

Introduction

This Special Issue was born within the workshop “E-LEARNING E MATEMATICA NELLA FORMAZIONE UNIVERSITARIA E POST-UNIVERSITARIA: DA BUONE PRATICHE A LINEE DI RICERCA”, aimed at comparing and discussing on-going research studies and existing good practices in Italian universities on e-learning in mathematics teaching and to develop collaboratively significant lines of research in the field of mathematics teaching and learning.

The seminar arose from the belief that the spread of technology in our everyday life inevitably also affects the world of education. The use of technology in teaching and learning contexts can be an opportunity to intercept the interest of students, to follow their natural tendency to use digital resources, also for educational purposes. The technological scenario requires to investigate how educational institutions can take into account how to effectively organize students’ learning environment and learning paths, integrating classical and digital resources.

In the months after the workshop, due to the current pandemic, the school had to deal with the problem of distance teaching and learning. Teachers suddenly found themselves in a new unfamiliar teaching setting. The implementation of a simple ‘digitisation’ of face-to-face teaching has been demonstrated to be unsuccessful. There is an emerging need for a ‘digital culture’, which allows us to understand how face-to-face teaching can be reorganised to integrate and exploit the potential offered by the digital environment, bearing in mind that learning processes depend closely on the tools and methods through which they are mediated by the teacher. Since the workshop took place before the pandemic, the scientific reflection of the participants was not affected by an emergency. For this reason, the research presented in this special issue can provide effective tools for distance teaching and learning, which is still implemented in many University courses and at many school levels.

As natural spin-off of the workshop, the participants to the seminar have given rise to a working group of the Unione Matematica Italiana, called DIGiMATH. The working hypothesis of DIGiMATH is that specific technological resources and digital environments alongside face-to-face teaching can help to solve the problems identified at the various levels, provided that the integration of technology into teaching interventions is carefully designed in relation to the educational objectives.

The positive mood of the meeting fostered discussion and collaboration among the participants, allowing for broad discussion and fruitful elaboration of ideas. Some of the reflections we have made have been incorporated into the papers that we present in this Special Issue. We think that they are an important starting point for a further elaboration of proposals for practice and theoretical investigations, which will broadly involve all the community of educators and teachers in the near future.

Papers presented as plenary talks allow us to focus on some key aspects of the use of e-learning in mathematics education.

Albano and **Sabena** propose a theoretical reflection on the design of activities in e-learning environment. Starting from a case study concerning online formative assessment workshops with University students, they try to grasp the complexity of the didactical system, with particular reference to the role played by technology. To this aim, the authors uses the so called “e-learning tetrahedron”, which allows to highlight the actors within the didactical system and the relations occurred among them. Moreover, the tetrahedron model sheds light on the dynamicity of the actors, intended as roles that can be played by any agent involved in the teaching/learning process.

In his paper, **Ferrari** aims to show how the use of a platform in a university learning context can promote linguistic and content-related competences in the students. The author seeks to argue how the development of competence, specifically linguistic competences, can be a step forward in improving the quality of teaching, compared to content-based teaching alone. For reaching this purpose and to stress the importance of designing courses based on the development of competences, some example of useful items have been provided.

Taranto, Arzarello and **Robutti** examine three significant moments related to an online training experience for mathematics teachers within the Math MOOC UniTo project: development of a teacher learning model in MOOC following the analysis of the various experiences designed and delivered within the project; the role of researchers involved in the MOOC experience; how the online training received by mathematics teachers impacts on their students in the classroom. The authors also discuss the relevance of their research in the mathematics education context outlining an open question about a possible new picture of the didactical framework.

Three main strands can be identified in the submitted contributions.

