FUTURE MIDDLE SCHOOL TEACHERS' BELIEFS ABOUT ALGEBRA: INCIDENCE OF THE CULTURAL BACKGROUND

Nicolina A. MALARA,
Department of Mathematics - University of Modena & Reggio Emilia - Italy

We report about the beliefs on algebra expressed by future middle school teachers at the beginning of a training course devoted to the approach to this discipline as a language for producing thought. We analyse qualitatively the collected data and we show some interesting differences among graduates in "mathematics", "physics" and "other science". The analysis also highlights a deep interlacement between affective factors and image of algebra. Finally we give some short indications on the course and we sketch its influences on the trainees' beliefs.

BACKGROUND, HYPOTESIS AND AIM OF THE STUDY

In the last decade many studies have been centred on the teacher, main 'variable' of the teaching/learning process (Krainer & al. 1998) and related to different aspects: disciplinary knowledge (Fennema & Fraenke 1992), problem solving (Roddick &Al. 2000), the ability of understanding of students' mathematical thinking and performances (Even & Tirosh 2002) and also, more in general, the teachers' beliefs, attitudes, emotions and values (MacLeod 1992; Thompson 1992, Vinner 1997, Zan 2000). Other studies have underlined the need and the importance of the teacher' awareness on these aspects (Mason 1998, Jawroski 1998, Malara & Zan 2002). This paper belongs to this frame; it rises in the context of the Italian studies where - for the recent opening of university courses for primary teachers and of post-graduate training schools for teaching in secondary schools –there is now a big attention towards the teacher (Navarra 2001, Iannece & Tortora 2002, Malara 2001). The paper concerns the beliefs about Algebra in future middle school teachers (pupils' age: 11-14) collected at the beginning of an Algebra training course. In the paper we focus on the differences emerged as to the cultural background of the trainees[1], we reflect on distorted beliefs and we sketch our interventions to correct them.

Algebra is a complex and mani-sided discipline and many are the studies on the problems of its teaching and learning (see for instance Chick & al. 2001), also because school approach to algebra is usually centred mainly on the syntactical study of algebraic objects, instead of the construction of algebraic language for promoting modelization, solving problems and proofs (Arzarello et al. 1993, Malara 1999, Radford 2000, Menzel 2001). Therefore it is important that teachers become aware of the complexity of Algebra and know the main steps of its historical evolution.

Our paper is based on the hypothesis that only bringing future teachers to express their ideas about algebra it is possible to lay bare ground their possible conceptual rigidities, misconceptions, cultural lacks, difficulties so that they can: a) became aware of their gaps through open discussions; b) reconstruct their knowledge and beliefs through opportune shared experiences; c) promote in Algebra teaching a meaningful constructive learning leading the students to understand the reasons of its theoretical study. The inquiry reported in this paper has been done in January 2002 through a test made of five free-answer questions (answer time: 4 hours). This test was proposed to 47 trainees (9 graduated in mathematics, 9 in physics and 29 in various sciences).

THE FIVE QUESTIONS

The test was contained the following five question: 1. *What does the word 'algebra' evoke to you?* 2. *Why was Algebra born?, 3. Why must Algebra be taught?,* 4. *Which are the main difficulties of Algebra?* 5. *Which is the best age to start the approach to Algebra?* The first question was conceived in order to investigate on the future teachers' beliefs on algebra and the correlated affective aspects. The second question aimed at investigating whether the future teachers had an evolutionary idea of algebra (and more in general of mathematics) and they have some historical-epistemological knowledge. The third question aimed at investigating on their awareness of the meaning of the discipline as to the mathematic-scientific development. The fourth question aimed at evidenciating the learning difficulties they can hypothise and indirectly the ones they lived. The fifth question was functional to introducing the question of early algebra (see the papers devoted to it in Chick & al. 2001) ed also to spread our experimental long term studies for approaching algebraic thought (Malara 1999, Malara & Iaderosa 1999, Malara & Navarra in these proc.).

THE ANSWERS

For each question of the test we have hypothised a set of possible answers on which we have organized the quantitative data (see table 1). Even if these data can give for each question a first idea of the different beliefs of the trainees, however they cannot say anything about the trainees' feelings

[1] In Italian middle schools the teaching of mathematics and science is jointed, so teachers may have a degree in any scientific discipline (mathematics, physics, chemistry, natural science, biology, geology ect).

underlying the answers. So we analyse the data qualitatively, reporting also some excerpts of the trainees' protocols[2].

