ARGUMENTATION AND PROVING IN MULTICULTURAL CLASSES: A LINGUISTIC-CULTURAL APPROACH WITH CHINESE STUDENTS

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The paper explore, through different theoretical/experimental investigations, a particular complex field of study as the analysis of the teaching/learning phases in multicultural setting in class. Through a linguistic-cultural approach it wants to be a further close examination on the didactic thematic related to a possible comparison between East and West and particularly between Chinese mathematical thought and the Italian one, in some particular aspects related to the phase of argumentation and proving. To carry out the research, we chose to investigated in a qualitative and quantitative way on the behaviour of Chinese and Italian students of different ages (13/15 years old) attending the Secondary School of Palermo, on a number theory problem as pre-algebraic problem ("the game of prime numbers").

1. THEORETICAL PERSPECTIVE AND PURPOSE

The research works published in the last years on argumentation and conjecturation are now manifold; some of these researches essentially contemplate a historical-epistemological analysis of the demonstration and therefore they describe its formative value for students; others instead treat the problematic through a critical discussion of the learning environments of the demonstration phase, they are therefore particularly focused on the delicate passage from the phase of the reasoning to the demonstration one. (Balacheff, 1988; Duval, 1991, 1992; Chazan, 1993; Malara & Gherpelli, 1994; Moore, 1994; Thurston, 1995; Boero, Garuti & Mariotti, 1996; Hanna, 1996; Harel & Sowder, 1996; Simon, 1996; DeVillers & Furinghetti, 1997; Lolli, 1997). Our study is inserted in this field of research as a further reflection on the treated problematic, taking also into account a possible didactic multicultural milieu: real context in the today's situation of school. In the scholastic year 2007/08 the pupils with not Italian citizen inserted in the national scholastic system corresponding to 574.133 unities. The phenomenon of the immigrations, in growth in our country, is continually reflected on the Italian school; the impact was so bigger that the affiliate student of foreign origin has seen to increase of over 500.000 unities in the last ten years. One of the most interesting didactic problems that today are defined in our classes is then the possibility to compare ourselves with the "diversability" in situations of multiculturalism, a reality in our society, in continuous socio-cultural change, and therefore central point for the research in didactics. To analyze the cognitive styles in different cultures, underlining those that can be the schemes of reasoning, the behaviours, the beliefs and the conceptions in respect to the acquisition of a particular concept, it is certainly a complex operation, but it can be

the key for a more careful didactics to the different abilities and therefore to the respect of the other. In the last years, the research didactics of mathematics appeared very sensitive to the treated problematic: different research studies allowed to put in evidence the role of the history of the mathematics as tool of observation and analysis of teaching/learning situations in a multicultural setting; the role of the language and the logic in the development of the discipline and the autonomous mathematical thought; the role of the socio-cultural context of the milieu in which the situation of multicultural teaching/learning situation is inserted and analyzed. In this sense, what we analyze when we discuss about of the binomials *mathematics-culture*, fundamental relation for this work and for a large tradition of research in didactic of mathematics, is not only the presentation of specific techniques trough witch a certain group of people treat the mathematical knowledge, but a critic discussion of possible correlations between the epistemological context of these people and the treated mathematical concepts that are elaborated and obtained in classes. This is the approach that we are following, even if in a first approximation, in our work. Our research is therefore aimed to explore a particularly complex field of study in which it is possible to find numerous "variable" in game, interconnected among them as for instance the *knowledge*, the *sociocultural context* in which the knowledge is inserted and on which the mediation happens, the *epistemology of the considered cultures*, the *schemes of reasoning* typical of the origin culture of the students etc. In the specific case of the considered didactic problematic, the paper specifically wants to be a further close examination on the didactic thematic related to a possible comparison between the Chinese mathematical thought[1] and the Italian one on the phases of argumentations and proving in a pre-algebraic mathematical context. According to us, the "variables" previously discussed, analyzed in two different cultures as that Chinese and that Italian, could result, in fact indeed interesting in many significant aspects (logical-argumentative, algorithmic, linguistic etc.). Aim of this article is so to augment, in an experimental way, if some of the cultural differences verified in the Italian and Chinese cultures (we refer to the historical-epistemological aspect of the two cultures) could tacitly influence the didactical sphere of the discipline, the teaching/learning phases, putting in evidence analogies and differences on the basic nature of the cognitive process utilized by the *cultural different students*, inserted in the same classes, on the mathematical (pre-algebraic) problem solving. The first research questions from which we started for the experimental work were so: *It is possible to find, in a experimental way, an equivalence between the analogies and differences verifiable in the Chinese and Italian historical cultural tradition and the patterns of reasoning found today in multicultural situations of teaching/learning of mathematics? The definition of these differences referred to the phases of argumentation and proving, can be significance for the mediation of particular*

