The Lack of Geometric Comprehension in Integral Instruction

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Abstract: In this study, students of the Science, Engineering and Education Faculties who have studied integral calculus during one school term were given an exam which involved a series of integration problems. What assessment of the students' performance on this exam reveals, however, is that students lack a full understanding of the geometric interpretation of integration. It is difficult for them to find the correct answer because they resort to the formulas that they have memorized. This finding indicates the deficiency in using solely formulaic manipulations and the importance of emphasizing geometric interpretations while teaching integral calculus.

Integral calculus is one of the basic concepts of analysis. Moreover, integral calculus has numerous application fields in other branches of science. Therefore, teaching concepts of integration is of vital importance for students of engineering. It is also essential for students of mathematics in the Education Faculty because all these students are prospective high school teachers.

Geometric support in teaching the concepts of mathematics is equally essential [1]. Generally geometric interpretation is not covered when teaching integral calculus in universities. Instead, a teaching method based on set integration formulas is applied. Consequentially, when a student is trying to solve a simple area problem, he may struggle long and hard with complex integration formulas, and still not find the correct solution for the problem.

To find out what deficiencies exist in the students' understanding, students of the Science, Engineering and Education Faculties were given an exam which involved a series of integral problems. All the integration problems were carefully chosen to find out whether the students were able to interpret the concepts of integration geometrically.

1. The Exam

The experimental group consisted of a total of 159 students of whom 75 students were from the Science Faculty, 30 from the Engineering Faculty, and 54 from the Education Faculty. This experimental group was given a five-question exam for which there was no time limit. Below are the five questions that the experimental group was asked :

1. $\int_{-1}^{1} \sqrt{1-x^2}\,dx = ?$
2. $\int_{-3}^{2} |x|\,dx = ?$
3. $\int_{1}^{3} \operatorname{sgn}(x^2-4)\,dx = ?$
4. Find the area of the region bounded by $y = 2x - x^2$, $y = 0$ and $x = -1$.
5. Find $\int_{0}^{5} f(x)dx$, where $f(x) = \begin{cases} 3x & if \quad 0 < x < 1 \\ 3 & if \quad 1 \le x < 4 \\ 15-3x & if \quad 4 \le x < 5 \end{cases}$.

These questions were designed to determine what deficiencies, if any, existed in teaching using solely formulaic manipulations [2]. The first, second and fifth were such integration questions as can be solved by means of the areas of simple geometric shapes like the circle, triangle and trapezoid. The third question involved the integral of a function which takes positive and negative values. In contrast, the fourth question concerned the area below the parabola curve. The aim of the third and fourth questions was to find out how the students consider the relation between integral and area.

2. Results and Discussions

The experimental group's performance is shown in Table 1, Table 2 and Table 3.

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	The Number of The Students, Who			
Question Number	Used The Geometric Interpretation of Integration	Used Integration Formulas	Gave Wrong Answer	Gave No Answer
1	42	9	23	1
2	7	51	15	2
3	2	35	27	11
4	-	4	65	6
5	5	56	14	-

Table 1: Performance of Students of The Science Faculty

	The Number of The Students, Who			
Question Number	Used The Geometric Interpretation of Integration	Used Integration Formulas	Gave Wrong Answer	Gave No Answer
1	11	1	14	4
2	4	7	17	2
3	1	11	9	9
4	-	4	25	1
5	-	17	13	-

Table 2 : Performance of Students of The Engineering Faculty

	The Number of The Students, Who			
Question Number	Used The Geometric Interpretation of Integration	Used Integration Formulas	Gave Wrong Answer	Gave No Answer
1	2	-	37	15
2	3	27	19	5
3	-	27	14	13
4	-	-	48	6
5	2	25	24	3

Table 3: Performance of Students of The Education Faculty

On the first question, 35% of the students gave the correct solution by using the area of a circle, and 6% of them by using memorized integration formulas; 47% of the students did not find the correct solution. Analysis of the latter's incorrect answers reveals that although all the students used the techniques of integration, they still could not find the correct solution. For the second question, 8% of the experimental group solved the problem quickly by using the area of a triangle, and 53% of them gave the correct answer by using integral calculus. On the other hand, 32%, who also used integration formulas did not solve the problem correctly. Similar results were obtained for the fifth question. The most striking finding of the exam regards the fourth question. 87% of the experimental group could not solve that problem. The analysis of all their solutions reveals that the students could not determine the area asked in the question. What all these findings show is that integration is not generally considered as geometrical. For that reason, it becomes difficult for even easily solved integration students to answer problems correctly because they depend on memorized formulas. This indicates the deficiency in teaching using solely formulaic manipulations and the importance of emphasizing geometric comprehension.

References

[1] N. Mahir, N. Çetin, S. Canbek, Comparison of Numbers, (2000), Mathematics for Living.

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