The first question

The answers reveal a fragmented and technical vision of Algebra to be strongly dominant. Algebra is often identified with literal transformations, the study of equations, or worse with directed numbers. This idea is prevalent among the graduates in science, TB(s) writes: *A. as <u>art of doing calculations</u>, solving equations in an appropriate and speed way according to the rules of this discipline.*

But this idea, paradoxically, appears also among the graduates in mathematics, FZ(m) simply writes: A.= *calculations, formulas, expressions*. Among these, however, a conception of the discipline aimed at order and organization prevails, for instance CF(m) writes: *the term 'A.' made me think of all that is number and calculation, that is I conceive A. as a well ordered, formalized discipline with a high grade of abstraction, which allows to solve problems and can be applied to concrete situations. Sometimes, then, I consider A. as a way to 'play' with numbers.*

Among the graduates in physics a different idea emerges, focusing on relational vision and considering Algebra as language for mathematization, BB(ph) writes: *a+3b =7, well, A. evokes to me a relationship between mathematical entities through opportune symbols*. CB(ph) writes: *The solution of a physics or chemistry or geometry problem presupposes the knowledge and the mastery of A., it is the language of scientific disciplines*.

Table 1: The quantitative data

Future teachers' degree	Maths n = 9.	physics n = 9	Science n = 29	Total n = 47	%
Idea of algebra					
Study of expressions and equations	6	4	18	28	60
Language for codifying relationships	1	3		4	8
Theoretical contents useful for applications	1	2	5	8	17
Algebraic structures	1				2
other			6	6	13
Reasons for the birth of algebra					
to solve practical or economical questions	1	2	2	5	10.5
to mathematize and to study complex situations	1	1	3	5	10.5
to generalize – to abstract - to systematize	1		5	6	13
to support the study of other disciplines	1	1	2	4	8
I do not know*	3	3	15	21	45
other	2	2	2	6	13
Reasons for studying algebra					
to acquire technical abilities		1	4	5	10.5
to acquire a language useful for science	1	1	4	6	13
to acquire useful theoretical knowledge	1	2	3	6	13
to mathematize – to apply algebra to other sciences	1	3	8	12	25.5
to generalize – to abstract – to produce thought	4		5	9	19
other	2	2	5	9	19
Difficulties in Algebra					
Technical – calculative difficulties	3	3	6	12	25.5
difficulties of translations into formula			3	3	6.4
logic – interpretative difficulties	4	1	4	9	19.2
generalization – abstraction difficulties	2	3	7	12	25.5
other		2	9	11	23.4
'Best' age to approach Algebra					
9-11	5	2	6	13	27.5
12-14		4	12	16	34
15	4	1	7	12	25.5
I do not know*		2	4	6	13

- This item has added owing to the high number of answers received

[2] To make reading easier, we quote each excerpt by inserting after the author'initials the letter m, ph o s in brackets (with the meaning respectively of: graduates in mathematics, physics, various sciences) to evidenciate his/her background. In each excerpt we substitute the word "Algebra" with "A.".

