, since the two seem commonly regarded as lying on opposite ends of a pole. For many people, the increased evidence of technology in modern societies is seen as a dehumanizing influence, and quite opposed to a notion of humanity. The very idea of technology seems to rely on removing the human from activity and that of humanism seems antithetical to that of technology. Within education, early use of technology such as computers focused on the (mistaken) idea that computers might be able to teach pupils; indeed, in the early seventies, there was even talk of replacing some teachers with computers. Outside the sphere of education, similar fears were common, perhaps starting as early as the Luddites in England, and perhaps better grounded in reality.

Even today, there is a sinister side to technology in the minds of many people. However, all technologies are not the same, and this paper will suggest that a reappraisal of this view of technology as dehumanizing and antithetical to the human spirit is needed. Since Wheeler's paper, technologies have become smaller, more powerful, more portable, less expensive and thus more likely to be useful to and available to pupils learning mathematics in school. In this paper, we sketch three different roles played by technology in mathematics education, specifically calculators, and appraise their significance for a humanistic perspective.

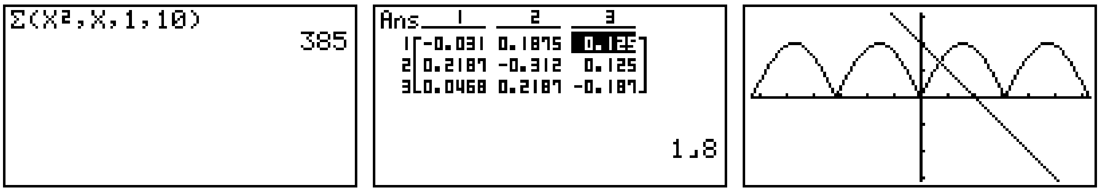
A computational role

In a remarkably short span of time, technologies for performing extensive mathematical calculations have been developed and have at the same time become affordable for many students in affluent western countries. While Wheeler was referring in 1982 to calculators that performed arithmetic, including arithmetic with numbers that previously were accessible only in table books (such as trigonometric and exponential functions), modern advances have increased the capabilities of calculators very substantially. As well as arithmetic, modern graphics calculators can perform the necessary computation associated with almost any mathematical tasks encountered in the secondary school. Over the 17 years since they first appeared, graphics calculator development has been in this precise direction, increasingly tailored to the needs of secondary school mathematics; in this sense, they arguably are the first (and only) example of a technology developed specific ally for secondary school mathematics.

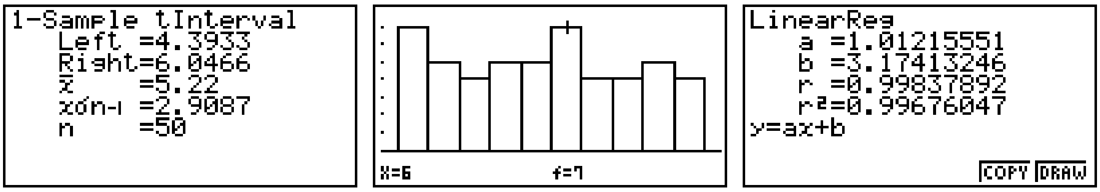
Examples seem necessary to support such a claim, but space precludes a complete treatment. The screen dumps below show numerical computations on a Casio cfx-9850GB PLUS graphics calculator involving the (numerical) solution of an equation, the evaluation of a definite integral and the product of a pair of complex numbers.



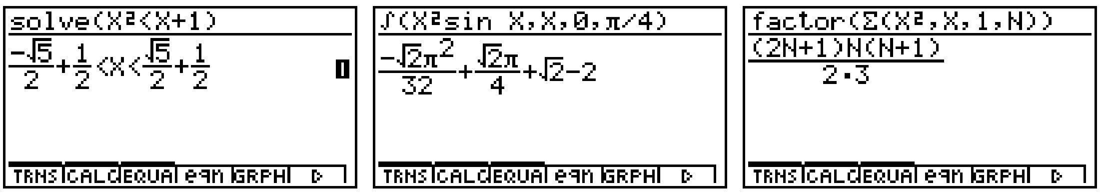
Many other examples are possible, such as those in the next three screens, showing calculator versions of summation of a series (the sum of the squares of the first ten integers), the inversion of a matrix and graphing of functions.



