DRAWING BY EQUATIONS

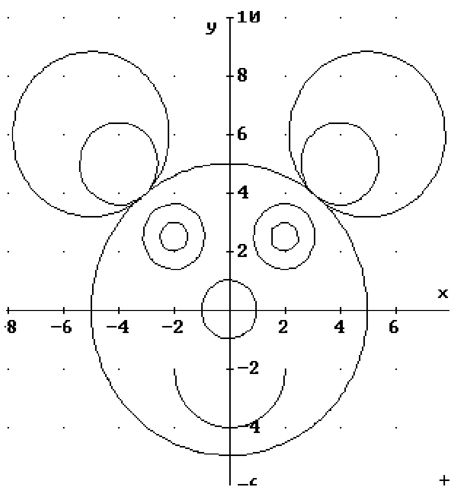
Valeria Facchini, Francesca Gialanella, Maria Talamo, Annalaura Trampetti

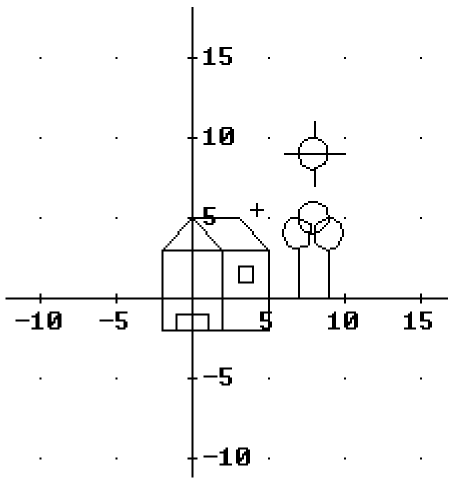
Nucleo di Ricerca Didattica
Dipartimento di Matematica e Applicazioni, Università "Federico II", Napoli, Italia

The research in Mathematical Education has often pointed out the opportunity of creating a context in which students can find a higher motivation in studying Mathematics.
In fact the teaching - learning process is enhanced by the involvement of students' creativity and interest: stimulating situations encourage them to use the concepts already learned in new contexts and to discover the need of new notions and the links existing between them.
Moreover the use of technology allows students to verify their conjectures immediately and to adjust them in order to attain the goal.
This paper is the report of a didactic approach to Analytic Geometry with the use of *DERIVE*, experimented in four High School classes.
The idea came from a student who challenged his teacher to draw a little bear on a TI-92 [2] and it was also developed as a classroom activity by a colleague, who used a TI-92 and parametric equations [1]. On the contrary, we experienced the use of Cartesian equations and led the activity in three different steps.

First step: *"seeing equations"*

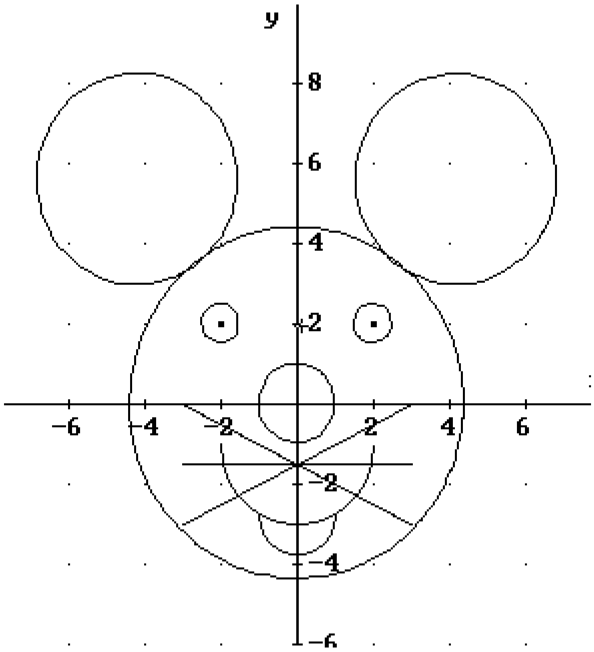
Students were required to use *Derive* - working in pairs - to draw Mickey Mouse and other pictures, giving the equations of their parts.





They already knew the equations of a circle and of some isometries, such as translations, reflections about x-axis, y-axis, and lines parallel to x and y-axes.
Using *Derive* they could plot the chosen equations and modify them in a short time, so as to have the expected result. This approach allowed them to understand easily the meaning of the terms of an equation.

