Mathematics Teachers and Students: How can we improve the human side of their relationship?

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Summary
After an introduction on the importance of redefining the didactic system in the terms of a more global interpretation of the personal relationships which intertwine in it, we look at the general principals of "cooperative learning", an educational strategy by social mediation in which the resources for the construction of knowledge are the students, who are called upon both to accomplish a disciplinary task and to develop social abilities. The role of the teacher remains fundamental, being the organiser and facilitator of the entire process.
After the presentation of some didactic experiences carried out in the upper secondary school with the cooperative groups, some elements of the evaluation of the experiences carried out are put forth, also through the judgments expressed by the students themselves. At the end, note is made of some open problems which it seems necessary to confront.

1. Introduction

Before entering into the heart of the presentation, it seems to me important to begin with an observation. The question expressed in the title of this talk presupposes that the necessity of such an improvement is shared by all. It is my conviction, as a matter of fact, that, in most cases, the personal relationship between teachers and students during the entire teaching-learning process is not at all satisfying and that this is even truer for the mathematics teacher with respect to teachers in other disciplines and in relation to upper middle school students rather than in relation to pupils in the compulsory school. This is not the moment, however, in which I intend to back up these convictions of mine.
The objective of this talk will be reached if, following what I have presented, someone will reflect on the reality of a mathematics lesson, then deciding in full liberty to put into practice, or not, some form of change in the direction which will have been illustrated.
I clarify, only that when I say "personal relationship" between the teacher and pupils, I do not mean to refer to a relationship centred exclusively on the discipline in which the teacher is a specialist (for example mathematics). Rather, I mean to refer to the relationship between people, understood in the more global sense of the term, in the absolute conviction that it can never reduce itself to a specific disciplinary aspect, but it invests every other aspect of the people involved, in a certain sense all of their history.
This idea, that I have come to understand ever more clearly through my personal and professional experiences, mostly in recent years, presents a notable resonance with a relational-systemic epistemology that, developed in the second half of the twentieth century beginning with the areas of engineering, cybernetics, physics and neurobiology, has strongly influenced the field of psycho-therapy and successively that of all the human sciences; in particular psychology, pedagogy, and sociology (A. Munari, 2001).
"System" means a complex of elements which inter-react and what is interesting is that each element assumes its own meaning in relationship to these interactions (Bertalannfy, 1968). However, the relationships do not exist only amongst the elements within the system, but the relationships of a system with the environment in which it lives are important; the environment also is another system with which the first must co-evolve. From a systemic perspective it is necessary,

therefore, to place the accent on the relationships between the various elements. It is impossible to consider a single element isolated from the others.
This concept was largely developed by constructivists, beginning with Piaget, and an important spin-off was, after the 60s, the affirmation of "active pedagogy", which brought to light the importance of contexts, concrete experiences and a careful organisation of spaces and times, in the learning process.
The passage from practical activity to its representation, that is, the construction of knowledge, is not all that simple and as a consequence the concept of cognition has been reformulated both as "situated cognition" (R. E. Nunez, 1999) giving relevance to the context, and as "distributed cognition" (K. Crawford, 1997) giving relevance to interrelationship and to sharing.
Today, given the number of studies on motivations, on the importance of the resources of people, on constructive and destructive emotions, on communicative ability, on organisational climates, et cetera, we are able to maintain that a more accurate rereading of the entire didactic system (teacher – pupil – knowledge – environment, Brousseau, 1997) is necessary; placing in discussion, above all, with which sort of relationships is it necessary to concern ourselves.
Having had the occasion to deepen some of these themes with psychologists, pedagogues, sociologists and philosophers, I hope to be able to offer a contribution in the direction of an effective change in the network of human relationships on which communication is founded and on which, among others, the didactic processes are developed, in particular those relative to mathematics education, with which I have concerned myself for years.
A teaching-learning model which is founded on relational-systemic epistemology, but seems very promising in the direction of interesting developments is that of Cooperative Learning. That which follows is dedicated to the central ideas of this educational modality, to some experiences carried out in class on this model, to a qualitative evaluation of the first results obtained and to some open problems.

