FACILITATING PROBLEM SOLVING: CHILDREN ARGUE THEIR WAY TO A MULTIPLICATIVE STRUCTURE WITH THE AID OF SELECTED CULTURAL TOOLS

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Abstract: In this paper we propose a way of facilitating children's performance on ratio and proportion tasks. An instrument that was designed to reveal common errors on the topic of ratio was given as a test to pupils aged 10-14. Next, discussion groups were formed by children that provided a range of responses on the test items. As an example, we present here the case of one discussion group for an item called "Campers". A pictorial representation of the problem was presented during the discussion as a tool that would provoke more productive arguments. We believe that children arguing over a provocative diagnostic item with the help of an appropriate tool can enhance their problem solving capabilities and provoke learning.

Introduction and background

Ratio and proportion are difficult concepts for pupils to learn and teachers to teach. Since learning about ratios is essential for students' performance not only in mathematics but in other subjects and real life situations as well, activities that could facilitate this learning are needed.

We believe that children's errors and misconceptions can be the starting point for the effective teaching of ratio. Bell et al (1983) reported cognitive conflict as a way of developing understanding in mathematics and in previous work we found that argument in discussion between conflicting positions is seen as one important source of such conflict (Williams and Ryan, 2001). More specifically, Pesci (1998) stresses that "the role of the discussion conducted properly by the teacher, is considered fundamental and decisive to the comprehension of mathematic concepts, in our case to the conscious construction of proportional reasoning".

Methodology

A general methodology that was used in previous work by Ryan and Williams (2001) has as a starting point an unresolved or not trivially resolvable problem, which can be the source of discussion. The existence of such a problem for a particular group of children can be established through prior testing which provides a range of student responses and methods of solution.

The children are then set the task of persuading each other by clear explanation and reasonable argument of the answer. The giving of reasons, justifications and informal "proof" is the rationale for the discussion. The researcher, adopting the teacher's role, can establish rules for the children's argument in order to facilitate genuine participation in discussion. Moreover, she tries to ensure that the arguments for the errors are clearly voiced. In eliciting and sustaining argument, we include the eliciting of a variety of "answers" and arguments, asking children to listen and sometimes paraphrase others' views, seeking further clarification of arguments, seeking alternatives, and seeking reasons and "backing".

An important role in productive argument may be played by tools in practice, which may provoke productive backing. Consequently, the researcher tries to ensure that potentially productive tools are introduced at some point. Finally, children are asked whether and why they have changed their mind and how they would summarise what they have learnt.

For this work, firstly we developed a diagnostic instrument with the aim to reveal pupils' common errors and misconceptions in proportional reasoning tasks. We constructed the instrument using problems that were selected having as criterion their "diagnostic value": their potential to provoke a variety of responses from the pupils, including errors stemming from misconceptions already identified in the literature.

Two versions of this instrument were constructed. The first version ("W") contains all the items presented as mere written statements. The second version ("P") contains the same items supplemented by "models" thought to be of service to children's proportional reasoning. The two versions were equated through common items in order for us to be able to compare the difficulty of the parallel items for the children.

The study sample (N=232) was of Year 6, 7, 8 and 9 pupils (aged 10 to 14). In each class, half of the pupils were given the W version of the test and half the P version.

Then, from selected classes of this sample, we formed discussion groups to work on the diagnostically most challenging items of the test. The pupils were chosen on the basis that they had provided a range of responses on the relevant item.

