A comparison of strategies adopted by primary students in four cities of China in solving mathematical problems

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Abstract "Virtually all young children like mathematics. They do mathematics naturally, by discovering patterns and making conjectures based on observation. Unfortunately, as children become socialized by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed and memory" (Everybody Counts . 1989. 43-44). In fact, it is very interesting to explore how primary pupils tackle unfamiliar mathematical problems by using different strategies. Pupils of P.4 and P.5 levels from four regions of China, including Hong Kong, Hangzhou, Nanging and Shenzhen were selected and invited to carry out a test of four mathematical problems. The time allotted for each test was 40 minutes. The results showed that different performances on the choice of problem-solving strategies of different groups of pupils had been measured.

Introduction.

In ancient China, people called mathematics the "Suan Xue" that means "the knowledge of computations." Traditionally, Chinese students especially at Primary levels concentrate on practicing of their computational skills. However, there is a new trend that mathematics teachers of Mainland China emphasized more on high-order thinking and problem solving (Zhang, 1998). In fact, from 2000 onwards, a major reformation on mathematics education has been implemented in China (NMCS, 2000) and problem solving is one of the integral parts in such a curriculum reform.

Hong Kong entered into a new era when China resumed the exercise of sovereignty over Hong Kong on 1 July, 1997. Under the principle of "one country, two systems", the Hong Kong Special Administrative Region (HKSAR) government shall on its own formulate policies on the development and improvement of education (BL Articles). In recent years, Hong Kong's relationship with the Mainland China has strengthened, not only in terms of business ties but also in the flow of people. In fact, the Mainland is the major source of the Hong Kong's immigrant population. As a result, the group of young immigrants undergoes tremendous pressure to adapt to educational, cultural and socio-economic system of Hong Kong. Apparently, their schoolteachers also face the problem when they teach the pupils from varying backgrounds and cultures.

From a constructivist perspective, the primary responsibilities of the teacher is to create and maintain a collaborative problem solving environment, where students are allowed to construct their own knowledge, and the teacher acts as a facilitator (NCTM, 2000). To achieve this, it is worthwhile to explore the developmental status of students' thinking and reasoning. Indeed, teachers' knowledge of students' thinking has a substantial impact on their classroom instruction and, hence, on students' learning (Gardner, 1999). The present pilot study was intended to explore some useful information about the performances of students, both from the Mainland China and Hong Kong, on problem solving. It is hoped that the results of the study can provide diagnostic and decision-making information about how students' mathematics learning can be improved. According to the Standards for School Mathematics 2000, 'problem solving' means engaging in a task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through

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this process, they will often develop new mathematical understandings.

This study was an exploratory investigation into the relative performances of Primary 4 and Primary 5 students from the four cities of China, including Hong Kong, Hangzhou, Nanjing and Shenzhen, on problem solving. Besides the number of correct solutions obtained by each student in a test paper was recorded, the process involved in solving the problem was also analyzed. In solving each of the problems, students were encouraged to write down their drafts as well as solutions. These visible written records were scrutinized so that the problem solving strategies used by different groups of students could be identified and compared. This study was thus designed to (a) investigate the difference in the performance between the Hong Kong students and students from the Mainland China on problem solving (b) compare the performances of students from the three different cities of the Mainland China on problem solving, and (c) examine how the selection of strategies by the four groups of students contributed to their performances.

Method. Two schools in each of the four cities of China including Hong Kong, Hanghou, Nanjing and Shenzhen were selected for the study. These four cities, which are located in southeastern China, are economically developed areas. The eight schools selected were typical of the area in terms of students' population and family backgrounds. After the schools were chosen, one teacher from each of the eight schools was selected on a volunteer basis. The teacher was then requested to choose randomly one P.4 class and one P.5 class of his/her school to participate into the study. There were altogether 683 primary pupils, of which 324 were of P.4 level and 359 were of P.5 level. (137 were from Hong Kong, 156 were from Hangzhou , 193 were from Nanjing and 197 were from Shenzhen.)

Two test papers I and II were designed. Each paper contains four non-routine mathematical problems (Appendix A). All participants possessed an adequate level of mathematical knowledge for the problems posed. The aim of the test papers is two-folded: (a) to measure pupils' ability on problem solving and (b) to analyze the kinds of strategies used by pupils from the selected cities. One of the two schools in each of the four cities was chosen at random to complete Paper I while the other school was requested to complete Paper II. The P.4 and the P.5 classes of the same school did the same test paper. The pupils were allowed to complete the test paper within 40 minutes. They were encouraged to write down their drafts and solutions in the space provided. After the test, each of the participants was requested to complete a questionnaire by which some useful information about the levels of the pupils' interest in the problems, and the levels of pupils' familiarity with each type of problems could be then collected. Research shows that interests are usually considered to be important antecedents of successful academic achievement (Krapp, 1999) and students' familiarities may reflect substantial training on problem solving had been emphasized.