2. The general principals of Cooperative Learning

Cooperative learning refers to a modality of democratic management of the teaching-learning process, centred essentially on the resources of the pupils. In recent decades it has been extensively diffused at the international level and it has been applied to a notable variety of categories of people, from pre-school age pupils to adults in professional contexts. In Italy, the diffusion, discussion and application of this educational modality was developed in the 80s and among the scholars engaged in this process there are M. Comoglio, of the Pontifical Salesian University in Rome and G. Chiari, of the University of Trento. Each of them is involved in both precise scientific research and in widespread activities of diffusion.
In a recent analysis (D.W. Johnson, R.T. Johnson, M.B. Stanne, 2000) regarding the efficiency of the various typologies of cooperative learning with respect to other modalities of teaching-learning, reference is made to over 900 research studies. Up to the present not only the experiences conducted in the cooperative mode are truly numerous, but also the scientific analyses of them and this highlights how broad ranging the phenomenon is.
The modalities that usually are considered as the antithesis to those of the cooperative type are the competitive ones, in which the objective is that of reaching a goal without the others reaching it and the individualistic type, in which the objective is that the individual reaches the goal independently of the fact that the others reach it. The analyses cited, in which research conducted in North America, Europe, the Middle East, Asia and Africa is considered, brought to light the greater validity of cooperative learning with respect to the others methods. There were multiple variables that were studied. On the one hand, they regard specific cognitive tasks, as for example memorisation, the ability to transfer knowledge and the production of complex reasoning, On the other hand, they regard attitudes toward themselves and toward others, as for example self-esteem,

psychological well-being, the reduction of stereotypes and prejudices and the development of social skills.

The characteristic that impressed me the most in Cooperative Learning is that it is a teaching-learning model in which scientific investigative processes are combined with the development of social skills. Therefore, the objectives to be reached play out not only at the disciplinary level but also at the social level, placing an inevitable emphasis on the relationships that are established amongst the people. This is, in my opinion, the central innovative aspect. The class teachers are always held responsible for the cognitive level reached by their students in the discipline in which they are specialised, but one cannot say the same thing in relation to the "climate" of the class at the psychological and social levels. The possible tensions and conflicts between specific individuals or groups of students, the convictions of the pupils about the values carried by the school, their attitudes toward the scholastic world or society in general have never been held subject to reflection on the part of the disciplinary teachers, much the less connectable to the teachers' own tasks.

It seems to me, therefore, that Cooperative Learning is an effective evolution in teaching-learning models, precisely for the global way and complexity with which it confronts the problem, in tune with what is expressed with regards to the systemic-relational approach, but also making itself responsible for the quality of the personal relationships within the educational processes.

In the sphere of the constructivist theories shared today, not only, for example, in the theory of didactic situations (of G. Brousseau, 1997) which is very centred on disciplinary cognitive relationships, but neither in the "Inquiry" (of R. Borasi, 1996) or in social constructivism (of P. Ernest, 1995, and of H. Bauersfeld, 1995) where the calls for the necessity of development of interpersonal relationships are more evident, the urgency of the reference to double polarity (discipline - relationship between people), assumed to be irremissible by Cooperative Learning, is not so explicit.

The research developed in Italy by the group coordinated by R. Zan (2000, 2002) also makes reference to the importance of a teaching-learning model in mathematics which keeps in mind the people involved in such a process in a global way, not leaving out of consideration, therefore, taking responsibility for the strictly personal characteristics of the individuals (teachers and pupils) involved in the process. The basic idea shared in such research, that the authors link very closely to the epistemology expressed by M. Polany (1958/1990), is that each cognitive act is an activity which always involves all the elements of the people, for example sensitivity, emotions, commitment, acceptance of risk, decisions, choices, beliefs, respect for others. It seems completely impossible, therefore, to be concerned with educational processes without a model which takes into account such articulated complexity.

Another reason, of a less general nature, but no less significant because of this, which has contributed to developing my enthusiasm for cooperative learning is the fact that in it class discussion is much better organised and therefore in the final analysis enabled.

The importance of arguing and discussing in class has already been fully underlined by didactic research, in particular at a national level. It's enough to remember, for example, the studies done by M. G. Bartolini Bussi and M. Boni (1991, 1995), the contributions of various research groups collected by L. Grugnetti, R. Iaderosa, M. Reggiani (1996) or the anaysis of specific didactic situations (R. Garuti et al., 1999, A. Pesci, 2000, 2002, M. A. Mariotti et al., 1998). What emerges from this research is that in each case the conducting of an appropriate class discussion is a very difficult job. Often, the teacher cannot manage to give everyone the possibility to express himself, is not adept at soliciting participation by those who are not used to doing so spontaneously, is not always able to collect the children's contributions which could give rise to productive developments, cannot manage to be sufficiently attentive to the social dynamics of the class or has difficulty in managing time in the most opportune way.