This idea appears also among the graduates in science, CC(s) writes: *I imagine A. as 'formalization' or modelization of real situations, as a representation.*
There are very partial visions, for instance several graduates in science identify Algebra with the study of directed numbers or the numerical ambits, BP(s) writes: *A. is the part of mathematics which concerns directed numbers, it evokes me 'rationality'".* SA(s) writes: *Soon it recalls me directed numbers and after the operations defined in the set of real numbers.*
The most interesting thing emerging from the protocols is the affective dimension related to the first approach to Algebra. Affective factors appear to be intertwined with the image of the discipline. For all the trainees except one (who conceives algebra as the study of algebraic structures), this discipline is associated to the first years of secondary school, of which they evoke the classroom atmospheres or the teacher. EB(ph) writes: *the memories connected with A. go back to the first years of upper secondary school and middle school. The thoughts coming across my mind are pages and pages of equations (in the worst case also LITERAL ONES, decompositions and fundamental theorems.* LG(s) writes: *The word A. reminds me of my middle school teacher, kilometric expressions and the 'surprise' of comparing my result with the one in the book; usually it was correct, because I was careful and precise in doing calculations.*
Methodicalness, order, and the precision requested in the application of rules or techniques is a constant reference, EB(ph) writes: *I remember the extreme attention and care that had to be applied in solving the given equation, also the perseverance and the diligence to arrive at the end*. This idea of ordered calculation is associated according to the cases with feelings of pleasure or frustration AM(ph) writes: *Thinking about A. I recall the pleasure of listening to my teacher and the pleasure of starting to solve expressions.* GM (s) writes: "*the word A. evokes me the feeling of frustration arising when the calculation did not succeed. LG(S): I was careful and precise in doing calculations, but when the result of the expression was wrong, it was a tragedy to find and correct the mistake.*
Some trainees state they have lived Algebra as a personal challenge, CB(s) writes: *the word A. evokes me a lot of afternoons spent in doing exercises; it is a nice memory because I considered the exercises as a challenge and if the result was not correct (when compared to the one in the book) I enjoyed looking for the mistake. The more the equations became complicated, the more I was excited by them.* EC(s) writes: *I saw myself in front of A. as in front a 'riddle', a rebus of a puzzle-weekly, the solution of which gives me an unbelievable satisfaction.*
Graduates in mathematics or physics see Algebra as a sure and reliable discipline, for instance BB(ph) writes: *Emotionally I have to do with something exact which leads to an absolute truth in the field, and then I can say that A. leads me to a security and certainty: given things cannot be otherwise*. But also the implications of a distorted image of A. (and in general of mathematics) emerge, AMA (m) writes*: at lyceum I have loved A. to an extent I wanted to study mathematics at university, but there I suffered a disappointment just at the examination of A.*
Among the graduates in science Algebra appears as an intricate and complex discipline, but also hermetic and full of fascination MV(s) writes*: I associate the word A. to the image of a 'BRAIN': because A. puts off to the idea of calculation, of reasoning and of elaboration.* GM(s) writes: *The word A seems to me a charming, perhaps because I do not know its meaning.*
The second question
As to this question more than half of the trainees having a science degree state they do not know how to answer. Frequently we have found answers like this: *Sincerely I do not know how A. was born* - unfortunately also among graduates in mathematics. However there is someone who has a clear idea of its historical evolution. The awareness of their cultural gap is testified by the fact they refer to the responsibility of their teachers, FM(m) writes: *I do not know how A. was born because none of my teachers has explained it to me and moreover because I have never have taught it so I have not made myself this question.*
As to the reasons of the birth of Algebra, a few of the graduates in mathematics or physics vaguely refer to practical needs (trading or construction problems) which involve complex calculations. Someone states that A. was born for representing geometry, others for representing reality. Among the graduates in science there is a distorted vision of the evolution of Algebra which reflects the inclusion of the various set numbers (Naturals, Integers, Rationals etc). Several have a static vision of the discipline.
Most trainees change 'why' with 'when' in the question, they (sometimes wrongly) declare the time of the birth of the discipline and refer to the cultural heritage by the Arabic. Each answer reflects the idea of algebra of author, sometimes appropriate and composite, but often partial and confused, mainly among the graduates in science; however we have to underline the freshness of their images and the expressive effort even in the naivety of the results. This is a sample of their statements.

CB(s): *I would say that A. was introduced by the Arabic, but I am not so sure; anyway I suppose that it was born with the aim of using letters where the numbers did not arrive.* Cs(s) *It was born in the Arabic world, probably for explaining practical phenomena and for being able to calculate various values in the most general way, then it would be born for seeing how two variables vary each other; then for generalizing and simplifying a practical problem, with its possible solutions* AL(s): *A. was born for generalizing certain procedures. For instance we write a=bxh/2 for the area of triangle; this is an algebraic expression which allows me to replace each letter with any numerical value (linked with the dimensions of the triangle), so we find the area of any triangle.*

T.B(s) *A. was born to succeed in solving mathematical problems, then to succeed in finding results in rational way (that is with logical processes) with the tools of calculations (the operations +, x, : , - , powers, roots etc) and with the rule of naïve set theory. Moreover: A. is the art of handling numbers according to 'algebraic' operations. The numbers can be of different nature, following the naïve set theory, then the object of A. are the different types on (naturals, relative integers etc.). A. was introduced by the Arabs in ancient age (I know that it is after Christ but I do not remember the right century).*

In some trainees there is the idea of algebra connected with the arrangement and rationalization of the known facts accumulated in time, PG(s) writes: *A. was born in Persia around the year Thousand and it was born for giving an order, a method to the knowledge of that time*. AB(s) *A. was born in the Arabic world as an answer to the research of meaningful and generalizing routes for the solutions of problems.*