We present one case of an item called "Campers". The plain version ('W') of the item is the following:
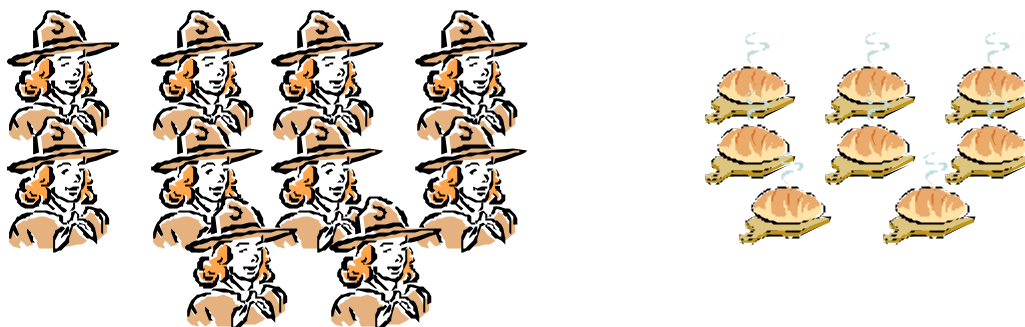
10 campers have camped at the "Blue Mountain" camp the previous week. Each day there are 8 loaves of bread available for them to eat. The loaves are provided by the camp's cook and the campers have to share the bread equally amongst them. This Monday 15 campers camped at the "Blue Mountain" camp. How many loaves are there available for them for the day?

The "models" version of the item is the following:

You can see below the campers that have camped at the "Blue Mountain" camp the previous week.

You can also see below the number of loaves of bread that are available for them to eat each day.

The loaves are provided by the camp's cook and the campers have to share the bread equally amongst them:

(a) This Monday 15 campers camped at the "Blue Mountain" camp. How many loaves are there available for them for the day? (The pictures above will help you find the answer)

Lamon (1989) suggests that the addition of a pictorial representation to a problem may facilitate proportional reasoning. The test scripts and the subsequent interviews of several pupils showed that they might have been helped by the supplementary pictures.

Three 11 years old pupils, "Mary", "Arpita" and "John" were selected form the same class to form a "discussion group" because they had provided three different responses on this item: "11", "12" and "13". All of them were given the "non model" version of the test. The discussion lasted 50 minutes and was audio taped. Below, we present selected extracts from the transcription.

Presentation of the group discussion

Initially, the pupils recalled their test response having in front of them their test scripts. They were also invited to present an argument for their response to the group.

Mary explained that the answer "11" was a guess: since 10 campers had 8 loaves available she just guessed that 15 campers should have 11 loaves.

John explained his answer "13": "The first one it says that it was 8 loaves but the campers are 10. So when it was 15 campers…there is 8 and 2 makes 10 so there must have been have been 2 left so when there is 15 campers there is 2 left…that's why I got the 13."

John used the "constant difference" or "additive" strategy to obtain his answer. This is a frequently used error strategy where "…the relationship within the ratios is computed by subtracting one term from another, and then the difference is applied to the second ratio." (Tourniaire & Pulos 1985, p.186)

Arpita explained how she got the answer "12" as: "…10 campers had 8 loaves…and another 5 campers…15 campers…so…and 8 divided by 2…each equals 4…because 5 is half of 10…and…if you half 10 campers…5…you'll get 4 loaves…and there is 15…so you do 8 add 4 which equals 12."

The researcher made sure that each pupil understood their classmates' positions and then distributed the sheet that is shown in Figure 1, as a tool that was supposed to facilitate discussion and pupils' proportional thinking. She explained that pupils could use it as an aid to their explanations and they could write or draw on it anything they wanted.

Then she invited the pupils to consider all three answers and explanations and share with each other their opinions. All of them discarded Mary's answer. John and Mary seemed to think that Arpita's method was the correct although they could not provide clear arguments:

John: I think the 12 is right

Researcher: Why?

John: Because it's more planned than the other two. So it's more like as you get a right answer.

Mary: I think Arpita's method is right because it's right...she has stuff to prove it...she has calculations to prove it.

Researcher: So why don't you accept these calculations? [Shows the calculations on John's method] We have calculations here...we say 10 minus 8 is 2...and then 15 minus 2 is 13.

Mary: But that seems more right...it looks better than that.

After all this discussion, Arpita appeared not to be sure about her method:

Researcher: OK. Arpita, what do you think?