Through cooperative groups, at least in the interpretation that we have given in our experiences, we have ascertained that the class discussion is very simplified, in that the final phase of discussion comes about above all amongst groups (and therefore on a limited number of proposals), after each

single group within itself has already discussed and shared a final product. The involvement of the children comes about therefore in two successive times; one within the single group and the other during the class discussion, facilitating both the production of what is requested and the phase of collective comparison and discussion. All of this will be clearer in the following paragraph, when I will give more detail on the practices which we adopted for carrying out the cooperative groups in class.

Having generally clarified the motivations which pushed me to develop and experiment with cooperative learning projects, I would like, first of all, to recall some central ideas on which this educational movement has developed.

The researchers who are concerned with tracing the historic development of Cooperative Learning often make reference to J. Dewey, K. Lewin and M. Deutsch as the fathers of this educational model. According to Dewey's educational philosophy, (1943, first edition 1899) it is essential that one thinks about the teaching-learning process considering both the cognitive aspects and the motivational and socio-interactive ones. The school, in all its processes, should function as a democratic society and the students, as citizens in a democratic society, should take part in the planning of their scholastic environment and of their learning activities, to be developed for the most part in a collaborative way.

Lewin (1935) and Deutsch (1949), whose contributions in the area of group social psychology are held to be fundamental, also agree on the necessity of setting up education in a collaborative way in order to improve society. Lewin underlined, in a specific way, the importance of interactions with others and of the organisational characteristics of the environment as determining elements for the interpretation of human behaviour. That which we do in a given context is profoundly influenced by how this context is organised and by the ways in which the various individuals behave amongst themselves.

R. T. Johnson and D. W. Johnson (1980), who in the 70s gave life to a vast research movement and the diffusion of Cooperative Learning, make reference to Lewin, Deutsch and Vygotsky. The concept of positive interdependence introduced by Deutsch, in particular, was considered by them as a fundamental ingredient for the creating of a significant Cooperative Learning experience. (We'll return to this concept shortly.)

Y. Sharan and S Sharan, (1992) considered to be amongst the principal exponents of Cooperative Learning, also make reference to Dewey and Lewin. Another theoretical foundation of theirs is given by J. Piaget and by all the constructivist school, which places the accent on the importance of conflict as a crucial moment for the construction of knowledge on the part of the individual. During a group debate, the students should learn to exploit, in a positive way, the conflicts and situations of interrelation as targeted occasions, both for disciplinary learning and for the development of cooperative work modalities.

Today there are many researchers who are concerned with studying Cooperative Learning models and classifying their various typologies. Some authors, for instance, trace six different typologies (M. Comoglio, M. A. Cardoso, 2000) while others (R. T. Johnson et al, 2000) identify ten.

The objective of this talk is neither to illustrate the multiplicity of such models nor to present the different nomenclature that identifies them. Rather, it is to highlight some central characteristics both because they are amongst the most shared and because they are those connected to the didactic experimentation work that we have carried out and that will be presented in 3.

Among the conditions that are held necessary for Cooperative Learning, there is, first of all, *positive interdependence*, which is reached when the members of the group understand that collaboration is such that individual members cannot exist without collective success and, as a consequence, the failure of only one element of the group is failure for all. Each one must be deeply convinced of being able to give a useful and indispensable personal contribution to the realisation of the common project and in this way he develops a strong sense of responsibility which translates into greater personal commitment, with positive consequences for learning and on his ability to work in a group.

Another important condition is the definition and *assignment of roles* to each component of the cooperative group. The division of social and disciplinary skills amongst the members of the group encourages collaboration and interdependence, assures that individual abilities are utilized for the common work and reduces the possibility that someone refuses to cooperate or tends to dominate the others.

In this frame it is essential to clarify the difference between the *status* of an individual and the *role* attributed to him. The *role* is assigned in a hierarchical way by an authority; for example by the teacher. *Status*, instead is that by which a person is recognised by society; not only with reference to his intellectual gifts or other personal characteristics, but also his social condition. Tied to the characteristics of *status* are the general expectations of skill, shared not only by the group but also by the individual himself and this could be an obstacle in relation to the objectives that one desires to reach in cooperative work. The one who is considered to be at a "low" level tends to intervene less than one who is considered to be at a "high" level and therefore has fewer occasions to develop his skill, ultimately, solidifying his "low" level. (E. G. Cohen, 1984)

