Simulation and Statistical Exploration of Data (e.g. Fair Die or Unfair Die)
Test of Hypothesis on Fair Die (Simulation of Chi Square Tests)

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Abstract:
The considered experiments help our students better to understand the randomness and the statistic methods of every day life. At first we initialize the random number generator of our **CASIO FX 2.0 PLUS** in the RUN-menu by the help of **Ran# 0**. Let us begin with an experiment on a die which has been rolled **N = 100** times. Each face does not appear an equal number of times. Is there something wrong with the die? In **M = 250** of such experiments the chi square variable is computed, i.e. the die is rolled **N * M = 25000** times by the help of the **CASIO FX 2.0 PLUS**. We check up that indeed chi square variables are simulated (only in the case of a fair die). The chi square variable is a statistical measure on the difference between the expected outcome and the actual outcome. The probability theory tells us that we should expect each face of the die to appear **N / 6** times. But in actuality this usually does not happen. By the help of the **CASIO FX 2.0 PLUS** we simulate **M = 250** chi square variable to answer the question „What is the significance of the chi square test?" and „How close to zero must the chi square variable be to conclude to have a fair die?". Here we simulate an unfair die with the probability distribution **P(X = k) = 2 / 11** for k=1,2,3,4,5 and **P(X = k) = 1 / 11** for k=6.

Keywords: chi square goodness of fit test, random number generator, simulation and exploration of data

1. Discussion on the considered problem:

The **chi square goodness of fit test** computes the chi square variable C, which we have simulated **M** (= 250) times (**M** experiments), to decide the hypothesis on the fairness of the rolled die.

The **null hypothesis** is **P(X = k) = 1 /** 6 for all **k = 1, 2, 3, ..., 6.**
(The **alternative** let be
P(X = k) = 2 / 11 for all **k = 1, 2, 3, 4, 5,** and **P(X = k) = 1 / 11** for **k = 6**)

If in one experiment we roll the die **N** (=100) times, we have
the **expected frequencies** **List 11 = {N/6, N/6, N/6, N/6, N/6, N/6}** and
the **observed frequencies** **List 12 = {H_1, H_2, H_3, H_4, H_5, H_6}** with **$H_1 + H_2 + ... + H_6 = N$**

E.g. let be **List 11 = {100/6, 100/6, 100/6, 100/6, 100/6, 100/6}, List 12 = {15, 10, 18, 22, 17, 18}**

We compute in the RUN-menu **C = chi square value = Sum((List 12 – List 11)^2 / List 11) = 4.76**

Practically by the help of **one** chi square value we have to decide between the null hypothesis and the alternative.
What is with the error of first kind, if we decide against the null hypothesis and the null hypothesis was valid?
What is with the error of second kind, if we decide for the null hypothesis and the null hypothesis was false?

Remember:
We consider **the probability of the error** of the first or of the second kind.

By the help of chi square distribution (5 degrees of freedom) we know:

P(C $\geq$ 4.76 | null hypothesis is valid) =
= 1 - Int(sqrt(X^3 * e^(-X) / (18pi)), 0, 4.76, 10^(-6)) = 0.446 = 0.45 = alpha

(Here the simulation shows **P(C $\geq$ 4.76 | null hypothesis is valid) = 112/250 = 0.448 = 0.45**)

Thus alpha% = 45% of our experiments give a chi square value of a fair die, which is at least 4.76! If we decide against the null hypothesis, than the probability of the error of first kind is alpha% = 45% !

On the other hand by the help of our simulation,

P(C $\leq$ 4.76 | our alternative is valid) = 45/250 = 0.18 = beta

Thus if we decide (because of C $\leq$ 4.76) not against the null hypothesis, then the probability of the error of the second kind is beta% = 18% !

