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# ANALYSIS OF THE DIDACTICAL CONTRACTS IN 10th GRADE MATH CLASSES

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## Abstract

This paper presents and describes two didactical contracts that characterize the identified typical interactions in two math classes of tenth grade, during the study of the polynomial factorization. *We introduce a new kind of contract, different from those already considered: the "potential adidactical contract ". It seems necessary, because it describes others practises observed in classrooms, intended to promote better mathematical performances in the students.*

**Keywords:** didactical contracts, math classes, didactic of mathematics

## Introduction.

During the year 2006, as part of a field study of a doctoral thesis, one of the authors of the article made a series of observations during five months, in four math classes of tenth grade. This article emphasizes the study of two kind of didactical contracts (ostension and mayeutic socratic contract) identified in the classes of the participant teachers (named Sam and Ron, each of them in charge of two classes), which characterize the typical teaching practices in Costa Rica.

We describe and analyze the interactions that take place during the teaching of the topic of polynomial factorization through common factor and cross multiplication method, principally under the lens of the established didactical contracts. Then, we describe a kind of contracts that we've denominated "adidactical", due to the nature of skills they encourage, inferred from certain types of exchanges absent in previous contracts. Although the observations did not evidence the establishment of this contract, we consider it is possible that the class interactions evolve to the kind of exchanges the didactical contracts describe. Finally, it refers to some of the conclusions of the performed analysis.

## 1. Reference Notion

The didactical contract is described by Brousseau (1998) as the set of the rules that determines what the students and the teacher "have the responsibility to carry on, and what each one is responsible in some way"[1]. (Brousseau, 1998, p. 61). Extending this definition, Sadovsky (2005) describes this contract as a keen game in which the teacher communicates "sometimes explicit and many other times implicitly, through words and also through gestures, attitudes and silences, aspects related to the functioning of the mathematical affair that is treated in the class"[2]

[1] The translations in the paper are free, performed by the authors.

[2] According to the author, this set of obligations [reciprocal] is not exactly a contract. It can't be explicit. This certainly that pupils would find the way to solve a problem is a fallacy. A contract of this kind is sentence to fail.

(p. 37). So that during this process, “meanings are negotiated, mutual expectatives are transmitted, methods of performing are suggested or infered, mathematical norms are communicated or interpretated (in an explicit or implicit way)” (p. 38).
During the *VIII Ecole d’été de Didactique des Mathématique* in 1995 in France, Brousseau presented in the course 2: *Les stratégies de l’enseignant et les phénomènes typiques de l’activité didactique*[3], a typology of possible contracts in the class. According to the author, such contracts

concern, first, the emission of knowledge – communication, validity, novelty, value, interest or cultural status – and the conditions in which these could manifest, be received, learned, reproduced, etc. (Brousseau, 1996, p. 17).

Among the contracts presented by the author, the *ostension* contract and the “*mayeutic socratic*” contract are of particular interest for the performed analysis, since characterize the teaching practice in traditional classes. From these discussions about the indicators of these contracts, their implications in the mathematical formation of the students, and supported by the contributions of the Theory of Didactic Situations (Brousseau, 1986) and the recently notion called “courses of study and research[4]” (Chevallard, 2005; Barquero et al., 2007) we’ve conceived a possible evolution of the teaching practices towards what we call practices characterized by a potential adidactical contract.

In the following section we present and describe each type of contract. Also we illustrate the first two of them by extracts from the tenth grade classes observed[5].

## 2. Analysis of teaching practices since the didactical contracts

The current practices that we observe in math classes in Costa Rica are most of them characterized by the present of a traditional didactical situation. These traditional situations are describes by the following facts:

- Teacher gives an oral exposition about the studied topic
- Teacher indicates to the students what exactly they have to write
- Students listen to him and take notes
- Students try to understand it by doing exercises
- Students need to be strongly supported in their learning

These practices provoke in the students a passive role that generates different limitations in their performance, so it’s necessary an evolution of these practices to get that the students accept and assume the responsibility to participate on the process of teaching and learning math. This means, to propose them from potential adidactical situation to effective adidactical situation where:

- Teacher proposes a problematic-situation which allows the students to construct the new knowledge as the optimal solution

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[3] “Teachers strategies and typical phenomena of didactical activities”.