With the attributing of a *role* to a student, full realisation is given to his autonomy; that is, he is allowed to take decisions, to evaluate and to control. When several roles work together contemporaneously, a situation of equal authority is established, making everyone a protagonist. In this way everyone puts in action his personality, emotions, his ability to decide and to manage his various skills. The recognition of a *role* on the part of the classmates, which comes about out of the consideration for the difficulties of the person and is fulfilled through interpersonal relationships, encourages the over-coming of possible problems (as for example, low self-esteem, the lack of regulation, the sense of inefficiency) which with *status* alone would not be possible to confront (L. Vianello, 2002)

Another component which is held essential in the fulfilling of Cooperative Learning regards *social abilities*. An efficient management of interpersonal relationships requires that the students know how to sustain a leadership role within the group, take decisions, express themselves and listen, ask and give information, stimulate discussions, know how to mediate and to share, know how to encourage and to help, facilitate communication, create a climate of trust and resolve possible conflicts. These abilities must be taught with the same awareness and care with which disciplinary abilities are taught.

In relation to the problem of *evaluation*, Cooperative Learning requires that the results reached are considered both at the individual level and at the group level. Group work always concludes with an individual evaluation of the disciplinary and relational aspects, usually entrusted to the teacher, but it is also important a discussion phase in which each group can, by comparison with the others, evaluate what has been produced. The evaluation of the group, as such, constitutes a strong motivation for its members to improve the quality of their collaborative work and this, inevitably, translates into individual progress.

From what has been shown, it clearly emerges how essential the role of the class teacher is. Along with the disciplinary skills, the social skills assume a decisive importance. In relation to them, the teacher must take decisions about the formation of the groups, developing in the students the social skills already mentioned, check the appropriateness of the group work, intervene with timely suggestions, encourage discussion, promote interventions and evaluate the results obtained.

To conclude this section it might be interesting to show, in synthesis, what the disciplinary and relational type advantages are, which the literature highlights as characteristic of Cooperative Learning.

From the disciplinary point of view:

- greater motivation toward the contents, better relation to the discipline and increase of personal work;
- greater autonomy in the acquisition and use of knowledge;
- improvement of metacognitive ability and consequent strengthening of study strategies;
- better critical ability and ability to synthesize;

- increase of the sense of self-efficiency.

From the relational point of view:

- better ability to work in a group in the carrying out of a common project;
- increase in the opportunities to share feelings, aspirations, difficulties and satisfactions with classmates;
- increase in positive relationships with classmates;
- better ability to confront interpersonal problems and to resolve conflict situations;
- increase of the possibilities to try different roles with the consequence of developing a better sense of one's self;
- more tolerance and ability to understand and accept others.

3. Some didactic experiences: starting points for reflection

The didactic experiences that we have carried out setting up a cooperative work modality regarded three classes in the upper middle school: a second class in a technical industrial institute (in which the Pythagorean theorem and its demonstration were proposed), a first class in a linguistic high school (in which several plane isometries were studied with the Cabri software) and a fourth class in a scientific high school (where the definitions and properties of exponential and logarithmic functions were examined). In all, 67 students (from 14 to 17 years old) were involved, three mathematics teachers of the classes and three university students who were getting their degrees in mathematics.

These experiences were carried out during the last scholastic year and this is not the place to present them in detail. Two of them, in particular, were studied in depth, as they were the subject of degree thesis work for a degree in mathematics (A. Fattori, 2001, G. Farina, 2002).

My proposal now, is to present the principal qualitative data which emerged from the analyses of these experiences, also from the words of the same students and teachers who were involved.

It is necessary, first of all, to clarify how we carried out the cooperative work, interpreting in our classes, with few modifications, the suggestions of L. Vianello, a psychologist who collaborates with the Cà Foscari University of Venice and who has been working for a decade in the experimentation of cooperative learning, both in the elementary school and in the secondary school.

The organisational framework put forth by Vianello and founded on the general principals of Cooperative Learning recalled in the previous paragraph, is very precise and this allowed us to carry it out quite easily.