The Big Mouse



x^2+y^2-1=0

x^2+y^2-4*ABS(x)-4*y+775/100=0

x^2+y^2-42/5*ABS(x)-56/5*y+42=0

x^2+y^2-1936/100=0

y=-1-SQRT(4-x^2)

y=-11/4-SQRT(1-x^2)

VECTOR([x,1/2*x-3/2],x,-3,3,0.05)

VECTOR([x,-1/2*x-3/2],x,-3,3,0.05)

VECTOR([x,y+3/2=0],x,-3,3,0.05)

[[-2,2],[2,2]]

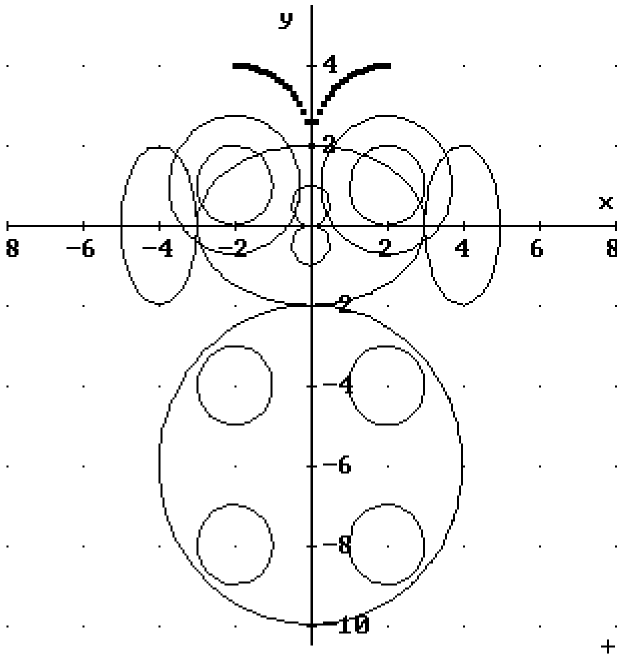
Students were stimulated to learn new notions, necessary to draw some parts of the picture (mouth, ears, …), such as the effect of the absolute value of x in the graph of an equation, or the way to represent line and conic segments or regions of a plane.
During this part of the activity the teacher answered students' questions, encouraged their attempts, suggested new ways to fulfil the task, somehow, she could adapt the teaching - learning process to the need of each student.

Second step: ***"… we all are artists"***

The results of the groups were compared and then, during a discussion in the classroom, the teacher asked to reduce the number of equations chosen to draw the pictures, by using the new concepts learned during the previous step.
Moreover she suggested to consider some points:

- an equation containing the absolute value of x or y can express some symmetries of the graph;
- an equation - although it can be "seen" - may represent no points, when it has no solutions;
- algebraic tools must be appreciated for their power and - at the same time - for their conciseness.

Baby Alien

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x^2/9+y^2/4=1            Face
x^2+(y+6)^2=16           Body
ABS(y)=x^2+y^2           Nose-Mouth
(ABS(x)-2)^2+(y-1)^2=3   Eyes
(ABS(x)-2)^2+(y-1)^2=1   Eyes
(ABS(x)-4)^2+y^2/4=1     Ears
VECTOR([x,2-SQRT(x-4)*SQRT(-x)],
x,0,2,0.1)               Left antenna
VECTOR([x,2-SQRT(-x-4)*SQRT(x)],x,
-2,0,0.1)                Right antenna
(ABS(x)-2)^2+(y+4)^2=1   Hands
(ABS(x)-2)^2+(y+8)^2=1   Foot
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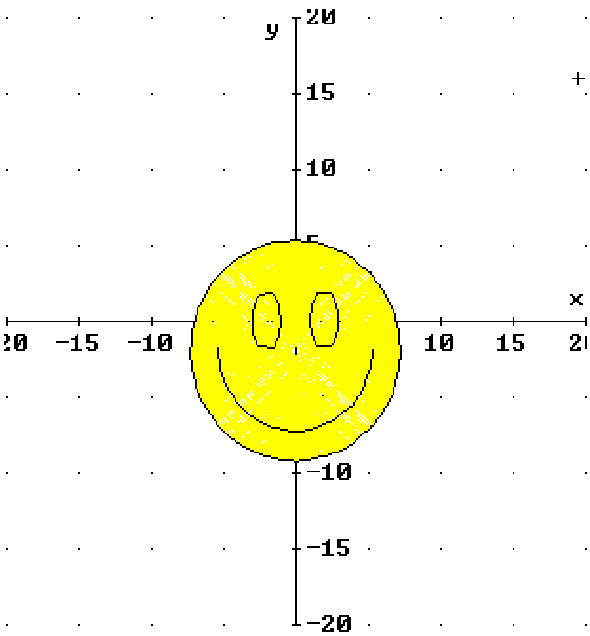
Some students, playing with the coefficients of the equations, discovered that a circle becomes an ellipse simply changing the coefficient of x^2 or y^2. It was the occasion to introduce in a "natural" way ellipses and some other geometric transformations such as strains about x- axis or y-axis.
Something similar happened with line segments, curve segments, families of curves, translated curves: a real need led to the introduction of new concepts.