Each pupil's response to a problem was marked according to two analysis schemes: a holistic scoring scheme and a cognitive analysis scheme. The details of these kinds of scoring can be found in other studies (Cai, Magone, Wang & Lane, 1996). In the holistic scoring scheme, each response was assigned a numerical score either 1 or 0. "1" indicates a correct answer while "0" indicates an incorrect answer or incomplete answer. A qualitative analysis of each response to the problem focused on critical cognitive aspects such as solution strategies and mathematical representations. These components have been identified as significant dimensions in mathematical problem solving (English & Halford, 1995). The representations and strategies that students employed and the success of those applications can offer important information regarding students' mathematical thinking and

reasoning processes. **Results.** The mean scores of the sample students, including both P.4 and P.5, of each of the eight schools in the four cities were shown in Table 1.

Mean Scores (S.D.)			
Regions	**Hong Kong (n = 74)**	**Hangzhou (n = 77)**	**Nanjing (n = 109)**	**Shenzhen (n = 105)**
Paper I	1.22 (S.D. = 1.04)	1.79 (S.D. = 0.86)	1.68 (S.D. = 1.07)	1.45 (S.D. = 0.98)
Regions	**Hong Kong (n = 63)**	**Hangzhou (n = 79)**	**Nanjing (n = 84)**	**Shenzhen (n = 92)**
Paper II	1.20 (S.D. = 1.01)	2.73 (S.D. = 1.02)	2.46 (S.D. = 1.26)	2.04 (S.D. = 1.18)

Table 1: Mean Scores of the sample students

It happened that the two rank orders of the four cities with respect to their sample students' performance in the two papers were the same. The sample students in Hangzhou achieved the best results. The other sample students in Nanjing, Shenzhen, and Hong Kong ranked the second, the third and the fourth respectively. The sample students in Hong Kong had a significantly lower mean score than any of the other three samples (See Table 2).

	Cities	Hong Kong	Hangzhou	Nanjing	Shenzhen
Paper I	**Mean (S.D.)**	1.22 (1.04)	1.79 (0.86)	1.68 (1.07)	1.45 (0.98)
	t-value **(Test value = 1.22, p < 0.05)**	---	5.81	4.51	2.38
Paper II	**Mean (S.D.)**	1.20 (1.01)	2.73 (1.02)	2.46 (1.26)	2.04 (1.18)
	t-value **(Test value = 1.20, p < 0.01)**	---	13.35	9.23	6.88

Table 2: Results of t-tests for a comparison of the mean scores

Table 3 shows the percentage of students who thought they had seen the similar problems with those problems in the test paper. It was found that the sample students in Hong Kong were less sophisticated with the problems than the students in Mainland China. It might explain why the overall performance of the sample students in Hong Kong was not as good as the others.

	Cities	Hong Kong	Hangzhou	Nanjing	Shenzhen
Paper I	**Problem 1**	27%	42.8%	58.7%	49.5%
	Problem 2	24.3%	36.4%	71.6%	56.2%
	Problem 3	28.4%	55.8%	65.1%	57.2%
	Problem 4	29.7%	48%	61.5%	53.4%
	Mean	27.4%	45.8%	64.2%	54.1%
Paper II	**Problem 1**	39.6%	67.1%	50%	67.4%
	Problem 2	36.5%	63.3%	51.2%	47.9%
	Problem 3	42.8%	68.3%	55.9%	59.8%
	Problem 4	47.6%	91.1%	63.1%	71.8%
	Mean	41.6%	72.5%	55.1%	61.7%

Table 3: The Percentages of students who had seen the similar problems

It is reasonable to think that a higher percentage of students who are more familiar with the test problems in a sample should obtain a better achievement of the test. However, the results of the study are not in line with this assertion. The sample students of Shenzhen did not perform as good as the students of Hangzhou and the students of Nanjing for Paper I and Paper II respectively though they were more familiar with the test problems than the other samples.

Table 4: Mean values showing the level of students' interest in each problem

Table 4 recorded the sample students' responses to the questionnaire about whether they found the individual problems interesting or not. All the positive mean values in the table indicate that the students of the eight sample schools were, on the average, interested in the test problems. Two schools, which got the lowest overall mean values 0.20 and 0.25, were both from the same city Hong Kong. Comparatively, the sample students in Mainland

China were more interested in the problems than the students in Hong Kong. Nevertheless, one strange point should be stressed that the sample students in Hangzhou did not have a high overall mean value as compared with the students in Nanjing and Shenzhen, although they obtained the best achievement in both of the test Paper I and II. Surprisingly, the sample students in Shenzhen who ranked third in the overall performance had the highest overall mean value of interest level for the two papers I and II (0.46 and 0.7 respectively).