1) *Teaching-learning mathematics at University level*

The paper of **Lepellere, Zucconi, Al Asbahi** and **Carminati** is focused on the design of two specific interactive tools regarding matrix representation of geometric transformations and eigenvectors, realized with the aim of improving students' sense-making in dynamic geometry environments. The theory of semiotic mediation is indicated as the theoretical framework of reference to foster the development of mathematical thinking. A first integration of the tools in a course of Linear Algebra is then presented. The activity sees students engaged in mathematical discussions aimed at exploring conjectures and arguments of their own thought.

Pierrri's paper addresses students' difficulties in secondary-tertiary transition. The author investigates how peer to peer online activities, in a context of formative assessment, can be an effective educational strategy to overcome these difficulties and to support the developing of (advanced) mathematical thinking both at cognitive and at metacognitive level. The results regarding activities aimed to enhance formative assessment strategies and support the development of continuous reflections on students' learning processes with first year Bachelor students are discussed.

Telloni's paper describes the design of two individualized routes of digital tasks for university students: the first one is a linear sequence of tasks on the coordination of multiple representations of subsets of the plane; the second one is a tree of tasks on elementary and conditional probability. The paths foresee the students' reconstruction of the interaction within the digital environment, their reflections on the difficulties encountered, on the role of feedback and hints. The author discusses the results of pilot studies, carried out with voluntary engineering students, focused on students' perception of task paths and their impact on learning.

2) *Transition from high school to University*

Di Gennaro's paper aims to investigate how the use of e-learning platforms in a University education context can improve students' learning by promoting metacognitive processes and critical thinking development. Therefore, peer review activities have been designed within a remedial course of Algebra and Geometry for engineering students, by using the *workshop* resource of Moodle e-learning platform. The first results of the experimentation conducted are quite positive both from a cognitive and emotional-affective point of view.

In his paper, **Rizzo** analyses the correlation between attendance at the 2018 online remedial course for students with Additional Learning Obligations (OFA) of the University of Milan Natural Science degree

programme and their formative success, defined as passing the exam in the shortest possible time. The author observes a significant correlation between passing the test at the end of the remedial course and passing the exam for the freshmen of the 2018-2019 academic year. The author concludes by highlighting the need for a re-elaboration of the data with other students to confirm or refute the results obtained.

3) *Teachers professional development*

In **Dello Iacono**'s paper, the effectiveness of the peer review methodology in online environment is investigated. The author shows how an e-learning platform can be used to implement the peer review methodology in order to develop teacher design capabilities both for pre-service mathematics teachers and for in-service ones. The data discussed by the author shows that peer review methodology seems to be an effective strategy that allows in-service and pre-service teachers to change their praxeologies and to improve their role as educational designers.

Montone, and her colleagues **Romita**, **Fiorentino** and **Pertichino**, deal with a training course for high school mathematics teachers carried out by means of the design and the analysis of teaching activities in digital platforms both in training phase and in the teaching practice. They investigate how the teachers relate with digital resources and how they grasp their potential to re-design the mathematics curriculum. They also discuss about how the analysis of interactions among students and teachers in a digital environment can help to define a teacher training activity useful for the re-design of the classroom practice.

The proposal to publish this volume as a special issue of *the Quaderni di Ricerca in Didattica* was immediately accepted with enthusiasm and great interest by the entire Editorial Committee of the journal. The topics addressed during the Conference organized by the University of Salerno and the papers that have been discussed and now published in this volume are interesting for many researchers in Mathematics Education and Mathematics teachers who read our journal.

Today, more than in the past, the world of Research and School is going through a crucial changing moment that forces Researchers, Experts and Teachers from all over the world to a shared reflection about the new scenarios that we are living in Education but not only. We are sure that the inputs offered, in a competent and exhaustive way, by the authors of the different papers included in this volume will be a point of reference for the literature in Mathematics Education and, in particular, for future investigations on E-Learning and teaching/learning Mathematics.

The Editors:

Giovannina Albano (University of Salerno)

Ferdinando Arzarello (University of Torino)

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Anna Pierri (University of Salerno)

Cristina Sabena (University of Torino)