Smile



1) $(x + 2)^2 + y^2/4 = 1$ first eye

2) $(x - 2)^2 + y^2/4 = 1$ second eye

3) $x^2 + y^2 + 4y - 49 = 0$ face

4) $y = - \mathrm{SQRT}(-x^2 - 4y + 24)$ SMILE

5) VECTOR([$x^2 + y^2 + 4y - k = 0$], k, - 4, 49, 0.1) Colour

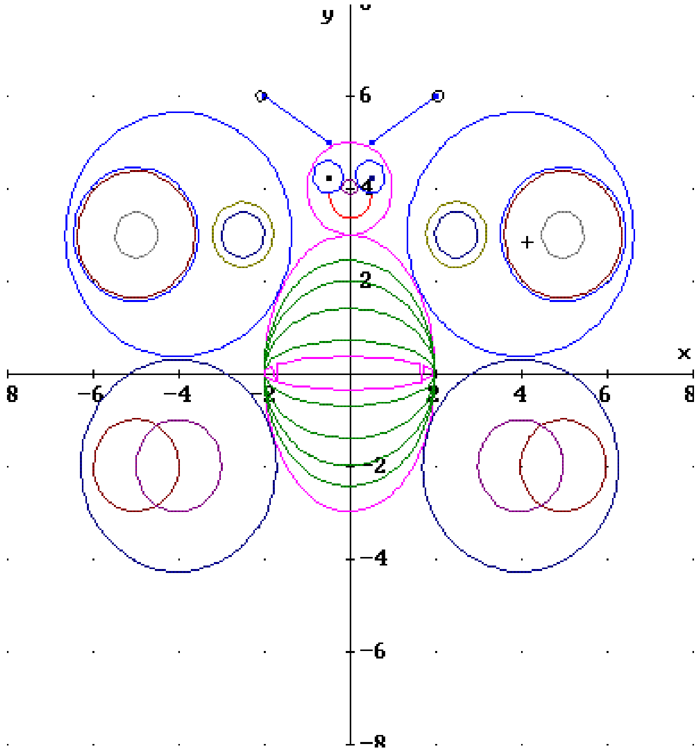
Definitely, with this approach, it was possible to deal with different topics at the same time, to show connections among them, to use some notions in contexts different from usual ones.

Third step: ***"… communicating by equations"***
As a final synthesis of the experience, student were required to write some instructions to make their classmates draw a pic ture they had never seen before. Not only had they to prepare the correct and clearly written sequence of instructions, but they had also to use geometric transformations to describe the drawing process so as to start from a little number of elements of the picture.

Therefore, students had to analyse their pictures, investigate on the geometric links among their parts, choose few basic elements of the graph, describe how the other parts of the picture could be obtained by means of geometric transformations and finally write the sequence of instructions for their classmates.
On the other hand, the other students had to find out the equations of the transformations and apply them correctly to get the unknown picture.
This step was indeed one of the most creative, quite a challenge, and some productions were very nice and funny.

Conclusion

Surely the main result of this didactic approach were, on one hand, the interest shown by the students during the activity and the fact that also those who usually found more difficulties in studying Maths produced good works: an example is the following butterfly.
On the other hand, as yet highlighted, it allowed the teacher to adapt her action to the characteristics of each student.



As a conclusion we like to report the comment of a student, really satisfied with his work, which is also a sentence pronounced by John Nash - si licet comparare parva cum magna - in the film 'A beautiful mind':

"Also Mathematics is a form of art"

References

[1] Cappuccio S. - "Matematica, gioco e tecnologia", *Atti del Convegno "Didattica della Matematica e rinnovamento curricolare"* (Castel S.Pietro, 2001), Pitagora Editrice.
[2] Impedovo M. - La matematica nella scuola di tutti: percorsi didattici e ipotesi di rinnovamento, *Atti del Congresso ADT 2000 (Montesilvano),* CD-Rom ADT, Ghisetti e Corvi Multimedia.