Each cooperative group is made up of 5 or 6 people, according to the number of pupils in the class. Within a group, each one participates in the solution of a disciplinary task, which can be assigned by the teacher or created by the class itself, but, moreover, each one must perform one of the following five roles:

coordinator: this is the pupil who is *orientated to the task* and must make sure that his group reaches the best result possible. He, therefore, is concerned with translating the task into an appropriate work plan, making sure that no-one is lost in the secondary aspects of the problem, making the point of the situation and urging the group to take decisions;

psychologist: this is the pupil who is *orientated to the group* and is responsible for the communicative climate. He must, therefore, make sure that everyone participates positively in the solution of the task, encouraging anyone who seems to be in difficulty, making sure that the various interventions are balanced in times and ways and playing down any possible conflicts;

notary: he is the *memory* of the group and is responsible for the verbalisation of the results of the group. During the work, he repeats the shared decisions, asks for the confirmation of partial formulations of the results and of the final report, agreeing with all of the components of the group, but overall with the spokesperson;

spokesperson: he is the manager, for the group, of the *oral report* on the results of the collaborative work carried out. He arranges, with the notary, the final version of the results reached and reads them to the entire class in the final presentation phase;
observer: he is responsible for the observation of the *interactive process* of the group. He observes whether or not each one carries out the task actively and appropriately, for example without predominating, whether or not each one suitable performs correctly his role and if the phases of the work are all accomplished. He takes notes on what he has observed and communicates them to the entire class in the final discussion phase.

In the case that it should be necessary to organise groups with six members, it is preferable that the role of observer is performed by two people, in such a way as to increase the number of notes about the interactive process of the group. On the other hand, in case it is necessary to form groups of four people, it is preferable to combine the roles of notary and spokesperson, due to the analogous nature of their tasks.
It should be noted that the observer participates in the solution of the disciplinary task but the performing of his role is not explicit. He observes the behaviour of his classmates and takes notes but does not mention them to the group, he mentions them to the entire class at the end. The observer must know well the skills required by his classmates for carrying out their various roles and he is a very important figure because his position allows him to express judgements about the others. If, for example, in a class there is a pupil in difficulty due to low self-esteem, it might be useful to have him take on this role. Finding himself in a situation recognised by a group of his peers, which legitimises him as observer and evaluator of his classmates, he could develop skills which will go on to positively influence his self-esteem.
It is also important that for each task the roles do not remain fixed but are rotated, in such a way that each person can have different experiences and develop different social abilities, managing also to better perceive his own skills and talents.
The role foreseen for the teacher is that of *supervisor*. Beyond the organisation of the work outside of the class (choice of the disciplinary task, choice of the criteria for the construction of the group, preparation of the didactic material), in class, during the cooperative work, he must not give suggestions relative to the solution of the task assigned but be particularly attentive to the interrelational processes. If he realises, for example, that someone is not appropriately performing his role, he approaches him and, whispering, so that only the person concerned can hear, gives him a few useful suggestions. In this way, the teacher becomes a "personal assistant" of his students, creating a more personal and equal relationship.
One technical note, which however is not without importance, is that each one, to facilitate the recognition of his own role, both on the part of the teacher and on the part of his classmates, has a card with the exact name of the role he takes on. This means an effective external sign which encourages both the assumption and the recognition of the role.
At the end of the group work there is a class discussion in which all the results obtained are shared, as well as any possible unresolved problems. This final phase foresees, first of all, the presentation of the spokespeople which is immediately followed by the presentation of the observers. Only at this point is the discussion opened up to the whole class and the debate developed both on the relative results of the disciplinary task assigned and on possible problems emerged during the performance of the roles.
It is, therefore, evident that opportunities for reflection, both on the discipline and on the interpersonal relationships that have been built, are continuously offered to the class.
At the conclusion of all the work, it is important that the children are invited to express their evaluation of the work done, for example, on a form prepared by the teacher expressly for this task, which, according to his decision, could be structured with precise questions or to provide for freer observations by the children.
From what has been shown, it clearly emerges that the structure of all the work is rather complex. Nevertheless, when it is clearly presented to the children, complete with the motivations which

brought the teacher to explore this educational path (motivations that obviously will be different from person to person) and with the disciplinary and social type advantages to which one wants to arrive, it usually happens that the expectation of the children becomes very strong, urging them to put themselves to the test, with commitment, and this facilitates the entire process. After the first phases, in which everyone must get used to correctly interpreting the roles and to balancing himself suitably between disciplinary task and social ability, one arrives at confronting the work with greater competence and a greater sense of responsibility.

The global judgments of the teachers about the didactic experiences done was fully positive on the merit of the students' participation, the commitment demonstrated and the disciplinary results attained. Their perplexities were relative to the management of time, the fear of not always being able to take appropriate decisions and above all the lack of preparation to identify, before managing, possible conflict situations in interpersonal relationships. The preparation of a teacher who is able to carry out an appropriate role in the area of cooperative groups is, in effect, an open problem. Even today, the disciplinary preparation in the training program of a teacher is decidedly predominant with respect to the construction of pedagogical, psychological and social skills, even if we are witnessing rapid and promising changes.