In some areas of mathematics, such as statistics, computation is central, so that a device for performing computational tasks is especially important. The next screens show the results of using a calculator to construct a confidence interval, draw a histogram and find a line of best fit, in each case starting from raw data.



Statistical tasks of these kinds are computationally tedious, so that all too frequently in the past, students have spent large proportions of their time essentially doing arithmetic to obtain the necessary result, instead of reflecting upon the meaning of their work, unfortunately.
Advances in hand-held technologies over the past decade have further invaded and redefined the nature of computation in school. The algebraic calculator (Kissane, 1999) allows graphics calculators like the Casio Algebra fx 2.0 to perform routine symbolic manipulation tasks of the kinds that have previously occupied very large amounts of human time. The next screens show three illustrative examples, in which exact results are obtained, in contrast to the numerical approximations available on a standard graphics calculator.



The significance of technology of these kinds is not just that machines can do such things, but that machines with such capabilities are relatively cheap and potentially available to a great many students in schools. Indeed, the term 'personal technology' seems quite appropriate for devices like these that may be available to humans at a personal, individual level, which is already the case in more affluent countries. Further, compared with computers, personal technologies of these kinds are potentially much more important for students at present and over the next few years, especially in less affluent countries. (Kissane, 1995). Armed with such technologies, pupils can undertake mathematical tasks that were previously inaccessible to them, as Kennedy (1995) has beautifully and whimsically illustrated.

From a humanistic perspective, personal technologies have the potential to restore the person to mathematics. Rather than mathematical work being accessible only to those patient and diligent enough to develop the many procedures for calculation needed for completing the foregoing tasks, technology may widen access to all pupils. Although attempts have been made (and will continue to be made) to preserve ancient mathematical rites of hand calculation, the democratizing device of a graphics calculator will allow pupils to decide for themselves what computation to perform, when to do so and why, subject only to the artificial constraints imposed by education systems and ideologies.

In less affluent countries, access to such technologies is of course more difficult at an individual, personal, level. But in such cases, access to the personal technology of the graphics calculator is a great deal more likely than access to and ownership of a (so-called) personal computer.

An influential role

A second role of technology is as an agent for reconsidering the balance of the mathematics curriculum. The mathematics curriculum is not written on tablets of stone and has always been subservient to the available technologies over time. Over the last thousand years, the development of a technology for place-value numeration and for associated arithmetic algorithms changed the accessibility of mathematics forever. Similarly, the development of logarithms has been described as doubling the working life of scientists of the day. In the more recent past, the people at this conference will have spent considerable amounts of their own school mathematics time learning to do things that are now readily accomplished by the personal technology of a graphics calculator.

A humanistic mathematics education must focus on how humans come to terms with the possibilities available to them, which involves at least helping them to use available technologies in proper ways and at proper times. But it also sharpens attention on what is most worth spending scarce educational time on. Personal technologies offer a prospect that some elements of the traditional curriculum may seem less important than previously, while others may be come more important or appear earlier. There are many examples of this phenomenon.

While it is probably universally agreed that spending scarce time developing expertise with calculations via logarithms or even via long division is no longer intelligent educationally (although a mere thirty years ago, these were standard practice), we are less in agreement regarding the importance of developing various methods of integration. While previously, it was possible to find maximum values of a function only after a study of differential calculus, pupils with graphics calculators can now do this numerically with ease several years earlier. Where statistical work was arithmetically complex, so that performing computations and representing data were each problematic, graphics calculators support such tasks with ease, allowing new opportunities to take data analysis seriously. Recently, Kissane (2002a) noted the range of ways of dealing with equations using graphics calculators, in contrast to the very limited repertoire provided by traditional analytic methods, arguing for a better balance in school curricula.

Technologies for purposes such as these have been available on computers for many years, with almost no discernible impact beyond the rhetorical on the school curriculum. What has changed recently is the prospect of individual pupils having regular personal access to such support, demanding (or at least allowing) that we consider the consequences. Of course, having a technology available is no guarantee that it is a good idea to use it. We may agree that something is too important conceptually to rely exclusively on a machine for doing it. Or we may find through careful empirical enquiry that a strong conceptual foundation for mathematics requires experiences without technology, at least in the early stages. We may even disagree amongst ourselves on what is important or not important to be done by hand. When technology is physically small, portable, relatively inexpensive and potentially available to most pupils, we are obliged to take questions of these kinds seriously.