Finally the questions:

Practically the chi square goodness of fit test works with a significance of alpha% = 5% , i.e. we need the quantil $C_{0.95}$ with P(C $\geq$ $C_{0.95}$) = 0.05 or P(C $\leq$ $C_{0.95}$) = 0.95 .
How to compute the quantil $C_{0.95}$ by the help of **CASIO FX 2.0 PLUS**?

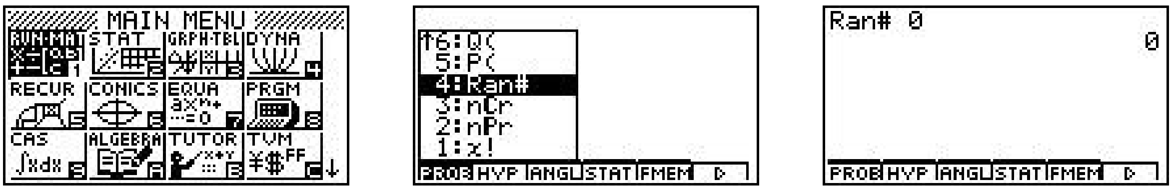
An other question ist the solution $\mathbf{C_{alpha}}$ of the Equation **alpha = beta**, i.e.
alpha = P(C $\geq$ C_{alpha} | null hypothesis is valid) = P(C $\leq$ C_{alpha} | our alternative is valid) = beta
and the computation of the error probability alpha (= beta) by the help of the **CASIO FX 2.0 PLUS**.

The **CASIO FX 2.0 PLUS** can not solve these problems in a direct manner (EQUA-menu or CAS-menu) but by the help of numerical integration and tabulation the (empirical) distribution functions of the simulated data in the RUN-menu. In the STAT-menu we can observe the functions in form of x-y-lines and search the solutions of the considered equations.

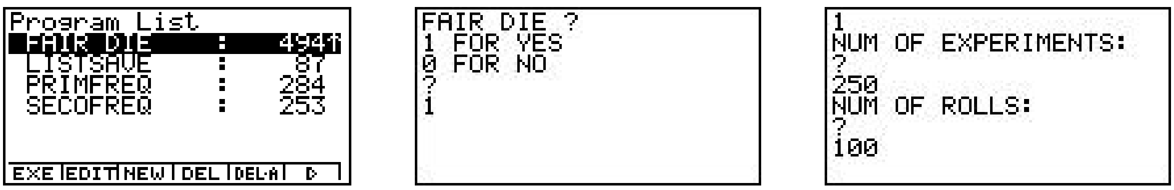
2. Sceenshots on the simulation and statistical/graphical exploration of data:

In the STAT-menu make the following SET UP: List File: File 1, Display: Fix3 :

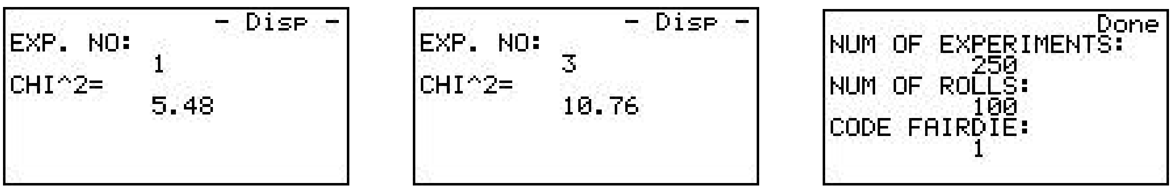
Start of the random number generator after resetting: RUN-menu: Ran# 0 and using the generator Ran# 1