In each of the three secondary school classes in which the experience was done, the children, at the conclusion of the work carried out, were asked their personal judgment about what had been proposed to them. Since, in each of the three cases, the class was used to expressing itself quite freely, even with critical judgments, it can be maintained that the evaluations which emerged were quite reliable and constitute some important indicators.
The students' evaluation was globally very positive. Most of them expressed the desire to repeat the experience even for the other scholastic disciplines and some proposed a direct comparison between cooperative work and the traditional lesson. Here, for example, are some meaningful quotations:
"…different from the normal lesson, where the teacher who explains is 'the protagonist', here we feel like 'the true protagonists'. We express our opinions, we formulate the properties by means of our deductions and considerations" (Patricia, I)
"…As work, it is beautiful, because doing a normal lesson is very monotonous because it is the prof who speaks, and those who understand follow and those who don't understand can't manage to follow. Instead, in this way, everyone can try to find the solution" (Paolo Be., II)
"…It was very different work from that done in the past, as a matter of fact, it was more entertaining and it involved me more, with respect to the old work.…I think that this is an efficient way to work, because you enjoy yourself but you learn." (Marco M., II)
"…the experience was certainly useful and interesting and certainly more entertaining than normal lessons" (Norma, IV)

In the protocols examined there are numerous positive observations about the disciplinary requests of the tasks, but the ones that regard interpersonal relationships brought into play by the modality of the work adopted, which for everyone was a strong novelty, are even more numerous. Here are some examples.
"…in my opinion, doing this group, it is easier to understand the things because the classmates help each other" (Maria, II)
"…this experience helped to reason about something more complex than usual" (Stefano. II)
"…we arrived at the conclusions by means of our own reasoning" (Fabiana, II)
"…this was a new adventure. The assigning of different roles, each with its own importance, characterised this activity, involving us in a surprising and unexpected way. Moreover, it helped us to appreciate geometry, a subject which is often boring and misunderstood… we managed to open up more to everyone and to create new likes and friendships" (Patricia, I)
"… it helped to discover sides of my classmates behaviour which I didn't know" (Guido, II)

"… the discussion at the end of each task was fundamental. We understood where we made mistakes and when we had made precise affirmations without anyone telling us if we had done something correctly or not. It was for me, therefore, a positive experience, entertaining, that above all helped us to express ideas in complete serenity" (Benedetta, I)
"It was a new and interesting way to learn geometry 'alone'. In the group you can make a mistake and manage to understand the error without feeling yourself judged by who has already studied everything, but by who could be mistaken like us, having the same knowledge" (Simona C., I)
"… I began to love geometry more…" (Valentina F., I)
"… this experience involved me in a subject with which I have always had difficulty, helping me to understand better and to conquer my timidity and my fear of making a mistake" (Elisa B., I)
"It was a beautiful experience. Finally mathematics is not such a weight for me. Moreover, I really liked comparing that which my group produced to that of the other groups" (Alex, IV)
"… I realised what collaboration in a group means and that is expressing your own opinions without dominating, listening to the considerations of everyone, discussing quietly and above all working not for a personal aim, but for a collective objective" (Carola, I)
"… each one had the same possibilities to express his opinions and even his doubts, in equal measure, without anyone standing out. I personally felt needed by the group in each role that I carried out" (Roberta, I)
"… the group work allowed the students with more difficulty to be able to have more and above all different help, seeing that it didn't come from the teacher but from a classmate." (Norma, IV: 6 pupils out of 25, in this class, expressed the same observation)
"… it was an interesting experience, in which each component of the group used all his abilities….. confronting together the contents of the problems permitted these to be understood and deepened by everyone" (Elena, IV)
"… unfortunately however, usually, it doesn't have the same complicity as that we have had during these months" (Paola, I)
Amongst the comments of the students, those which we can consider prevalently negative are very few (5 out of 67). These expressed, more precisely, perplexity, or their difficulties in overcoming a conflict in the group, as for example the following:
"The work was very beautiful, quite difficult and stimulating. The only defect was the group because we didn't work well together and there was laziness. Another thing is that the reason for this work is still unclear to me, but it's always better than doing a normal lesson" (Luca, II)
"The time that I liked less was the discussion, with all of the class, of the results obtained. I didn't like it because often the same things were repeated and it was boring. Of the group work, I liked the division into roles and the way of working in general. I only liked the subject a little" (Elisa, II)
"In my opinion, the work of cooperative learning has had negative aspects and positive aspects… for the negative aspects I encountered people who, in some roles, didn't carry out their work and didn't not participate" (Carlo, IV)
It might be interesting to note that the observations of perplexity were presented by good pupils, who were perhaps more hit by the fact that the work was not, for example, well organised in the division of the times or in the management of the final discussion, rather than by the advantages offered by collaborative work. Pupils with difficulties in learning or in relationships with their classmates presented observations in which the perception that the collaboration with their classmates could constitute the help of which they had need was very evident.