[1] With the use of the term mathematical thought we refer to the studies of Nisbett et alii. (2001) and Spagnolo (2002) as regard specifically the study of the Mathematics Education in Different Cultural Tradition in to Comparative Study of East Asia and the West.

mathematical knowledge into the whole classes between Chinese and Italian students? As regard also the choice of the problematic situation: *It is possible to give a-cultural "definition" of argumentation and proving? A particular mathematical context as that the Number theory, as pre-algebraic mathematical context, can help us, in this sense, to answerer to this question?* These questions are not simple and they would deserve greater close examinations of historical-epistemological nature. In this context we will treat the problematic only in a first approximation, underling, trough the experimental phase, as what constitutes the way to proving varies from culture to culture, as well as from age to age (*to prove for convincing*, *to prove to verify*, *to prove to generalize* etc.). *Which can be the reasoning dynamics putted in evidence by the Chinese and Italian students in the phase of reasoning and conjecturating in a no-standard pre-algebraic problem solving?* Aware of the impossibility to treat in a exhaustive way the framework of the variables in game, particularly "complicated" for the Chinese student inserted in our Italian classes, we think, in function of the experimental data discussed below, that could be useful to present briefly, as an instrument of observation and analysis of multicultural learning/ teaching situations with foreing Chinese students, the book of the *Nine Chapters on Mathematical Art* (*Jiuzhang suanshu*) as canon for the construction of mathematics (1st Cent. B.C.-1st Cent. A.D.). After that in the following paragraph, we discuss, even if in a first approximation, the role of written natural language in the possible development of the mathematics thought for the Chinese students (Di Paola-Spagnolo 2008). The main references in this sense are the works of Chemla (2003, 2004, 2007), Needman (1981) as regard particular typical reasoning scheme for the Chinese students. Parallel consideration has to be taken into account as regard the tern Socrate/Platone/Aristotele for the occidental civility and Confucius/Tao/Buddha for the oriental one. Terns that, according to us, could influence the Italian and Chinese mathematical thought.

1.1 THE "NINE CHAPTERS ON MATHEMATICAL ART (JIUZHANG SUANSHU)"

The *Jiuzhang suanshu* or *Nine Chapters on the Mathematical Art* is a practical handbook of mathematics consisting of 246 everyday problems of engineering, surveying, trade, and taxation. It played a fundamental role in the development of mathematics in China, not dissimilar to the role of Euclid's *Elements* in the western mathematics. It is so principal reference for mathematics in Chinese education, a canon both for the construction of mathematics (1st Cent. B.C.-1st Cent. A.D.) and for the teaching/learning of the same in the various historic periods. Among the most notable is the commentary of Liu Hui (263 A.D.) presented in the collection of the Mathematical Canons of the Tang Dynasty (618-907 A.D.). The key concept that organizes the description of the *Jiuzhang Suanshu* is the concept of "class" or "category" (*lei*) that plays a fundamental role in the commentaries. The elements that we find relevant to understand the specificity of the book and so the related Chinese