The influence of technology on the curriculum is not restricted to what can be replaced, or completed more efficiently, of course. A graphics calculator offers a possibility that aspects of mathematics previously inaccessible can be introduced in an intellectually responsible way. An example is the elementary study of chaos, which, among other things, opens up the prospect for pupils to begin to realize that mathematics is a human enterprise going on at the present time, rather than being only the work of the mathematicians who are long dead. This too seems to be an important and different potential contribution of technology to humanizing mathematics.

There is a considerable gap between 'influencing' and 'changing' the curriculum, of course. The school mathematics curriculum seems to be remarkably resistant to change, especially to change that has not been imposed from above, an issue discussed in some detail in Kissane (2002b). Nonetheless, the personal technology of the graphics calculator seems to offer fresh prospects for reappraising the balance of the mathematics curriculum, especially towards a curriculum that focuses on humans making decisions for themselves (including decisions about the appropriate use of technology).

The very existence of technology serves to remind us that calculation is what machines do well. But only humans can do mathematics and it is humans that drive the technology. Informed by an available technology such as a graphics calculator, we may be able to get closer to constructing a mathematics curriculum that focuses on the humans and their thinking instead of the mathematics itself, unlike most of the past.

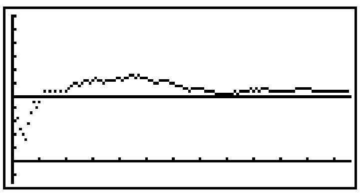
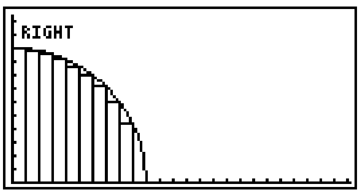
An experiential role

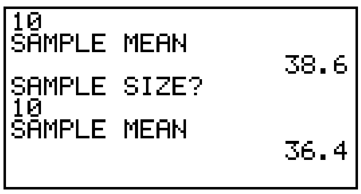
A third kind of connection between a humanistic approach to mathematics education and technology is that technology might be used to provide humans with fresh experiences that are not otherwise available. A conventional view of mathematics is that it proceeds in a logical and formal way, relying on the canons of deductive argument and proof, starting from agreed premises. Such a view has been quite alienating for many pupils over the years, mainly because they have not learned the differences between mathematics in the making and mathematics as a finished product. School mathematics curricula have rarely done a good job of making such a distinction, with the unfortunate consequence that very many people have left school convinced that were mathematically inept, despite the claims of mathematics educators like Wheeler (1982) and others that all can learn to think mathematically or to 'mathematise'.

Technology provides the possibility of restoring an exploratory or laboratory element to school mathematics, recognizing the importance of experimentation and informal exploration before formalisation. Furthermore, learning activity involving elements of these kinds may take place within classrooms as well as individually, widening still further the new possibilities opened.

A modern graphics calculator is an essentially interactive device, which responds to pupil inputs, demands interpretation and allows further inputs. Many examples of the exploitation of these characteristics have been generated and productively used by teachers over the last decade, resulting in widespread enthusiasm for the multiple perspectives offered, encapsulated in the 'rule of three' slogan. This refers to the importance of considering symbolic, graphical and numerical representations of functions, especially in the study of algebra and calculus, since these different perspectives contribute to making sense of the ideas involved. Such interactivity and easy movement between different representations is not available to most pupils without a personal technology such as a graphics calculator. The graphical perspective in particular has been enthusiastically greeted by many teachers and their pupils. (This has perhaps contributed to the unfortunate description of these devices as 'graphing' calculators, thereby inadvertently limiting their interpretation considerably.)

Examples of experiential roles of calculators are not limited to multiple representations of functions, however. Space precludes an exhaustive description of other possibilities, but three of them are reflected in the following screen dumps.