One can, therefore, conclude, both from the judgments of the teachers and from those of the students involved, that the experiences carried out had a decidedly positive impact, even though in all three cases it was the first experience in that sense, with all the uncertainties which that brings.
Following the wrap-up discussions carried out with the class teachers several open problems, that I would like to note here, were focalised.
As underlined in the previous paragraph, it is important that at the conclusion of the work, the teacher expresses both an individual evaluation and a group one. This task was indeed not simple,

above all in relation to the group. Usually the teacher is used to expressing evaluations on individual (oral or written) performances and, it is still not clear, in the case of the group, where the relational aspect is so important, what to observe and how to observe, to arrive at formulating an evaluation.
In a project developed in cooperative groups, the attention which must be given to the communications processes, both between teacher and pupils and between pupils themselves must be crucial. The personal relationships are the events which determine the quality of the educational process and which express themselves in communicative acts, translated into verbal and non-verbal languages (as for example, words, drawings, graphs, gestures, posture, tone of voice, etc.). We still do not have available analytical instruments which could read, in a global way, the didactic situation of a mathematics lesson with cooperative groups. The construction of observation grids which allow the mapping out, contemporaneously, of the various typologies of communicative actions which transpire on the didactic scene, keeping in mind the globality of the people involved (and therefore their motivations, convictions, expectations,…) appears urgent. This does not have to do with, for example, isolating single communicative forms, for example verbal language with respect to symbolic or graphic ones, but managing to observe how the coordination of the various languages comes about and how communication manages to make a specific contextual situation evolve, both at the disciplinary level and at the relational level. All this, obviously, could facilitate the task of the teacher, orientating him in a meaningful way.

It is evident, in conclusion, that with the cooperative work modality the work of the teacher becomes notably more complex than in a traditional lesson, delineating the necessity of skills which are still too distant from those normally acquired.
The challenge, nevertheless, seems very promising, in the sense of a strong redefinition of the centrality of the role of the teacher, understood primarily as educator in the broadest sense of the term beyond the exclusively disciplinary meaning to which, too often, he is usually ascribed.

References

Bartolini Bussi M. G., 1991, Social interaction and mathematical knowledge, *Proceedings PME 15*, Vol. 1, 1-16

Bartolini Bussi M. G., Boni M., 1995, Analisi dell'interazione verbale nella discussione matematica: un approccio vygotskiano, *L'insegnamento della matematica e delle scienze integrate*, vol. 18, n. 3, 221-256

Bauersfeld H., 1995, The Structuring of the Structures: Development and Function of Mathematizing as a Social Practice, in Steffe Leslie P. – Gale Jerry (Eds.), *Constructivism in Education*, Hillsdale, New Jersey, LEA, 137–158.

Bertalannfy L. v., 1968, *General System Theory*, G. Braziller, New York. Italian Transl.: *Teoria generale dei sistemi*, 1971, Istituto Librario Internazionale, Milano

Borasi R., 1996 *Reconceiving Mathematics Instruction: a Focus on Errors*, Ablex P.C., Norwood, NJ

Brousseau G, 1997, *Theory of Didactical Situations in Mathematics*, ed. and translated by N. Balacheff, M. Cooper, R. Shuterland, V. Warfield, Kluwer.

Chiari G., 1997, Gruppi e apprendimento cooperativo: un'alternativa ai gruppi di recupero, *Scuola Democratica*, n. 1, 24-34.