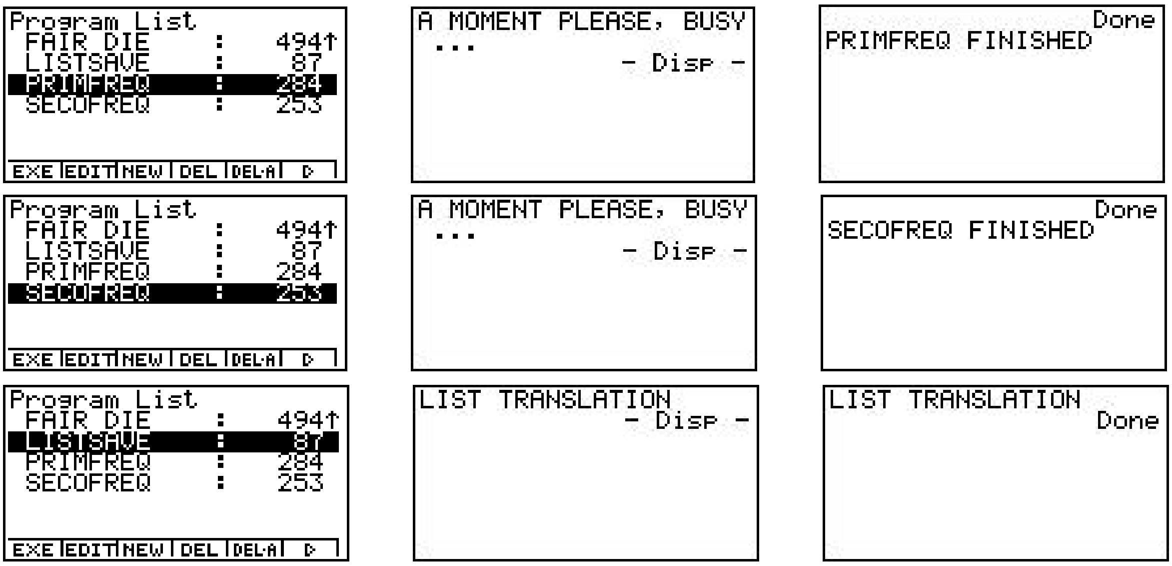
Start of the program **FAIR DIE** (Simulation of a fair die, code = 1)



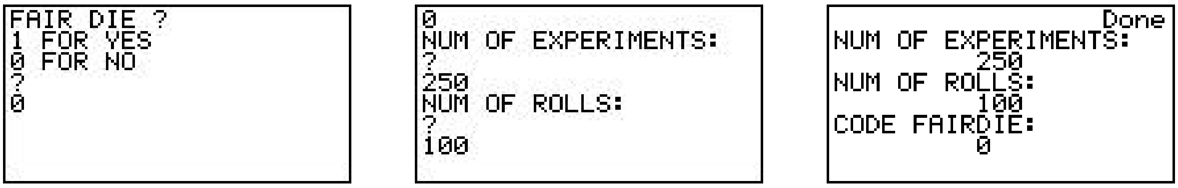
The CASIO FX 2.0 PLUS needs approximately 30min to generate 250 chi square data. Some chi square data:

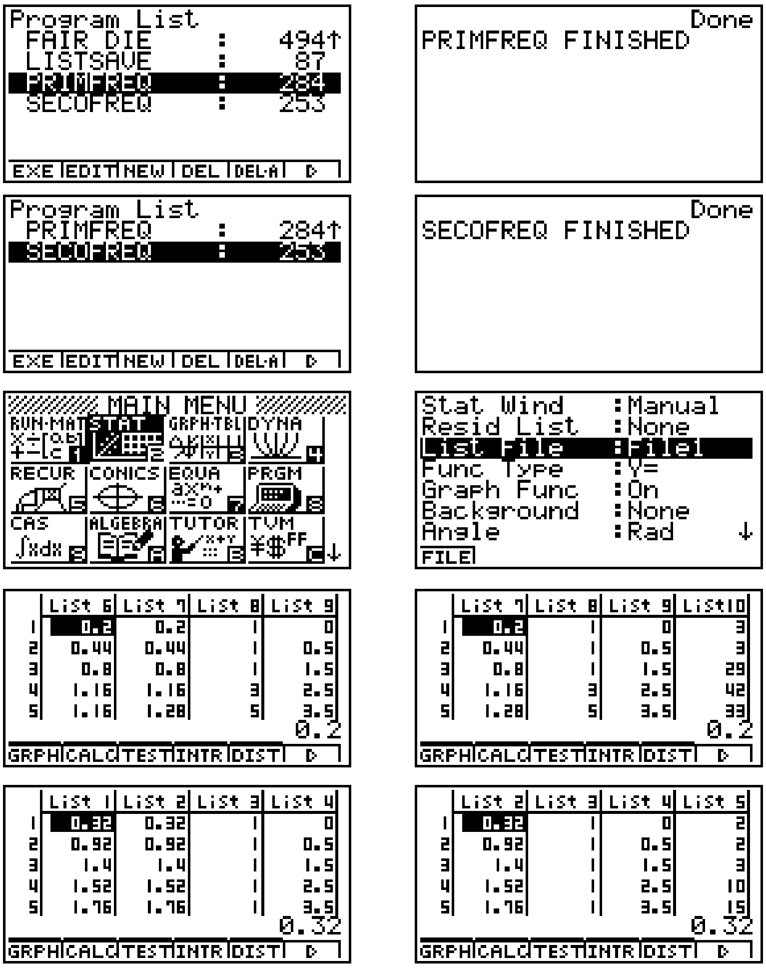


Now we generate the primary/secondary (grouped) data and frequencies to draw statistical graphics:



Start of the program **FAIR DIE** (second simulation, simulation of an unfair die, code = 0)
The CASIO FX 2.0 PLUS needs approximately 30min to generate 250 chi square data.





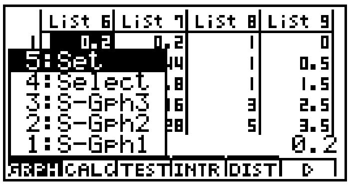
Next we consider the created lists:

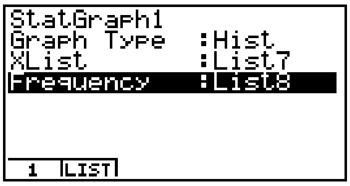
Remember, using List File 1

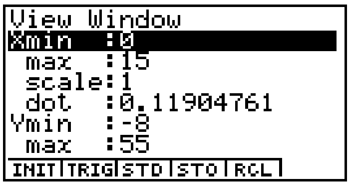
Chi square data (fair die)

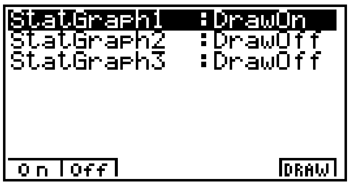
Chi square data (unfair die)

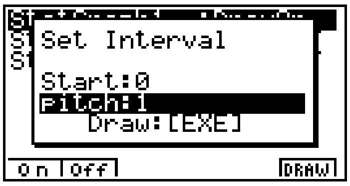
Now the definition and the histogram of the fair die chi square data follow:

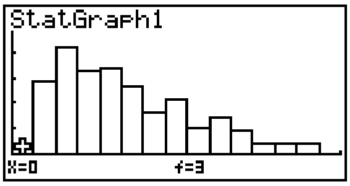


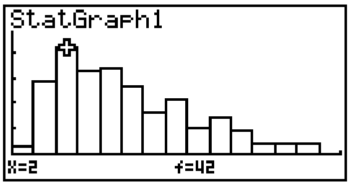


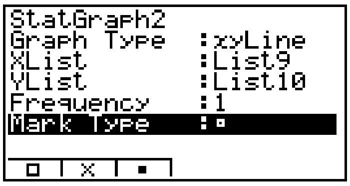


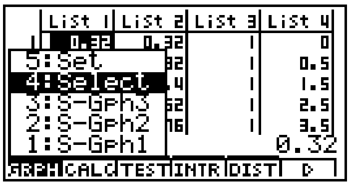


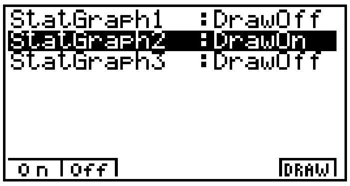


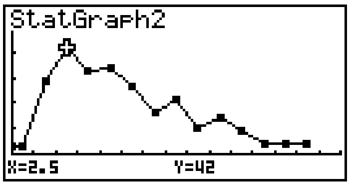




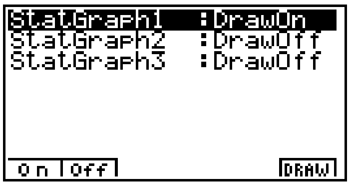


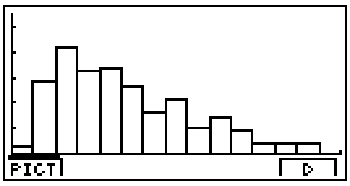


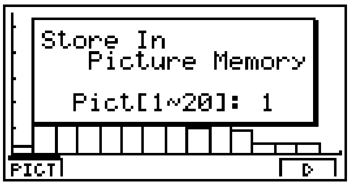




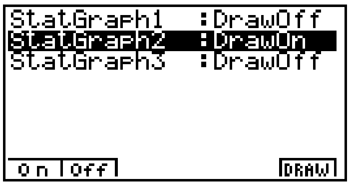
The frequency polygon

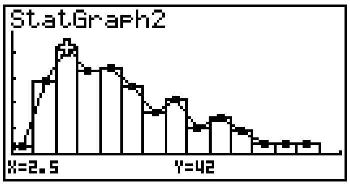






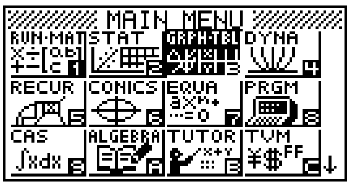
Store the histogram in the background picture 1

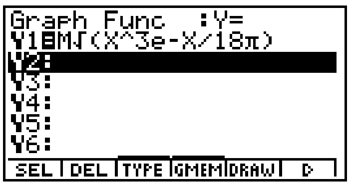


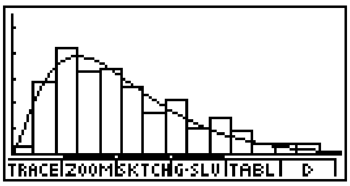


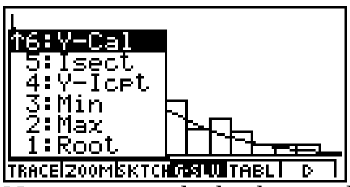
polygon and histogram together

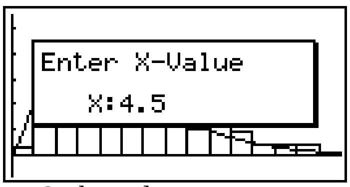
In the GRPH-TBL menu we draw the **chi square density function** with the background picture 1:

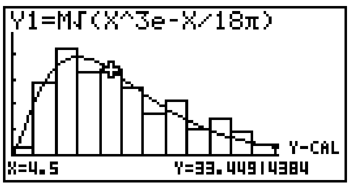




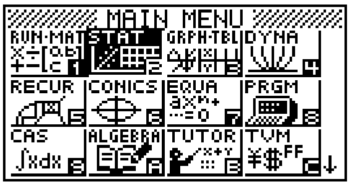


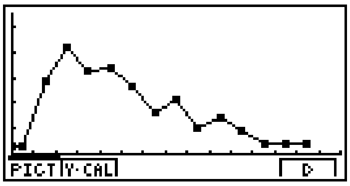


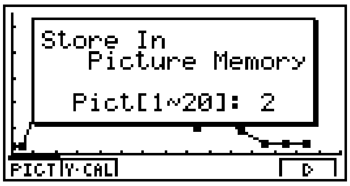


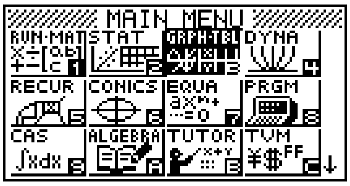


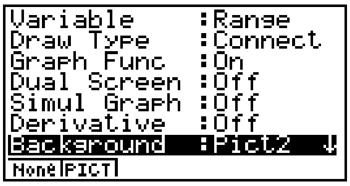
Next we store the background picture 2, the polygon:

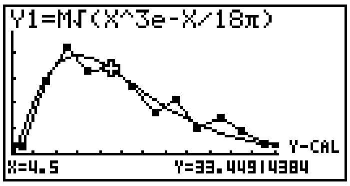






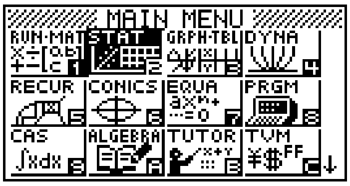


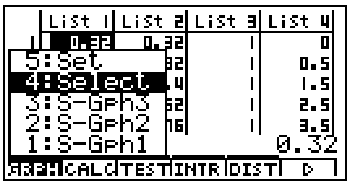


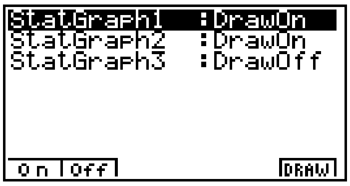


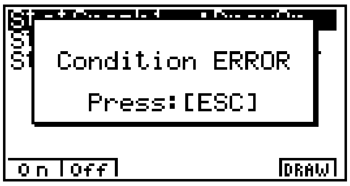
The chi square density function together with the polygon (fair die)

3. Now some error conditions:

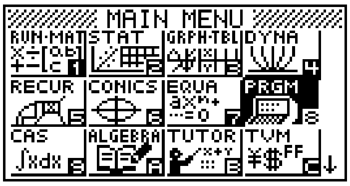


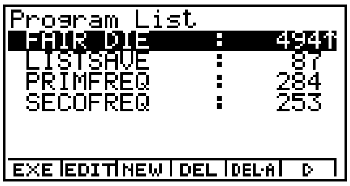


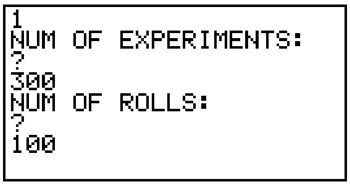


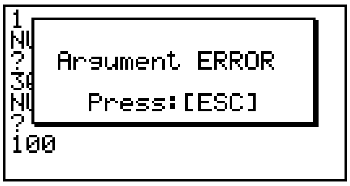


The FX 2.0 PLUS can’t draw two different statistical plots



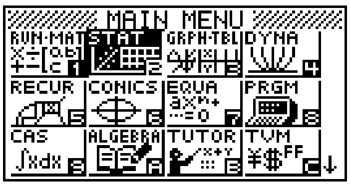


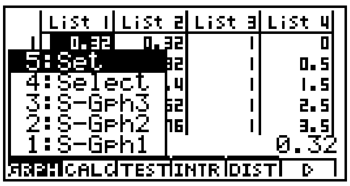


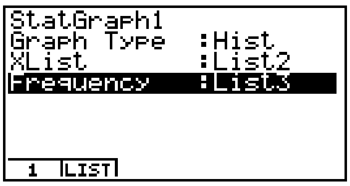


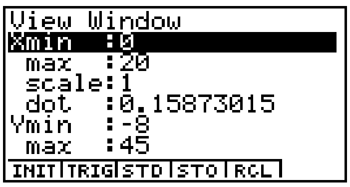
Too large lists with 300 elements are not possible

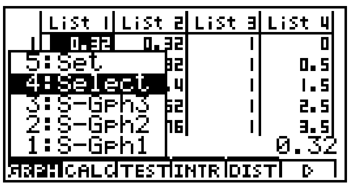
4. Screenshots of the chi square data of the unfair die:

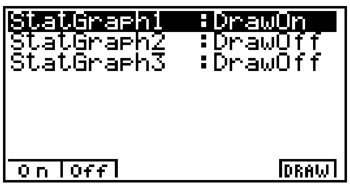




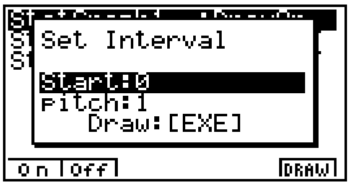


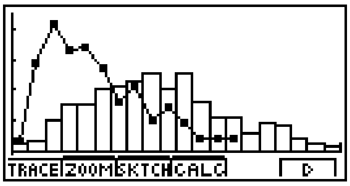




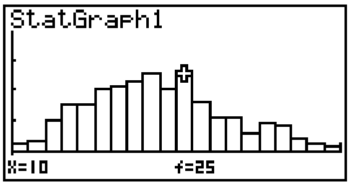


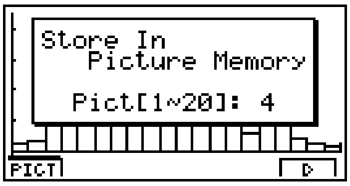
The polygon (background picture 3 in the new view window, fair die) together with the histogram of the chi square data (unfair die):



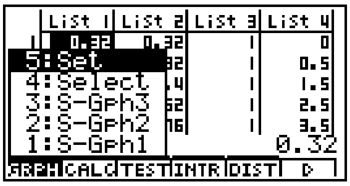


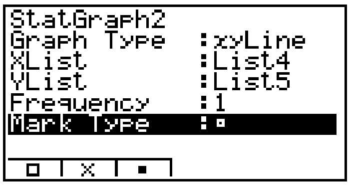
The histogram (unfair die) we store in picture 4:

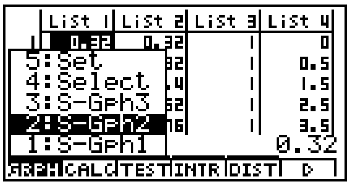


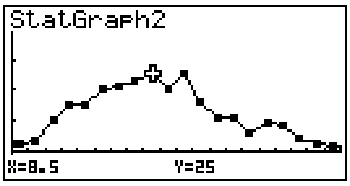


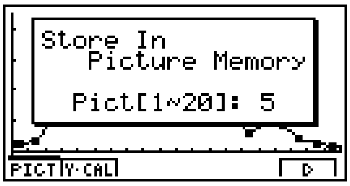
The polygon (unfair die) we store in picture 5:

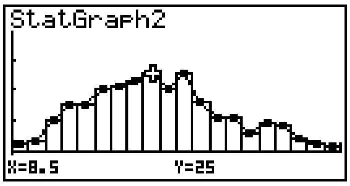






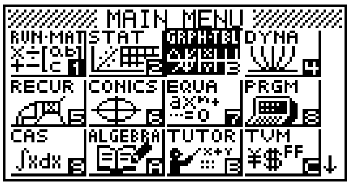


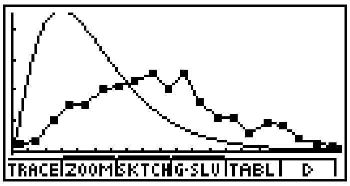




The polygon together with the background picture 4

Finally we draw the chi square density function (fair die) and the polygon of the unfair die:





Now we can see, that the computed data of an unfair die are not chi square data!

For more informations see internet (To read the pdf-file use the Acrobat Reader version 5.0.5.)**:**
http://www.informatik.htw-dresden.de/~paditz/Paper_Palermo2002.pdf
Program files you get by the help of the CASIO Program-Link FA-123 (Software) here:
http://www.informatik.htw-dresden.de/~paditz/FAIRDIE1.CAT

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