The first of these show a sample output from a small calculator program (called *Longrun*), designed to show the proportion of times a Bernoulli event occurred. (Kissane, Harradine & Boys, 1999). Such a program allows a pupil to get many opportunities to see how randomness is operating, perhaps prior to a formal treatment of the mathematical ideas involved; experience of such kinds is an important complement to analytical treatments. Such experience is easily shared in a classroom community among pupils, or even used in a demonstration setting, with whole class discussion involved. For all three environments (personal, small group or whole-class), the technology provides an experience not easily accessible otherwise.

The second screen also shows a calculator program, *Intarea*, (Dowsey & Tynan, 1998) being used to approximate Reimann sums. The program provides various approximations (only one of which is shown above) and allows pupils to interact by providing their own functions and choice of number of intervals. In this case, the technology provides important experience related to the concepts of integration and convergence that are difficult to replicate otherwise.

The third example chosen shows a calculator program used to select random samples from a population of values, as a first step towards understanding some critical aspects of sampling and statistics, related to the central concept of a sampling distribution, and building the conceptual foundation for difficult ideas related to confidence intervals and hypothesis tests. As with the other examples, experiences of this kind can be personal, shared or collective, depending on how the classroom is structured. Their stochastic nature provides a wealth of opportunities for pupil engagement and learning.

Well-constructed experiences with calculators allow many opportunities for pupils to interact with mathematics in new ways. Peter Galbraith (2002, p.16) recently reported a pupil who described her use of technology in terms of a partnership:

> *Because my calculator has become my best friend. His name is Wilbur. Me and Wilbur go on fantastical adventures together through Maths land. I don't know what I'd do without him. I love you Wilbur.*

It is perhaps in the experiential use of calculators that the humanistic element is most evident; this quotation from one student was chosen to emphasise explicitly the human element in graphics calculator use. The experiential use of calculators best reflects the calculator as a learning technology, and thus links it most closely to the human experience.

Conclusion

Humanising mathematics is an important project, although progress with it seems less evident than Wheeler's 1982 remarks suggest. A careful consideration of the roles that technology can play in mathematics education may allow us to make progress with the renaissance alluded to in the title of this conference. Increasingly, a mathematics education bereft of a place for electronic technology will be seen as inappropriate, regardless of the cultural roots of mathematics. If attention is paid to what mathematics is important for people to learn on an assumption of availability of technology, what computational tasks are safely and properly left to technology and what fresh opportunities for learning mathematics are provided by technology, we can make progress on restoring humans to their rightful place at the center of mathematics education, surely the main point of a humanistic renaissance.

References

Dowsey, J. & Tynan, D. 1998, *Intarea* [Available via http://www.casio.edu.shriro.com.au]

Galbraith, P. 2002, 'Life wasn't meant to be easy': Separating wheat from chaff in technology aided learning?, *Electronic Proceedings of the Second International Conference on Teaching Mathematics*, Hersonnisos, Crete.

Kennedy, D. 1995, Climbing around on the tree of mathematics, *The Mathematics Teacher*, 88, 6, 460-465.

Kissane, B. 1999, The algebraic calculator and mathematics education, In Yang, W-C, Wang, D., Chu, S-C & Fitz-Gerald, G (Eds) *Proceedings of the Fourth Asian Technology Conference on Mathematics*, (pp 123-132), Guangzhou, China: Asian Technology Conference in Mathematics.

Kissane, Barry 2002, Equations and graphics calculators. In D. Edge & Y. B. Har (Eds) *Mathematics Education for a Knowledge-Based Era: Proceedings of Second East Asia Regional Conference on Mathematics Education and Ninth Southeast Asian Conference on Mathematics Education*, Selected Papers, Volume 2, (pp 401-408) Association of Mathematics Educators, Singapore. (a)

Kissane, B. 2002, Technology and the curriculum: The case of the graphics calculator, *New Zealand Mathematics Magazine*, 39, 1, 64-84. (b)

Kissane, B. 1995, The importance of being accessible: The graphics calculator in mathematics education, In Fong, H.K. (Ed.) *Proceedings of the First Asian Technology Conference on Mathematics* (pp 161-170), Singapore: Association of Mathematics Educators.

Kissane, B., Harradine, A. & Boys, A. 1999, Mathematical Interactions: Data Analysis and Probability, Chatswood, New South Wales: Shriro.
Wheeler, D. 1982, Mathematisation: The universal capability, *Australian Mathematics Teacher*, 38(4), 23-25.