culture in reference to the Italians one (as Aristotelian/Euclidean tradition) are so: *the typology of the problematic situation* putted on the book, the *modus operandi* (*shu*) described, the evident *algorithmic approach* to the mathematical problem solving and contextually the generalization of the different problems opportunely categorized and classified, the *calculus instruments*, the *way to demonstrate* the correctness of the problems (in the Chinese sense of term) and its *epistemological values*. The structure of the book is gradually articulated from the simple given of a problem (*wen*) related to a particular category, to the solution of this, "generalized" step by step trough an analogical reasoning, trough a *variable mutation* strictly connected to the proposed contest in a holistic vision (Nisbett, 2001). It's through a work on the procedure, on the resolute algorithmic that is possible to define the situation classes. The solution process is almost an abdutive process where deduction and induction are together in a unique reasoning scheme. The perfection is defined in terms of simplicity and generality trough a global vision of the problematic. So, w*hat is he meaning of argumentation and proving, therefore, on the Jiuzhang suanshu?* According to the Chinese thought of the *Jiuzhang suanshu*, to prove the validity of a mathematical reasoning, means to "demonstrate" the correctness of a procedure (using for example simple properties of arithmetic operations) with an algorithm approach in a possible combination of an iteration and of chosen "conditionals" and assignment of variable. As Nisbett declare "*The social worlds of East and West today reflect to a substantial degree their origin in Chinese and Greek culture, respectively…the social differences influence cognitive processes…we might expect to find cognitive differences among contemporary peoples that parallel those found in ancient times*". (Nisbett, et alli., 2001). Some of the differences that Nisbett putted in evidence and that we found, even if in a first approximation, in our Secondary school classes in which Chinese and Italian pupils study together, are specifically referred to: the algorithmic approach used by the Chinese students to argument and prove mathematical problems, a different verbalization from the Italian students in terms of generalization and relationship between the mathematical field and the object in the phase of argumentation; the organization into categories and the covering rules, instead of the organizing in terms of similarities and relationships (typical for the Chinese culture); the evidence that the "Westerners students" emphasize to the non-contradiction, whereas Easterners emphasize on the "Middle Way" value of choice.

1.2 OBSERVATIONS ON THE CHINESE WRITTEN LANGUAGE

The organization of the Chinese written languages is complex: the different written characters are defined and classified in various categories or "*meta-rules*" of composition that are interesting from mathematical pint of view because, according to us, the activities of reading and writing these type of characters vehicle a holistic mathematical thought, strongly algorithmic and relational-functional (en example is the contemporary relation "part-whole" on a written *sign*) that colud influence the way of learning mathematics in many aspect, in particular in Algebra and

transversally on the phase of augmenting and proving. Needham reports the classification in six classes and he discuss them in this way:

a) *Xiàngxíng* (象形) "Forms of imagines, pictograms": tree 木; sun 日; moon 月; mountain 山; horse 马; bird 鸟;
b) *Zhǐshì* (指事), "Indicators of situation, indirect symbols";
c) *Huìyì* 会意., "Union of ideas, composition by association or logic composition". 80% of the ideograms are of the associative kind (Needman, 1981). They represent a sort of *mental equations* (Needham, 1981, pp. 35-36, vol. I) as semantic combinations of two or more characters that are composed by association. We could find different examples for this: 男 [*nán*] man= 田 [*tián*] "field" + 力 [*lì*] "strength; 林 [*lín*] (森林 *sēnlín*) forest = *tree* 木 + *tree* 木 (plus 木). Two 木 [*mù*] trees side by side; 休 [*xiū*] stop, rest = 亻(人 *rén*) + 木 [*mù*] tree. A person stopping to rest under a tree.
d) *Zhuǎnzhù* 转注, "Transferable sense, symbols that is possible to interpret reciprocally".
e) *Jiǎjiè* 假借, "Language or sound". These characters are defined in a determinate general manner: the radical is associated to a phonetic sign to indicate the category on which we have to find the meaning of the word. So a lot of words with the same sound are written without confusion. (Needham, 1981, p. 38). Example: 园 [*yuán*] garden = 口 (*wéi*) "surround", suggesting a garden fence, and (full form) 袁 *yuán* phonetic or (simple form:) 元 *yuán* phonetic; 袁 [*yuán*] or (simple form:) 元 [*yuán*] phonetic, and 辶(辵 *chuò*) "go" (to go far) = "far"
f) *Xíngshēng* 形声, "Loan, rented phonetics character". The formation is much similar to the precedent case but the way to construct the character is different