Arpita: I think...it might be that one right [shows John's method]...I am not sure.

Researcher: Why?

Arpita: Because right here is 10 campers…8 loaves…it's 2 difference to 10…so 15 campers…and there is 2 difference…so it should be 2 difference in here.

10 campers   **8 loaves**
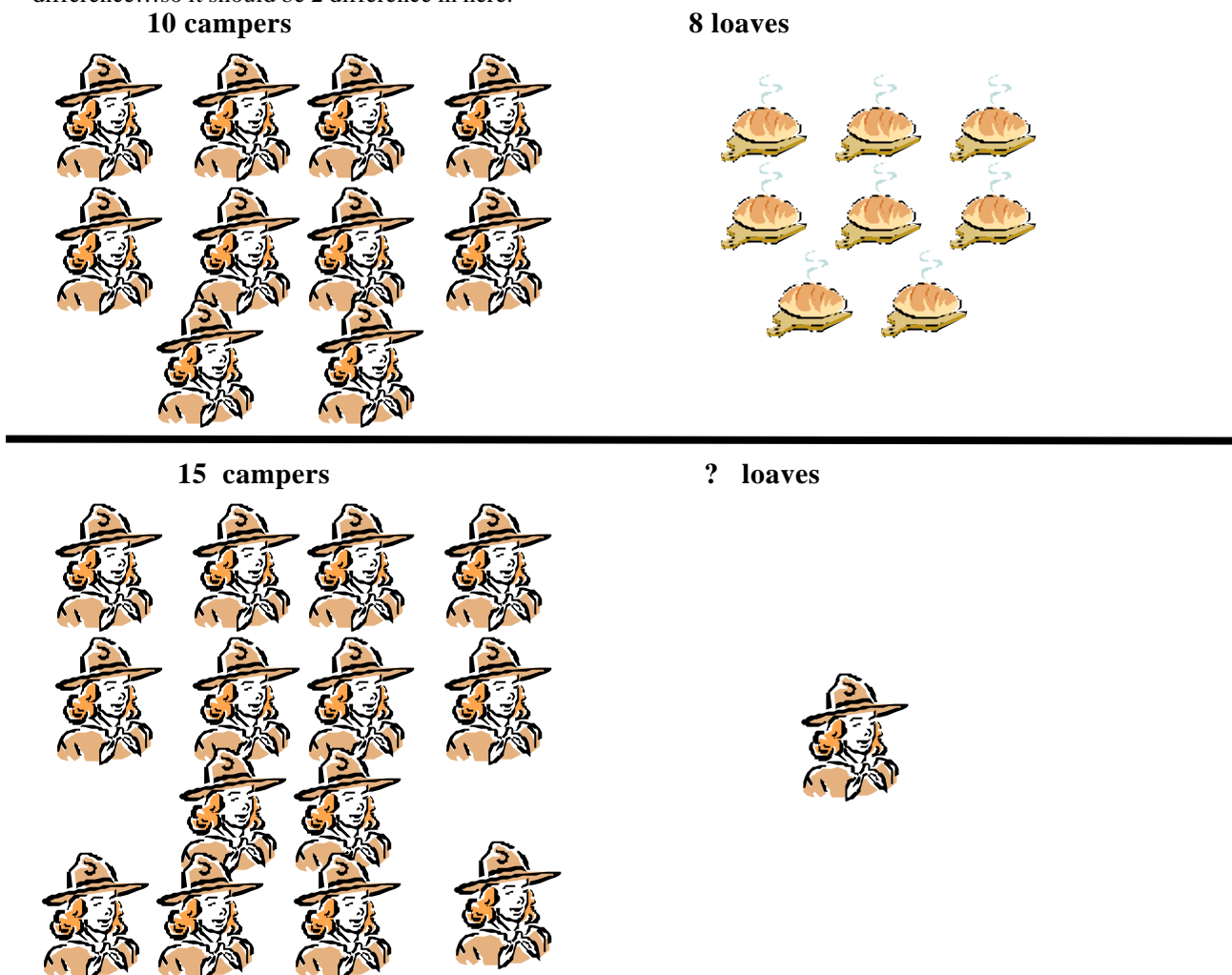
15 campers   **? loaves**

Figure   1

Then Mary seemed to have changed her mind again:

Mary: Um...13 could be right because...8 is 2 less than 10...and then 13 is 2 less than 15. And then it will be like the same...