[4] “The courses of study and research arise from the study of problematic issues whose resolution requires the construction of a [...] set of theoretical and practical knowledge associated and interconnected among them. This way, in the REI, the generating question appears as a possible way to experience the problem-solving activity in the classroom" (Barquero et al., 2007, p. 485).

[5] As we have indicated before the third type of contract is not evidence but we considere that is possible to control the teaching practices toward the specific type of dynamic which it describes.

- The responsibility "to get in" the knowledge to the class is not exclusive of the teacher
- Students formulate and validate strategies to solve a problem

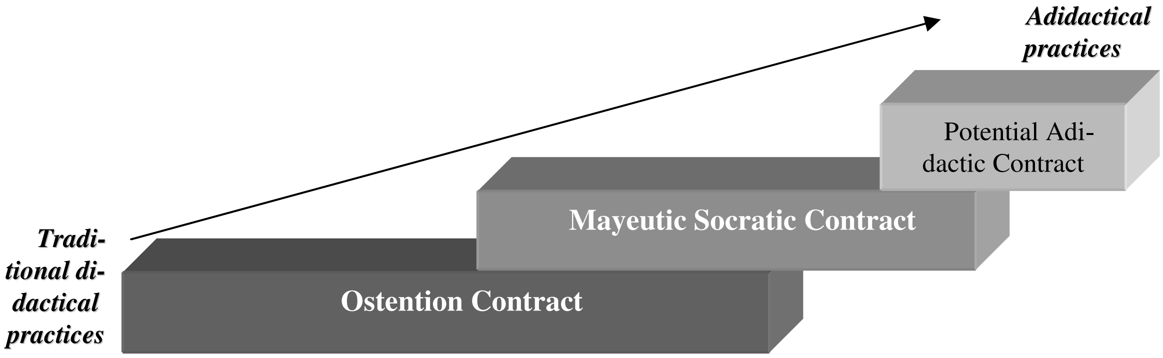



**Figure 1: Types of didactic contracts**

Obviously we recognize that practices don't show a unique type of contract, but a mixture of clauses from some of them; even though in some cases, it is possible to recognize the prevalence of a specific contract. However, to analyze these practices it is necessary to consider each contract separately one from the other. In this sense, we define for each contract indices that let us identify its presence in math practices.

### 2.1 Ostension Contract

During the practices where teacher shows an object (property, technique or example, etc.) and the students accept to see it "as a representative of a type, in which, they must recognize their elements in other circumstances" (Brousseau, 1996, p. 46) or objects; the exchanges between the actors are ruled by the **ostension contract**[6]. As the author indicates, the contract permits the teacher to communicate knowledge, avoiding the situations of action and formulation[7].

In general, in this kind of dynamic the teacher is always the responsible one for knowledge. Let's see an example taken from the first class about the cross-multiplication method to factorize polynomials. At the beginning of the lesson, Sam[8] announces the new topic. Then, he writes on the board the polynomial $x^2 + 5x + 6$, and asks if it is possible to factorize using one of the

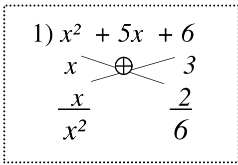
[6] "Ostension" refers to the act of the teacher who "show" an object to his pupils into a didactical intention, within the illusion that that act is enough for the pupils to see the same object that the teacher is seeing.

[7] "The sequence of the "situations of actions" constitutes the process by which the student builds strategies, that is, "teaches himself" a resolution method of his problem" (Brousseau 1998, p. 33). The formulation situations, "allow to progressively set a language that everyone understand and that considers the objects and the pertinent relations of the situation in an adequate manner (that is, allowing the useful reasoning and actions)" (Idem, p. 36).