Other two transversal interesting observation: are referred to:

I) the use of the contradiction in a contemporary of the opposites (From the point of view of Fuzzy Logic (Kosko, 1995) inside the language: 杯 [*bēi*] cup: from 木 [*mù*] "wood" and 不 [*bù*] "not" phonetic. From the association of these two characters and so from the idea of "opposition/contrast" born the concept of cup. Everybody knows that cups are 不 (not) made of 木 (wood).
II) the idea of a variable *as thing that varying* (Radford, 2000, 2003) and a parametric system inside the composition of many characters. Some simple examples could be referred to the use of the radicals [*kǒu*] (口) or [*tián*] (田).

These characteristics of the written language seem to put in evidence from a mathematical point of view, an internal research to a use of a *common strategy* to define "different characters" in which the radicals part assume a role of a parameter (in a mathematical sense of the terms) and vehicle the meaning or the sound of the character. The Algebra or pre-Algebra therefore is a possible carrying element for the construction of the characters of ideograms type for association. It seems to us a sort of research of a possible fundamental algorithmic to construct different "words" in a sort of generalization of the procedure with the aim to think "classes of signs" more

and more ample and sophisticated, proceeding for "*analogy*" and working for concepts of sound in a possible construction of a *algorithmic maps*. As also many neurosciences research confirm, the Chinese student learn to read and to write these *signs*, in a continuous parallelism, in a continuous relationship, between "serial thought" and "global thought" on each single sign, therefore, in a *relational-functional* aspect of the concept of variability (Radford, 2004), from the first years of their age[2].

2. METODOLOGY

Keeping into account the analyses reported before, referred to some key epistemological elements that, according to us, are really interesting for the study of the phase of argumentation and proving of the Chinese and Italian students of different ages on the resolution of particular mathematical pre-algebraic problems, we propose, in the paper, the description of the experimentation that we conducted with some Chinese and Italian students of the Lower and Upper Secondary school classes of Palermo. The descriptions of the results put into evidence some features that allow us to recognize some key aspect of argumentation conjecturation and proving typical of the different culture of the student involved, bringing back to some epistemological value of the own culture (philosophy, language, logical argumentative schemes). In the article, will be stressed the attention particularly on the Chinese student and the analogies and differences that we found in their reasoning schemes in relation to the Italian ones.

To carry out this research we chose to investigate on a pre-algebraic problem formalized as a Number Theory problems called "*the game of prime numbers*". The choice was made for two reasons: firstly, because, according to us, these kind of problems could represents nodal points from which the students derive the conceptions of the letters as unknowns or "*things which are varying*" (Malisani 2005, Radford 2004), interesting aspect, as we describe before, not only from the mathematical point of view (idea of generalities and truthiness of an assertion) but also strictly linked to the written Chinese language and so to its internal structure; secondly, this kind of problems, as no typical mathematical problems, could give us the possibility to analyze the free phase of argumentation, conjecuration and phase of proving from different viewpoints: *reasoning schemes*, *strategy of solution* and *possible generalization*, *justification of the strategies* etc. These are the *behaviours variable* that we analysed in a qualitative and quantitative way.