The third question

As to the third question only two or three trainees motivated the study of algebra for solving problems or making proofs. The main reasons given are vague: 1) to organize and synthesize one's (mainly among the graduated in physics); 2) to train logical reasoning (mainly among the graduated in mathematics); 3) to generate flexibility and mental openness (only among the graduated in sciences). A common idea (above all mainly among the graduates in science) is that pupils need to study Algebra for learning techniques of calculation or theoretical results to be applied on modelling reality. EC(s) writes: the study of A. is important either because it is applied to many fields (not mathematical fields but where mathematical laws are needed: for instance, a mechanical designer has to tune up a sequence of 'calculations' to project an engine, using for instance an equation,…), or because it is a tool for growing, for 'opening the mind', for acquiring techniques and algorithms which can be used in everyday life. For supporting this thesis RS(ph) resorts to the analogy with music, she writes: *In general, we cannot think of studying something without the tools; an example is given by music. Before we can play an instrument at a certain level we need to know how to read music, we have to know how to pitch the note and the arrangement of the instrument, this series of notions requires a certain time to be assimilated. So before we know how to play an instrument with a certain mastery we need to acquire the tools which allow to do it. Similarly, if we wish to set out a theory, a model of equations, or a theorem, first we have to master the basic notions of A.*

Among the trainees there is someone who, in tune with the expressed idea of Algebra, states it has to be studied for improving attention, precision, reflection and order.

The fourth question

Among the trainees there is the common idea that the difficulties in Algebra are due to the need of attention in doing calculations, for instance LG(s) writes: *Many of the mistakes made by the pupils are due to inattention, they do not succeed in combining the operations we are doing, they loose the signs or do not write the letters (2a becomes a)*. Beyond this there is a big differentiation among the trainees' beliefs. Again, the ideas of mathematics or physics trainees are different from the ones by the graduated in science.

For the graduates in mathematics the quoted difficulties are of logical-interpretative kind, even if they also consider difficulties in calculations and of generalization/abstraction. GA(m) writes: *we meet the main difficulties in logic, because it is difficult for the pupils to connect common language and symbolic language*. Someone mentions also the difficulties due to the mental fixity induced by a certain teaching of mathematics, LD (m) writes: *I consider mainly the effort to go out from rigid and fixed mental schemes*. Not by chance, just the trainees graduated in science express a wider range of difficulties. Among them the prevalent idea is that the bigger difficulties are due to the inability to generalize and abstract, AB(s) writes: *I believe that the passage from the number to the letter or, in any case, from specific to generic or general is not so obvious and foregone and so it can be represent a difficulty.* Some trainees, almost surely going over their own experience again, see the biggest difficulties in managing the negative numbers, for instance SB writes: *In my view, the first difficulty is the meeting with the negative part of numbers, but above all the working with positive and negative numbers (mainly addition and subtraction), distinguishing among negative numbers which are greater or lesser then others. And also: learning to manage letters, grouping them, 'isolating' them and finding their values through equations.*

Others correlate the difficulties with the teacher's behaviour, for instance TB(s) writes: *learning difficulties of A. depend on the way followed by the teacher in facing the topic, if in a convincing and also amusing way (there are questions which can enjoy the pupils).*

There is someone who refers to the difficulties due to the lack of motivation for a serious study of which the aims are unknown. BB(ph) writes: *The difficulties are abstraction and concentration. A further problem could be the lack of stimuli for arriving at a certain goal which seems far from one's daily life*

Other statements are more explicit on this side; they focus on the lack of meaning for the pupils. ES(s) writes: *I believe, at least for my own experience, that the main difficulty in learning algebra is to be able to see its usefulness, releasing it of the abstraction of calculation in itself. It is a difficulty you have also on teaching mathematics, i.e. to succeed in leading the students to understand its utility*. SA(s) writes: *The main difficulties in learning Algebra are to understand why one has to study it. At the beginning it is presented as something abstract, especially during middle school. Therefore one finds it difficult to learn something of which he does not find links with reality.* This is also confirmed by this excerpt which refers to a practice of Algebra teaching as blind application of rules, AMA(m) writes: "*I can say why it was so difficult for me at the lyceum to understand the rule of the square of a binomial: because I wanted only rules like the ones I had learned at compulsory school: my study was mnemonic. It was unknown for me to try to understand, to reason, to wonder why.*