Cohen E. G., 1984, Talking and working toghether: Status, interation, and learning, in P.Peterson, L. C. Wilkinson, M. Hallinan (Eds.), *The social context of instruction: group organization and group processes*, New York, Academic Press, 171-188

Comoglio M., 1996, Apprendimento cooperativo e insegnamento reciproco: strategie per favorire apprendimento e interazione sociale, in Vianello R., Cornoldi C., *Metacognizione disturbi di apprendimento e handicap*, Ed. Junior, Bergamo, 77-105

Comoglio M., Cardoso M. A., 2000, *Insegnamento e apprendimento in gruppo: il cooperative learning*, LAS, Roma
Crawford K., 1997, Distributed cognition, thecnology and chance, *Proceedings PME 23*, Vol. 2, 137-144
Deutsch M., 1949, A theory of cooperation and competition, *Human Relations*, 2, 129-152
Dewey J., 1943, *The school and society*, Chicago, University of Chicago Press
Ernest P., 1995 The one and the Many, in Steffe Leslie P. – Gale Jerry (Eds.), *Constructivism in Education*, Hillsdale, New Jersey: LEA, 459–486.
Farina G., 2002, *Le isometrie con Cabri–Géomètre: un'esperienza di apprendimento cooperativo nella scuola secondaria superiore*, Tesi di laurea in Matematica, Università di Pavia, A.A. 2001/2002
Fattori A., 2001, *Il teorema di Pitagora nella scuola secondaria superiore: un'esperienza di apprendimento cooperativo*, Tesi di laurea in Matematica, Università di Pavia, A.A. 2000/2001
Garuti R., Chiappini G., Boero P., 1999, Bringing the voice of Plato in the classroom to detect and overcome conceptual mistakes, *Proceedings PME 23*, Vol. 3, 9-16.
Grugnetti L., Iaderosa R., Reggiani M, (Eds.), 1996, *Argomentare e dimostrare nella scuola media. XV Convegno nazionale dei Nuclei di Ricerca in Didattica della Matematica per la Scuola Media*, Salica Terme, 18-20 aprile 1996
Johnson D. W., Johnson R. T., 2002, An Overview of Cooperative Learning, www.clcrc.com/pages/overviewpaper.html
Johnson D. W., Johnson R. T., Holubec E. J., 1994, *The nuts and bolts of cooperative learning*, Interaction Book Company. Italian trans.: 1996, *Apprendimento cooperativo in classe*, Erickson, Trento
Johnson D. W., Johnson R. T., Stanne M. B., 2000, Cooperative learning methods: A Meta-Analysis, www.clcrc.com/pages/cl-methods.html
Lewin K., 1935, *A dynamic theory of personality*, McGraw-Hill, New York
Mariotti M. A., Bartolini Bussi M. G., 1998, From drawing to construction: teacher's mediation within the Cabri environment, *Proceedings PME 22*, Vol. 3, 247-254
Munari A., 2001, Lo sviluppo del pensiero sistemico e le sue influenze sulle scienze umane, notes from a course, iuss, University of Pavia
Nunez R. E., 1999, Embodied cognition as grounding for situatedness and context in Mathematics Education, *Educational Studies in Mathematics*, 39, 45-65
Pesci A., 2000, Promoting mathematical investigation in class: reflections on the role of the teachers on the basis of experiences conducted in the secondary school, A. Rogerson (Ed.), *Proceedings of the International Conference "Mathematics for Living"*, Amman, 234-239
Pesci A., 2002, *Lo sviluppo del pensiero proporzionale nella discussione di classe*, Pitagora, Bologna
Polanyi M., 1958, *Personal Knowledge*, The University of Chicago Press, Chicago. Italian trans.: 1990, *La conoscenza personale*, Rusconi
Sharan Y, Sharan S., 1992, *Expanding cooperative learning through investigation*, Teachers College Columbia University, New York. Italian trans.: 1998, *Gli alunni fanno ricerca, L'apprendimento in gruppi cooperativi*, Erickson, Trento
Tressoldi E., 1996, Apprendimento cooperativo e insegnamento reciproco: strategie per favorire un apprendimento attivo ed indipendente e l'educazione alle relazioni interpersonali, in Vianello R., Cornoldi C., *Metacognizione disturbi di apprendimento e handicap*, Ed. Junior, Bergamo, 108-115
Vianello L., 2002, La relazione tra intelligenze ed autonomia, *Grimed XI, Osservare, valutare, orientare gli alunni in difficoltà:"per non fare parti uguali tra disuguali"*, Castel San Pietro Terme, 1-2/3/2002
Zan R., 2000, A metacognitive intervention in Mathematics at University level, *International Journal of Mathematics Education in Science and Technology*, vol. 31, n. 1, 143-150.
Zan R., 2002, Verso una teoria per le difficoltà in matematica, *Seminario Nazionale di Ricerca in Didattica della Matematica*, Pisa, 31/1-2/2 2002