The given mathematical problem has a historical background, namely the determination of all the prime numbers which are sums of two squares. The first formulation of the theorem was expressed by Pierre de Fermat (1601-1665) into a letter to Merin Mersenne (1588-1648). The problematic situation presented in class

[2] Same consideration can be made for Chinese student that live and study in China and in for the Chinese pupils that live and study in Italy. In China they study their own written language at school (first ages of Primary school), in Italy this aspect is referred to the family.

arose in an informal manner during a lesson about the Pythagorean theorem and its converse. The simple observation of some examples of triples as $5^2 = 3^2 + 4^2$ or $13^2 = 5^2 + 12^2$ got the teacher to ask if a same relation can hold for prime numbers too, namely, when *a prime number can be written as a sum of two squares*? For example, 5 and 13 are sums of two squares, since $5 = 1^2 + 2^2$, $13 = 2^2 + 3^2$, while 3 and 19 cannot be written as a sum of two squares, since, for example, one can check, for 19, that none of the differences $19 – 1^2 = 18$, $19 – 2^2 = 15$, $19 – 3^2 = 10$, or $19 – 4^2 = 3$ is a square. This question, apparently simple, greatly interested the pupils, therefore the teacher posed a general question ***to find the generic form of all the prime numbers which can be written as a sum of two squares***. The proposed situation problem has, according to us, three interesting characteristics: it uses the concrete experience field of arithmetic, semantic and syntactic register that is well known by all the Secondary school students and so significant in term procedure of solution and justification; it is an no-standard open problem, and finally, it has, in its linguistic formulation, the possibility "to favour" on the students the production of argumentation and conjecturation in natural language or in other different semiotic register chosen in a free way by the students with the unique aim to find a "general rule". The strategies adopted by the students to solve this problem could be different.

The experimentation was carried out at three different public Upper Secondary Schools and in an Lower secondary school (student's age13/15) in Palermo. The classes involved were the third classes of the Lower Secondary schools and the first year of the Upper Secondary school. Everyone was able to understand the language by which the text of the problem was expressed[3]. The experimentation was conducted in groups (Italian and Chinese) after a first phase of autonomous work of conjecturation. So, the students after a brief moment of autonomous conjecturation and discussion in small groups, shared their approached strategies to the problems, discussing with all the student of the class. The procedure was choice to favour, at final step of the experimentation, the mediation of knowledge and the validation of the procedure of solution of the mathematical problem/game. The role of the teacher was as a mediator and coordination. As regards the methodology of investigation, the questionnaire reported the text of the problem/game was distributed at the same time to all the classes with a table of the first 500 prime numbers and a table of the first 500 squares. The time availed to the students was 120 minutes. The students were free to use all instruments that they need (paper and pencil, calculators, computers etc.). The methodological framework is relative to the Theory of the Didactic Situations of G. Brousseau (1998). As regards the adopted statistic methods, we make reference to the works of Spagnolo et alii (2008) on the implicative and factorial analysis. In order to quantify student's answers to the question, the research was carried out by using an a-priori analysis about the possible behaviours of pupils.

[3] The text of the problem was express in Italian. All the Chinese students involved in the experimentation were able to understand Italian language and to read and write it in a correct way. All the Chinese students involved in the experimental phase frequented the Italian scholastic system form the first year of school.

4. ANALYSES OF THE RESULTS AND FIRST CONCLUSION

Through the use of the software CHIC (Classification Hiérarchique Implicative et Cohésive), the quantitative analysis of the collected data of the proposed mathematical situation, was been read not only in relationship to the relevant historical epistemological aspect underlined previously in the paper, but also, in a more ampler way, connecting other relevant works realized by the GRIM of Palermo in other different mathematical contexts and conducted in multicultural milieu with Chinese and Italians student of different ages presented in our classes in Palermo.

