WHY AND HOW I USE COMPUTER PROGRAMS DURING THE LESSONS OF MATHEMATICS.

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New technologies – calculators and computers - are used in almost every Polish school. Some young people have computers in their homes. Internet cafes are open in many places. We can say that technology of information exists in the life of almost every young person. It is very attractive for them; it provides quick access to any information, quick contact with people from different places in the world, graphic and sound effects, nice effortless amusement and the possibility of performing "undo" – without being blamed for mistakes. Unfortunately, most students use the computer as a source of recreation and a tool of virtual social communication, sometimes as an intelligent typewriter and sporadically as a modern tool which can help learn school subjects. What can we do to encourage students to treat the computer as a tool helping them to learn? I think that we should make use of the fondness young people have for computers and use them during the lessons of different subjects.

I have been working as a teacher of mathematics for many years. Now my main place of work is university but I am also working as a teacher in secondary school. For about five years I have been using computer programs in those moments of teaching when it is helpful and methodologically valid.

I would like to start with some practical remarks about using computers in the teaching of mathematics:

1. A lesson of mathematics with the computer cannot be a lesson in which the computer is a "star". It is a tool which helps achieve a mathematical aim of the lesson.
2. Preparing and conducting a lesson with the computer is more difficult than a traditional lesson. The teacher has to make sure that mathematical aims of the lesson are achieved (especially doing deduction and drawing conclusions) and, at the same time, has to watch over the correct use of computers and computer programs.
3. The computer cannot substitute the teacher.

It isn't necessary to have complex computer programs if we want to use computers during the teaching of mathematics. It is enough if we have:

- a simple, user-friendly program for drawing graphs of functions (after typing formula of function) – e.g. I use *Graphic Calculus* when I teach the following topics:
  - Transformations of graphs.
  - Properties of quadratic function.
  - Graphs of polynomial functions.
  - Solving problems (text exercises) by graphic methods. - and many others.
- any spreadsheet - e.g. I use *Excel* when I teach the topics:
  - Solving quadratic equations with a parameter.
  - Geometric interpretation of system of two linear equation's.
  - Horner's algorithm.
  - Properties of sequences.
  - Limits of sequences. - and many others.
- a program for interactive learning and teaching geometry – I use *Cabri II* when I teach such topics as:
  - Properties of different geometric figures.
  - Geometric transformations.
  - Mutual positions of the two lines in the coordinate plane. - and many others.
- a program for visualizing intersections of solids by the planes – *unfortunately I haven't good program for these topics*.

I use computer programs during the lessons of mathematics in the following situations:

- when students have the opportunity to be "explorers of mathematics",
- when they are assigned to do a simulation of some scientific processes,
- when they are able to do their work faster and more precisely by the computer than with pencil and paper.

I would like to give some examples of the plans of the lessons in which computers are implemented.

Example 1.

Graphs of polynomial functions.

(Note: This lesson comes after the lesson on topic: **Polynomial equations** *and precedes the lesson on topic:* **Polynomials inequalities**. *Students can already find zeros of polynomials – solve the equation* $\mathbf{a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0 = 0}$. *In each lesson about polynomials we use the formula of polynomial like in previous sentence. I teach solving polynomial inequalities by finding zeros of polynomial, drafting a simplistic graph and reading the signs of polynomial values from this graph.)*

Age of students: 17 . Time of the lesson: 45 minutes. Aims:

- general:
  - to improve the ability of using *Graphic Calculus*,
  - to enlarge knowledge about polynomials,
  - to form a positive attitude to solving problems and inquiring,
  - to facilitate accurate, logical and critical thinking,
- detailed –after the lesson students can:
  - describe how the graph of polynomial function depends on its zeros,
  - describe how the degree of polynomial and the sign of coefficient a_n influence the polynomial values in infinity ($-\infty$ and $+\infty$),
  - draft a graph of polynomial knowing its zeros and sign of coefficient a_n.

Methods: discussion, brain storming session, work with computers.
Forms of work: plenary session and work in pairs (with computers).
Procedure of the lesson:

1. Introductory part: Students revise:

- the idea of "zero of polynomial function",
- the connection of the zero of any function with its graph,
- the rules of using *Graphic Calculus*, especially needed on the lesson option *Draw Graphs*.

2. Main part:

Students work with computers in pairs. Before working with computers students are given their task: they have to observe what the influence of degree of polynomial, multiple of zeros and sign of coefficient a_n on different attributes of its graph is.
They use *GC*: draw and watch graphs of polynomials:

- $y = (x-1)(x-2)(x-3)$ - *three single zeros, odd degree of polynomial,* $a_3>0$
- $y = -3 (x-1)(x-2)(x-3)$ - *three single zeros, odd degree of polynomial ,* $a_3<0$
- $y = (x-1)^2(x+3)$ – *one single zero, one double zero, odd degree of polynomial,* $a_3>0$
- $y = -2 (x-1)^2(x+3)$ – *one single zero, one double zero, odd degree of polynomial,* $a_3<0$
- $y = -2 (x+3)^3 (x-2)^2$ – *one triple zero, one double zero, odd degree of polynomial* $a_5<0$

(Note: The teacher or students can build other examples of polynomials. The form of these polynomials saves time needed to find zeros and help to observe the connection between zeros and graph.)