[8] One of the two teachers observed.

methods[9] studied up to now. Since it is not possible to use any of those, he indicates that the cross-multiplication method must be used.

| | |
|---|---|
| PROF=Sam | […] let use the cross-multiplication method. Consists as follows. Will seek two expressions that multiplied result in this $x^2$, two numbers or two expressions, right? And two that multiplied result in a 6. |
| S=F | $x \bullet x$. |
| PROF=Sam | Well, here we can put $x \bullet x$. Also, we could have written $x^2 \bullet 1$ right, result in $x^2$. Well…so you have already decided to write $x \bullet x$. Then write two that results in, 6. |
| Ss | $3 \bullet 2$ |
| PROF=Sam | So I have to think about all factors of 6. $6 \times 1$, $1 \times 6$, $3 \times 2$, $2 \times 3$. Now, which of those are helpful to me, in what should I be based on to choose that pair. Then, the numbers I write here, I have to combine them with those I wrote here, or expressions, in such a way that when I cross or multiply them I get as a result that 5 $x$, or the one in the center let's say. I cross multiply and add it; so let say that if I write here, $6 \bullet 1$. $6 \bullet 1$, 6. Right, it is correct. Now, $x \bullet 1$, how much will it result in? $x$ |
| Ss | $X$ |




Notice that Sam's affirmation: "Well, here we can put $x \bullet x$. Also, we could have written $x^2 \bullet 1$ right, result in $x^2$. Well…so you have already decided to write $x \bullet x$." illustrates another clause of the contract identified. It is not a students' decision, since they lack of the necessary criteria to make a proposal, according to the demands required by the technique. However, the teacher not only validates it given that it is the factorization needed to illustrate the technique, but also with the purpose to make evident that it is the result of a group work between the teacher and the students.

A strong indicator of this kind of contract is that the teacher is the main speaker:

- he announces the topic
- he presents a task that needs the kind of new objects for students
- he performs the task showing how to apply the new objects in order to get the task

From these responsibilities we deduce that the students keep a "passive participation", must recognize what the teacher presents and apply it to other tasks without making it explicit as the used technique.

The ostension contract in Sam's lessons –as well in Ron's lessons– seems to be articulated with a primitive "*mayeutic socratic*" contract that will be covered in the next section.

### 2.2 Mayeutic Socratic Contract

The interactions between teachers and students that are ruled by this contract consist basically in a dynamic of questions and answers among the actors of the didactic system. We had observed, in several occasions during this dynamic, that the teacher modifies his questions, taking any rhetoric way (analogy, metaphor, etc.), to obtain the answer he expects (Brousseau, 1996).

[9] In other words, factorisation using the common factor, by grouping "two and two", or using any "notable product": difference of two squares, perfect squares, etc.

The most characterized style of the *mayeutic* from Sam, consists in enunciating certain proposition, finishing his intervention with one of these expressions: "yes?", "no?", "yes or no?" In this way, the role to affirm or deny is assigned to the student. For example, let's see certain extracts from the introductory class to factorization[10], the class about the common factor:

| | | |
|---|---|---|
| PROF=Sam | Why this last one is called complete factorization and the other two aren't, if they also are factorized, that is they are also multiplying… | $12 = 3{\cdot}4$<br>$= 2{\cdot}6$<br>$= 2{\cdot}2{\cdot}3$ |
| S=F | I cannot factorize another term. | |
| PROF=Sam | Because I cannot factorize any other term, that is, it cannot represented<br>as a product any other term? **Or yes?** | |
| Ss | No, no. | |
| PROF=Sam | Well, I can write, for example this 2, written as 2 by 1. **Or no?** | |
| Ss | Yes. | |
| PROF=Sam | And I'm representing it as a product. So what you are probably telling me is that I cannot factorize in other way different than 1. Then itself by one. Why? For example, let's say here, 2 by 6, 6 can be factorized once more, **or no?** | |