This shows the relapse of their own experience on the way they conceive teaching. Among the graduates in mathematics or physics, some state that Algebra is not difficult and some of these, at the first experience of teaching, are amazed by the difficulties picked out in the pupils. Among the graduates in science there is someone who transfers his/her negative experiences to the pupils, GM(s) writes: *I believe that the pupils are terrified just by the same word, because they do not know it and they wonder with fear "what happens about us?"*

The fifth question

The data show that the more convenient range of age to approach algebra is 12 to 14 years. But among the graduates in mathematics nobody states this traditional range. Almost surely this depends on the fact that those who indicate an early approach refer to ideas learned in previous university courses.

A different origin have the answers of the graduates in science who support the idea of an early approach; this is correlated with a very poor idea of Algebra, which is identified with directed numbers, once first chapter of Algebra books and now topic in the syllabus for primary school. However a few of them conceive a meaningful early approach for minimizing the difficulty of an ex abrupto approach in middle school. This excerpt is a good example, SC(s): *In my opinion it should be started in primary school, perhaps shaped as a "game", but pupils must be accustomed to managing numbers and letters simultaneously, so they get the abstraction process as familiar as possible*.

The answers which have raised a certain perplexity are the ones who state they have not any idea about it.

SOME GENERAL REFLECTIONS

To sum up, we can say that the inquiry shows the trainees' fragmentized and distorted vision of Algebra; moreover, even when the vision is appropriate, it turns out to be reductive and quite inadequate to make teaching possible. Paradoxically, there are more pertinent beliefs among the graduates in physics than in the graduated in mathematics, since the first consider both the aspect of algebra as language to modelize and the one of theoretical body. Among the graduates in science there is a widespread cultural poorness even if sometimes pertinent visions appear, as well as interesting ideas for limiting the difficulties in the pupils. However, as to this point many trainees state the importance of giving meaning to this teaching, so as to make the pupils aware of the problems that it poses and to conceive an early and friendly approach which enables reasoning. A few are afraid that they may fail in teaching Algebra because of its difficulties.

It must be underlined that the test is extremely productive for the intrinsic power of breakthrough due to the metacognitive reflection required. Many protocols are read and discussed in following meeting and this reading allows an open and free comparison among the trainees. During the sharing, the single beliefs are reflected and interlace each other, constituting in the trainees' minds a mosaic of many little elements of awareness, that all together magnify themselves as the fragments of glass in a kaleidoscope. Moreover, it creates an atmosphere of full cooperation in the class, which is much different from the standard university lessons to which the trainees are accustomed. From a disciplinary point of view, there is a widespread feeling of curiosity, interest and participation for the things that are progressively come presented, particularly in the workshop.

The course (5 hours per week for 10 weeks) is focused on an approach to Algebra as a language. It is conceived in a dynamic way, alternating individual or small-group work on problems selected for introducing some theoretical question, the study of papers of literature on the main knots in Algebra teaching and the following group discussion. During the workshops, there emerge differences in the trainees' behaviour. In the spotting of the generation-rules of graphical and numerical sequences, of laws in elementary number theory and also in front of verbal problems with unknowns to be solved through arithmetical-intuitive strategies, the graduates in science are much more productive than the others; the graduates in mathematics show rigidity and blocks; when dealing with a proof problem, on the contrary, the graduates in mathematics or physics show a bigger productivity (Malara 2001), even if some of them meet difficulties. On these questions many of the graduates in science stall or limit themselves to some trials.

The collective discussion of reflection on the protocols is fundamental for getting the 'kaleidoscope effect', which is very useful from the cognitive point of view. Beside the individual effects on each trainee, the more productive result concerns the 'classroom culture' (Even & Tirosh, cit.) and the meaning given to Algebra as a tool of thought. At the end we have asked the trainees to produce a report on the changes in their beliefs of algebra. The answers were very satisfying; generally speaking, the trainees start by reporting their previous beliefs and highlight the new ideas: they suggest approaching algebra through stimulating questions which gradually bring the pupils to construct algebraic language and then the objects of Algebra. A crucial concern is the statement made by many (mainly graduated in science) trainees about their ability to realize a constructive teaching in the classroom, according to the model learned in the course, since they fear that the previous older models, which are more rooted in school, could prevail.