We hope that after watching the graphs students will conclude:

- The polynomial's graph is a curve which intersects the X-axis in points $(x_i, 0)$ if x_i is a zero of odd multiple , and reflects on the X-axis in points $(x_j, 0)$ if x_j is a zero of even multiple.
- If $a_n>0$ then polynomial values in infinity (when $x\to\infty$) are positive.
- If $a_n<0$ then polynomial values in infinity (when $x\to\infty$) are negative.

These conclusions will help students to draft the graphs of polynomials when solving polynomial inequalities.
Students should write down their conclusions in their exercise-books and try to draft the graphs of the following polynomials by themselves:

- $y = 2(x-1)(x-2)^2(x-3)$
- $y = -3 (x-1)^2(x+2)^3(x-3).$

3. Final part:

- Short recapitulation of the lesson.
- Getting the feedback from the students by asking questions: Is everything clear? What was the main difficulty?
- Setting the homework.

Example 2. **When are lines $y=a_1x+b_1$ and $y=a_2x+b_2$ perpendicular?**
Age of students: 15 . Time of the lesson: 45 minutes. Aims:

- general:
  - to improve the ability of using program *Cabri II*,
  - to form the skills of setting hypotheses and arguing,
  - to form a positive attitude to solving problems and inquiring,
  - to facilitate accurate, logical and critical thinking,
- detailed –after the lesson students can:
  - describe what is relationship between slopes of perpendicular lines,

- select formulas of perpendicular lines from the set of formulas of lines,
- calculate a slope of the line which is perpendicular to the line with a given formula.

Methods: discussion, brain storm, work with computers.
Forms of work: class work and work in pairs (with computers).
Procedure of the lesson:
1. Introductory part: The revision of the:

- known formulas of the line in coordinates plane,
- sense of coefficients *a* and *b* in the formula $y=ax+b$,
- possible mutual positions of two lines in the plane.
- rules of using *Cabri II*, especially needed in the lesson options: constructing lines, measuring angles etc.

2. Main Part:

- The teacher formulate the problem: **Is it possible to know if two lines are perpendicular when we know their formulas?**
- Students recall that there is a connection between the slopes of parallel lines, so maybe there is a connection between slopes of perpendicular lines.
- In the brain storm session students plan to solve the problem using options of the program *Cabri II*; one person can write down each step of the future work on the blackboard.
- In pairs students work with *Cabri II*, doing the following activities-instructions of the program:
    - Show axes,
    - Construct two lines,
    - Show equations of these lines,
    - Construct the point of intersection of the lines,
    - Mark the angle between lines with vertex in the point of their intersection,
    - Measure marked angle,
    - Move with one line until the angle between the lines will be the right angle and then write down slopes of two lines,
    - Repeat a few times those activities with different pairs of lines,
    - Try to find the relationship between the slopes of perpendicular lines in each event.
- The teacher helps students who need her help. **It is very important: each pair of students should find the relationship. It is their success in "discovering of mathematics".**
- Each pair of students reports to the class. They tell about their formulas of perpendicular lines and their conclusions.
- We hope that students will quickly formulate the conclusion:
  Lines $y=a_1x+b_1$ and $y=a_2x+b_2$ are perpendicular only when $a_1a_2=-1$.

(Note: It is only hypotesis. We have to prove it using the mathematical reasoning - maybe in the next lesson.)

3. Final part:

- Two exercises for consolidation:
    - there are several formulas of lines on the blackboard; students should to find among them formulas of perpendicular lines,
    - there are some formulas of lines on the blackboard; students should write formulas of an example of perpendicular lines to each given formula.
- Short recapitulation of the lesson.
- Getting the feedback from the students by asking questions: Is everything clear? What was the main difficulty?
- Setting the homework.

The end of the examples.

In both lessons the computer is used in a rather simple way. Undoubtedly the use of it is helpful. I was very interested in what my students' opinions about the lessons were, so I changed the form of homework after the lessons with computer.

This is an example of such homework:

Name: _____ Date: _____

Homework after the lesson on topic:
The circle circumscribed about the quadrilateral – using the program Cabri II.

Answer the following questions:

1. What was the lesson about?
2. What were you doing during the lesson? Describe your activities.
3. What do you remember from the lesson? What were the underlying mathematical principles?
4. What was good about the lesson?
5. What was bad about the lesson?
6. If you could, what would you change in the lesson?

(After each question there is the place for answer.)

Solve the following exercises:

Ex. 1. Can you circumscribe the circle about the quadrilateral with angles 100°, 120°, 50°, 90° ? Justify your answer.
Ex. 1. Can you circumscribe the circle about the quadrilateral with sides 3 cm, 3cm, 5 cm, 7 cm ? Justify your answer.

After solving the exercises answer the questions:

1. What was the most difficult for you in these exercises?
2. Why did you choose for solving the methods that you used?
3. Do you know any other methods for solving these exercises?
4. Could computer technology help you in these exercises? If so, how?

This kind of homework makes students think about the lesson after it has been completed, make them look for different methods in solving exercises and problems (without technology and using it), give them a chance to be active in planning the forms of their education.

Sample students' answers:

- "We had a chance to see if we were right in our assumptions."
- "I could see on the screen about which quadrilaterals we can circumscribe the circle. I could come to conclusion by myself."
- "Drawing on the screen is easier than in the exercise-book. I have a good visual memory, so distinct drawing and the fact, that I could do it myself, helped me to remember."

My teaching experience shows that using computers in the teaching process has various advantages: students are enabled to work more actively and creatively, their motivation to learn mathematics increases, which makes their success easier to achieve.