The researcher then tried to promote the sheet in Figure 1 as a tool for generating more productive arguments. She suggested explicitly a method of using the pictorial aid after her attempts for implicit introduction had failed.

Researcher: ...Can we do that? Can we find smaller groups and see how many loaves can they get?

John: You can't split it into 3, because it's 3 groups of 3 and that's 9.

Researcher: We can't split it because it's 10 campers...

John: And so if we split it into 2 there is 5 groups of 2 and we have 8 loaves and we can't do that

Researcher: So we can't split them into groups of 2...

The researcher's explicit input directed John on a simple trial and error method as the above extract shows. This method led the whole group to an agreement about an answer and a method to the problem:

John: We can't have groups of 4 because there will be 2 people left and we will have leftovers...if we have 2 groups of 5 we will be able to share it out.

Researcher: Why?

John: They will get 4 loaves...

Researcher: 4 loaves one group...

Arpita: And 4 loaves the other group.

Researcher: OK...how does this help us for the whole problem?
John: We need to split those [shows the picture of 15 campers on his piece of paper] into groups to get ...
Researcher: What groups?
Mary: Of 5.
Researcher: Let us see what happens.
[They all draw on their papers]
Arpita: Split them [shows the 15 campers in the picture] into groups and they've got 5 campers in each...so they should have 4 loaves...12
Researcher: How did you find 12?
John: Cause we've split that into groups of 5 and there's 8 loaves and they get 4 loaves each. So we've split that into 5 and we get 4 loaves each in the group again. So...yeah...12
Researcher: What do you think Mary?
Mary: The same

The children enjoyed solving the problem by using the pictures:
Arpita: This method [she shows the pictures on her paper] is easier...there is grouping in it.
Mary: If I had to do it again, I would use the grouping thing I guess.
Researcher: So what do you think? Would you like to have the pictures as well as the problem?
All: Yeah   and   *Arpita: It gets easier.*

Finally, the researcher tried to lead the pupils in generalising their grouping method:
Researcher: How did you find 12? [She means by using the "grouping" action]
John: Cause we've split that into groups of 5 and there's 8 loaves and they get 4 loaves each. So we've split that [he shows the picture of the 15 campers] into 5 and we get 4 loaves each in the group again. So...yeah...12
Researcher: So what did Arpita did here?
John: She's done 8 loaves and these 4 loaves each so she's divided the 8 by 2...so that's 5 groups with 4 loaves each...and then she got all of them together
Researcher: So is this method different than the one that we did by drawing?
John: Slightly

Unfortunately, due to time constraints it was not possible for the group to make a clear connection between the action of grouping and a more general multiplicative method.

Conclusion

In previous work, Williams and Ryan (2001) have shown how conflict groups have learnt through discussion, and have pointed out how the teacher can play a critical role in providing significant problems and ensuring that critical cultural tools, such as number lines, are made available.

In this paper we have shown how this approach can work in the context of children's multiplicative argumentation. The cultural device highlighted here is that of a sharing context and a diagram, which "begs" to be grouped (in Freudenthal's sense).

More work needed to be done with this particular group in terms of articulating a connection between grouping and a general multiplicative strategy. Nevertheless, we believe that this method as a whole helps pupils advance their thinking beyond the "additive" approach and thus facilitates problem solving in certain types of ratio and proportion tasks.

The next stage of our research will be to support these findings by analysing the rest of our data on group discussions and then to explore the implementation of these discussions as part of a regular classroom teaching session.

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