The following qualitative analysis of the solution processes and strategies espoused by the solvers provides in-depth information about how students think and solve the problems. Such information may help us to make progress in understanding the various performance trends among the sample students in the four cities.

Which strategies were used efficiently by most of the sample students in both Hong Kong and Mainland China in solving problems?

Drawing a diagram

Nearly all the sample students from the four cities were able to make use a diagram to clarify the conditions stated in the problem and to find the relationship among the relevant subjects. When tackling the Pocket Money problem, the Sweet Game problem, the Square Table problem and the Building's Height problem, students drew some relevant and useful diagrams onto the papers. Also in dealing with the Coloring A Log problem, many students attempted to draw a part of the log in order to find out the required pattern of colors.

Which strategies were used more efficiently by the sample students in Mainland China in solving problems, as compared with the sample students in Hong Kong?

Listing all possible outcomes

In solving the Car Accident problem, the sample students in Mainland China tried to list all the possible pairs of numbers for the last two digits. However, most of students in Hong Kong did not do the listing. Some solely approached the problem by a wild guess.

Making a table

In attacking the Sports' Day problem, 27.5% of the sample students in Mainland China drew a table and obtained a correct solution. It was worth noting that none of the scripts from the Hong Kong school included such a table in solving the problem. As a result, only 9.5% of the Hong Kong students attempted it successfully.

Using Logical reasoning

In solving the Car Accident problem and the Sports' Day problem, the students were required to use logical reasoning for making judgment and elimination. The percentages of students in Mainland who got the correct answers to these two problems (64% and 27.5%) were relatively higher than that in Hong Kong (50% and 9.5%).

Working backwards

Students were required to adopt the working backward strategy to solve the Sweet Game problem and the Building's Height problem. In fact, the percentages of students in Mainland who obtained the correct answer were 27% and 62.4% respectively while the relative percentages of students in Hong Kong were only 25 % and 11.1%.

Which strategies were used more efficiently by the sample students in Hong Kong in solving problems, as compared with the sample students in Mainland China?

Guess and Check

In attempting the Sheep & Ducks problem, students in Hong Kong tried to guess a number of ducks and found the number of sheep by subtraction. Then, they looked back to check whether the numbers fitted the conditions of the

problem or not. In contrast, most of the students in Mainland solved the problem by using a standard rule such as $(4 \times 12 - 38) \div 2$. It seemed that they had already learnt the method to solve the problem. None of the students in Mainland used generalized approaches such as guess and check for the solution.

Which strategies were used inefficiently by most of the sample students in both Hong Kong and Mainland China in solving problems?

Looking for a pattern

In dealing with the Coloring A Log problem, most of the sample students, including both in Mainland and Hong Kong, tried to generalize the pattern by looking at the diagrams. The part of the log in the diagram they drew was not long enough, which leaded to the wrong pattern they got. The percentages of students in Mainland and Hong Kong who obtained the correct answer to the problem were only 7.6% and 1.4% respectively.

Conclusions

In this study, the sample students in Hangzhou, Nanjing, and Shenzhen were more competitive than the students in Hong Kong. Among these three Mainland cities, Hangzhou's sample students performed the best and Shenzhen's performed the worst. This result is not surprising because it is known that the standard of mathematics achievement by Chinese students is one of the highest in the world. China obtained the championship in the HKMO contest in 2002. Presumably, the eight problems we particularly designed for the two test papers were "novel" to the sample students; however, a range from 24% to 91% of the eight groups of the sample students in the four cities claimed that they had seen the similar problems in the test papers. The sample students of Hong Kong were most unfamiliar with the problems, especially those in Paper I (See Table 3). Most probably, students in Hong Kong have not been exposed to such problems as frequently as that in Mainland China. This conclusion could be easily drawn by a number of evidences shown by the result of the present study. For instance, 71.8% of sample students in Mainland obtained the correct answer to the Sheep & Ducks problem while only 41.4% of sample students in Hong Kong solved it successfully. The students in Mainland generally solved this problem simply by applying a standard rule or algorithm for computation. On the contrary, none of the students in Hong Kong used this method; instead, they made use of problem solving strategies such as drawing a diagram and guess & check to get the answer. It should be noted that for those Mainland students who did not get the correct answer to the Sheep & Ducks problem did not try any other methods. Some of them even wrote down some meaningless numerical expressions for computation. Obviously, they tried to recall the standard rules they had learnt from their memory.