When the questions formulated by Sam are not the type of *mayeutic socratic*, the time he offers the students to think about and answer is short. In general, after two non expected answers, is Sam the one who answers. Students know in this way, that it is not necessary to make an effort to give the expected response; because if they don't know the answer, the teacher will tell them what should be known about the studied topic.
We could find other kinds of mayeutic socratic dynamics depends of the kind of interactions applied. For example, teacher may choose the questions in a way that students can find the answers by their own. In this sense, the organization of the questions has the purpose of modify the knowledge and convictions of the students. Let's see an example taken from Ron's class about the factorization of polynomials, where tries to evocate what characterizes a complete factorization.

| | | |
|---|---|---|
| PROF=Ron | And why is it called a complete factorization? | |
| S=S | Well..because it is reduced to…the exponentials | |
| S=D | To the maximum | |
| PROF=Ron | What is to reduce to the maximum or to factorize to the maximum possible? | |
| S=D | Hmm..because obtained the maximum common divisor | |
| PROF=Ron | Not again, cannot be a maximum common divisor, because, common to whom? […] | |
| S=D | Because 80 became 1. | |
| S=L | Yes, because it got up to 1. | |
| PROF=Ron | If I divide it by 20, and the result by 4 also results in 1 (13s). Mm? What does this has as special?, why called complete factorization? […] When factorizing, the idea is to multiply two numbers of what kind? Because 80 can also be obtained multiplying 40 by square root of 2 twice […]; but | $4^2 \cdot 5 = 80$<br>$2^4 \cdot 5 = 80$<br>$50 + 30 = 80$<br>$20 \cdot 4 = 80$<br>$16 \cdot 5 = 80$<br>$40 \cdot 2 = 80$ |

[10] During this lesson, factorization by common factor and grouping method were reviewed.

not interested in that, right? When asked to factorize numbers, we are only going to factorize it by the product of numbers…of what kind?

S=S Integers

PROF=Ron Integers. And positive, well. Only interested with the product of integer numbers? But in there, the 2 and 5 what characteristics they have? That do not have the 16 nor the 20, nor the 4, nor the 40…

S=M They are prime numbers.

In this example, is easy to recognize that teacher modifies questions according to the answers given by the students, given them a new tool to make the original question easier. Basically, sometimes in this type of dynamic, students try to guess the answer that the teacher wants to hear.

### 2.3 Potential Adidactical Contract

We associate the term “adidactical[11]” with the typical interpretation given by Brousseau in his theory (Didactical Situations). The notion of “adidactical” is described by Margolinas (1993, in Sadovsky, 2005) as:

The kind of intellectual obligation that the student has with the environment and do not allude to the silence of the teacher but to the fact that for give place to the production of knowledge, the teacher do not shows what are the knowledge the student may move (p. 25).

In this sense, the absence of this type of interaction between students and student-teacher previously described by authors like Brousseau (1986, 1987), Chevallard (2005) y Barquero et al. (2007), together with our experience, it has taken us to recognize the different interactions whose characteristics describe potential adidactical practices. We mean for example, when

- Teacher plans situations where the students evoke, formulate, rationalize and justify their propositions
- Teacher assigns to the pupils a great part of the responsibility of the knowledge
- Students accept the compromise to answer the questions and try to perform the task assigned by the teacher

This kind of contract offers new responsibilities to the students, responsibilities that are important to be identified. For example, in a potential adidactical dynamic, students elaborate questions and not wait the answers of them. Also, they are called to answer other students’ questions, and so the teacher is not the unique person who has the right answer in the class. Students must explain their suggestions and give contributions to rebuild the cognitive way that takes them to a result. In addition, they must be able to justify a technique and propose verifying strategies of it.

We formulate the hypothesis that these new responsibilities would let students to develop not only mathematic skills but also some others applied in contexts not necessarily associated with this discipline. For exemple,

- to create strategies to value ideas –making explicit and using reference points–
- to construct or to hear coherent arguments
- to refute arguments

[11] The election of this term has been taken from the discussion exposed by Hersant, M. & Perrin-Glorian, M-J. (2005) in their article about *Charaterization of an ordinary teaching practice with the help of the theory of didactic situations.*

- to be careful of not making abusive generalizations
- to be a good "listener"

The potential adidactical contract demands to teachers other kind of responsibilities that were not required in the ostension and mayeutic socratic contracts. Now, teachers not answer all the questions and they must contribute to questioning among students. Also, they must develop skills as to argue, to communicate precisely the ideas, to know about other ways of learning (metacognition). An essential aspect presents in practices regulated by a potential adidactical contract is that teacher (and students) makes evident regularly the application of these skills in other contexts.