The *game of the prime number* confirms in fact, even though in a first approximation, the results previously discussed in other our research works (Di Paola-Spagnolo 2008, Spagnolo, 2002). The comparison of the strategy putted in evidence for the solution and the justification of the situation/game by the Italian students and Chinese ones seem to show that the students present a different "kind of logic" in the following items: *problem reading data*, *data organization*, *"type" of language* used to describe the solution and hence *different schemes of reasoning in argumentation and conjecturation of the solution*. In particular, the collected data relative to the Chinese students, confirm a pragmatic behaviour, already highlighted in the works of Chemla (2001), behaviours strictly bound to procedural thinking, to algorithm through which students use in each single numerical case as a representative case of all the possible series, series connected through the construction of a possible fundamental resolute algorithm. This can represent a typical reasoning of construction adopted, according to us, in the written Chinese language and presented also in the history of the Chinese mathematics, as we sow before. For the Chinese students involved in the research, to prove the validity of the local arithmetic reasoning with the aim to generalize the procedure and find a generic form of all the prime numbers which can be written as a sum of two squares, means to express verbally step by step the procedure to construct these kind of numbers.

The Italian students after few random attempts and numerical error try to arrive instead directly at a possible formalization of the treated problematic but they point out some difficulties to work in a formalized setting and so they always re-consider again, as a help to their reasoning, the arithmetic strategies with a lot of attempts. The way to argument and prove some initial conjectuation on the problem/game is so a continuance balance between arithmetic and algebraic thought. In this sense, particularly, in the protocols they show a continuous re-consideration of some sequence of large prime numbers as single cases of their reasoning. Formalizing equals for them to consider that a property has to be true when they consider very large natural numbers. As regard this behaviour we could recognize two different interpretations. First, in order to prove the truth of the assertion concerning the posed problem, a student utilizes an "induction process" sui generis: i.e. he considers as initial quantities the examples presented in the text of the problem and he thinks that the decomposition of a prime number into the difference between two squares could

be true for all the prime numbers if he is able to verify it for some large numbers. Second, a student goes for abduction (according to Peirce) because he believes that verifying the truth of the assertion for a little set of large prime numbers the truth follows for the whole set of prime numbers. Regarding this aspect, the Chinese students putted in evidence, as first step, a holistic way to read and therefore to interpret the table of prime number gave to them. They sketched on it same particular single case and they tried to formalize a general form of prime numbers which can be written as a sum of two squares. General form never expressed in terms of formalized algebraic language (that it is instead possible to see on the Italian protocols). They showed initially attention to the particularity of each single case as a variable case in relation to the other possible case, reading the table in a unitary vision. We didn't find the same procedure with Italian students that try to formalize directly the general case of this type of number. The structure of the adopted procedure seem to put in evidence as gradually, from a first simple case given in the text of the problem, the Chinese students try to solve the question "generalizing" step by step, trough an analogical reasoning, trough a *variable mutation*, the proposed case and defining hence a "general" solution strictly connected to the proposed contest of the problem, in a holistic vision (Nisbett, 2001). The solution process is almost an abdutive process. According to us the reasoning putted in evidence by the Chinese students on the research of the common construction of these prime numbers that could be written as a sum of two squares could be an interesting example of the meta-rule "*Making homogeneous and making equal*" (Chemla, 2003) that is a fundamental rule in the Chinese Mathematic and Philosophy. The adopted algorithm once again is an operation which "makes equal" and "makes homogeneous" (in the reduction to unity). This fundamental algorithm is, during the process of argumentation, combine itself several times always arriving at a sure argument. As Liu Hui observes, in this joke of relationships between "serial thought" and "global thought" in reading and understanding a problem, particular attention must is given to the examination of the algorithm on the classes of possible homogeneous cases to be able to highlight its correctness. The mediation of the different reasoning schemes adopted by the Chinese and Italian students were discussed with the all classes in terms of possible approach to the problem. This final step of our experimental work was able to integrate in classes the pragmatic behaviour of the Chinese student with the deductive logic and the research of the generalization utilised by almost the all the Italian students. The student in a autonomous view observe critically the passage between the possibility to prove for convincing, to prove to verify (typical Chinese strategy), to prove to find the generalization (typical Italian strategy) etc. The showed results could be, according to us, very interesting as regards the didactic implications for the multicultural classes that present foreign students of Chinese ethnic.

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