Evidently, the sample students in Mainland showed greater interest to the problems than that in Hong Kong. It may be explained by the poor performance of the Hong Kong sample students in solving the problems. Nevertheless, the attitude of Mainland students towards learning Mathematics was positive and encouraging. We should not neglect the fact that the sample students in Hangzhou were familiar with the problems and achieved the best results among the four cities. However, a scrutiny of the questionnaires showed that they were not interested in the problems as much as the students in Nanjing and Shenzhen. It is a warning to both teachers and educators that whether it is worthwhile to "drill" students on problem solving. "Practice makes perfect" is a traditional belief in Chinese education. However, we should alert that routine practice on problem solving only leads to a set of rules or algorithms bounded to a set of problems. These sets of rules will limit students' thinking and further learning unless we can help students shift to more generalized approaches. Students' success on problem solving

depends very much on the flexibility of their thinking and reasoning.

A major objective of comparative studies is to understand students' mathematical thinking and reasoning and then to improve students' learning of mathematics. The present study provides a starting point for a deeper understanding of the various performances of students from different cities of China on problem solving. Additional studies are needed to investigate more about diverse views of teachers towards the 'weight' of problem solving in a mathematical curriculum and the ways to promote the teaching and learning of problem solving in classroom instruction.

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Appendix A

Paper 1

Problem 1 *(Pocket Money problem)*

Mrs. Smith has four children, Mary, David, Peter and Amy. On the first day of every month, she will give each of

her children some pocket money. On the 10th of the month, Peter used up all his pocket money and borrowed 25 dollars from David. On the 20th of the month, David wanted to buy a book and found that he didn't have enough money. Thus, he borrowed 15 dollars from Mary and 5 dollars from Amy. At the end of the month, Amy borrowed 5 dollars from Mary. Find the simplest way to settle their debts among themselves so that no one owes any money to others.

Problem 2 *(Sweet Game problem)*

Three girls are playing a game for their sweets. They play for 3 turns and in each turn; the loser must give the other two girls some sweets so that the winners will double their number of sweets that they have in the previous turn. The result is that all the girls lost one turn after the game. At the end of the game, each girl gets 40 sweets. Find the number of sweets each girl has before the game starts.

Problem 3 *(Car Accident problem)*

A car was involved in a traffic accident, but the driver drove away the car immediately after the accident had occurred. According to the following clues, find the plate number of the car which was involved in the accident:

Lewis   :   The alphabetic characters in the car plate are AB.

Kirsten :   The two left most numbers are equal .

David   :   The three right most numbers are even.

Charlie :   The sum of all the 4 numbers is equal to 26.

Jack   :   The last two digits form a number that is divisible by 8.

Problem 4 *(Coloring A Log problem)*

William and Bruce played a coloring game. They colored a 150cm log according to the following methods:

William : Starting from the left end of the log, he colored the log in red for the first 2 cm and left the next 2 cm uncolored. He then colored the log for next 2 cm and left the next 2 cm uncolored. The process continues until he reaches the other end of the log.

Bruce : Starting from the left end of the log, he left the first 3 cm uncolored and colored the log in green for the next 3 cm. Then he left the next 3 cm uncolored and colored the log in green for the next 3 cm. The process repeats until he reaches the other end of the log.

What is the total length of the log being uncolored?

Paper 2

Problem 1 *(Square Table problem)*

Herbert plans to invite seven friends to celebrate his birthday at his home. In his house, there are 4 square tables and they will be arranged to form a larger table. It is required that each side of the table can serve for only one person.

(a) Show how to arrange the four square tables so that all the 8 people could seat around the larger table and there will be no seats left?

(b) If Herbert would like to invite two more friends, show how to rearrange the 4 tables so that all the people could seat around the larger table and there will be no seats left?

Problem 2 *(Building's Height problem)*

The height of a building is unknown. A special rubber ball is dropped from the top of the building. Each time the ball hits the ground it bounces only half as high as the distance it falls. If the ball reached 2m high at the fourth

bounce, what should be the height of the building?

Problem 3 *(Sports' Day problem)*

Four students A, B, C and D participated in the four games on a school sports' day. The four games are 60 metres run, 200 metres run, high jump and far jump. Please decide in which game(s) did each of the four students participate according to the following information:

(1)   Student A, B and C each took part in two games but student D only took part in one game.

(2)   Among A, B, C and D, three of them took part in a common game.

(3)   A took part in long jump while D did not.

(4)   B did not take part in the 200 metres run .

(5)   C did not take part in any game which A or D took part in while B and C took part in a common game.

(6)   Student who took part in high jump would not participate in long jump and vice versa.

Problem 4 *(Sheep & Ducks problem)*

A farmer owns some sheep and ducks. He knows that the total number of sheep and ducks is equal to the present age of his son, and the total number of legs of the sheep and ducks is equal to his own age. If the present ages of the farmer and his son are respectively 38 and 12 years old, what are the number of the sheep and ducks respectively?