REFERENCES

Arzarello F., Bazzini L., Chiappini G.: 1993 Cognitive processes in algebraic thinking: towards a theoretical framework, *proc. PME XVII*, vol.1, 138-145

Chick,E., Stacey, K, Vincent, Jl., Vincent, Jn. (eds): 2001, *Proc. 12th ICMI Study 'The future of the teaching and learning of Algebra'*, Univ. Melbourne, Australia

Even, R., Tirosh, D.: 2002, Teacher knowledge and understanding of students' mathematical learning,, in English, L. (ed) *Handbook of International Research in Mathematics Education,* LEA, NJ, 219-240

Fennema, E. and Franke, M. L.: 1992, Teachers' Knowledge and its Impact, in Grows, D. (ed.) *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, 147-164

Iannece D., Tortora R.: 2002, Un tentativo di ricostruzione del pensiero matematico nella formazione dei maestri, in

Jaworski, B.: 1998, Mathematics Teacher Research: Process, Practice and the Development of Teaching, *Journal of Mathematics Teacher Education*, n.1, 3-31

Krainer K., Goffree F. Berger, P. (eds): 1998, On research in mathematics teacher education, in proc. CERME 1, Osnabruck<http://www.fmd.uni-osnabrueck.de/ebooks/erme/cerme1-proceedings/cerme1-proceedings.html>

MacLeod, D.B.: 1992, Research on affect in Mathematics Education: A reconceptualization, in Grows, D. A. (a cura di) *Handbook of Research on Mathematics Teaching and Learning*, Macmillan, 575-596

Malara, N. A.: 1999, Teaching and Learning of Algebra in Compulsory School: Questions and Results on a Long Term Research, in Rogerson, A. (ed), *Proc. Int. Conf. on Maths Education into 21st Century*, vol. 2, 68-78

Malara N.A.: 2001, The behaviour of future teachers dealing with proof problems in arithmetic, in Rogerson, A. (ed), *proc. Congr. Int. 'New Ideas in Mathematics Education'*, 162-166

Malara N.A, Iaderosa, R: 1999, Theory and Practice: a case of fruitful relationship for the Renewal of the Teaching and Learning of Algebra, in *Proc CIEAEM 50* , 38-54

Malara N.A., Navarra G.: 2001, "Brioshi" and other mediation tools employed in a teaching of arithmetic with the aim of approaching algebra as a language, proc. 12th *ICMI Study on Algebra*, vol. 2, 412-419

Malara, N.A., Navarra, G., Giacomin, A., Iaderosa, R.: 2002, *ArAl project: routes in arithemetic for promoting pre-algebraic thought,* Pitagora, Bologna (in presss), www.aral.too.it

Malara N.A, Zan, R.: 2000, The problematic Relationship between Theory and Practice, in English, L. (ed) *Handbook of International Research in Mathematics Education,* LEA, NJ, 553-580

Mason, J.: 1998, Enabling Teachers to Be Real Teachers: Necessary Levels of Awareness and Structure of Attention. *Journal of Mathematics Teacher Education*, 1, 243-267

Menzel, B.: 2001, Language conceptions of algebra are idiosincratic, in Chick, H. & al. (eds) 2001, *Proc. 12th ICMI Study 'The future of the teaching and learning of Algebra'*, Univ. Melbourne, Australia, 446-453

Navarra G.: 2001, Percorsi esplorativi di avvio al pensiero algebrico attraverso problemi; osservazione e rilevazione di difficoltà in insegnanti e allievi, *Atti 3° Conv. Naz. Internuclei* Univ. Napoli, 53-60

Radford L.: 2000, Signs and Meanings in Students' Emergent Algebraic Thinking: a Semiotic Analysis, *Educational Studies in Mathematics*, vol.42, n.3, 237-268

Thompson, A.: 1992, Teachers'beliefs and conceptions: a synthesis of the research, in Grows, D. A. (ed) *Handbook of Research on Mathematics Teaching and Learning*, Macmillan,105-128

Vinner, S.: 1997, From Intuition to Inhibition – Mathematics, Education and other Endangered Species, *Proc. PME21*, vol. 1, 63-80

Zan, R.: 2000, a) Le convinzioni; b) Le emozioni e le difficoltà in matematica I e II c) Gli atteggiamenti, e le difficoltà, *L'Insegnamento della Matematica e delle. Scienze. Integrate*, vol 23A, a) n.2 171-183; b) I n.3, 207-232, b) II n. 4, 328-345; d) n.5, 442-465