Clearly, these new tasks demand teacher to be a good observer about how his students learn mathematics, to be a good "listener" and require skills to manage situations that led to the students assume their new responsibilities.

The evolution from traditional ostension and mayeutic socratic contracts toward potential adidactical contract, has to stand out a change in terms of the principal actor in the didactical system that can be synthesize in the schema in Figure 2.

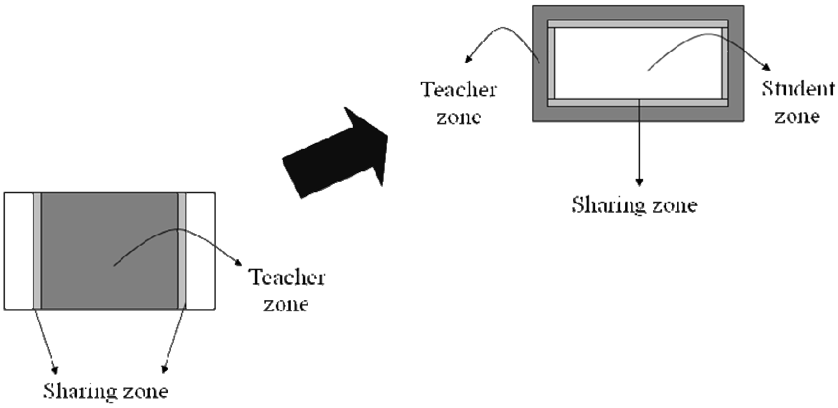



Figure 2: Evolution of the main student zone

In a potential adidactical dynamic, the teacher leaves the main place in the class, and begins to assume a supported role. The student becomes the main character, not only because he is the unique person that can build his own knowledge, but also the situation becomes an useful tool to get this.

### 3. Conclusions

In this article we have tried to characterize the teaching practices, using the notion of the didactical contract. As it has been observed, the teaching strategies of each professor encourage the setting of the ostension contract and the mayeutic socratic contract.

For both teachers, it was recognized an interrogative dynamic that describes the way in which the teacher presents the knowledge to the class. During this communicative dynamic, students are responsible for tasks such as: complete with one or two words the phrases enunciated by the teacher, enunciate the result of an indicated operation, reaffirm the teacher institutionalizations, etc. It is important to point out that this dynamic lacks of a closure that takes and orders the sequences of ideas that lead the students to the answer expected by the teacher.

Some elements of the contracts limit the students, in such a way that they can't assume other mathematical responsibilities. For example, the students must assume responsibilities such as: elaborate questions, answer other students' questions, explain their suggestions and contributions, rebuild the cognitive way that takes them to a result, justify a technique, propose verifying strategies of the technique, etc., that are included in a new type of contract. Our first tentative to denominate the contract that include these responsibilities as *potential adidactical*, is an open discussion. This article has the aim of to present some attempts, observed in classes, to make evolve the distribution of the responsibilities between professor and students, toward a greater freedom and responsibility for these ones. These distributions are interpreted like "contracts". The theory of the Didactical Contract affirms that these contracts are irremediably fictitious but essential, that it is impossible for the professor to withdraw himself from some obligations and that consequently the radical constructivism is impossible. But professors consider this approach as useful. It is necessary to describe the many existing attempts, including the experiments of Brousseau. The *potential adidactical contract* was not identified yet in the preceding articles.

The analysis and characterization of the didactical contracts is a useful and necessary tool that allows for reflection on the type of interactions present in teaching practices. It is consistent with the roles of the teacher. In this sense, it is a necessary first step for the proposal of new teaching practices.

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