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PERCEPTION OF THE NOTIONS OF CONSERVATION, COMPARISON AND MEASUREMENT OF THE AREA. A STUDY THROUGH ARGUMENTS IN THE CLASSROOM

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Abstract
This study examines university students' perceptions of how the area on plane regions is conserved, compared and measured and also the role of the teacher in this process. The data analysis comes from classroom interactions during the solution of a learning situation which involves geometrical transformations –convex polygons– and includes a semi-structured interview. The analysis is supported by the argumentation scheme proposed by Toulmin (1958) to reconstruct students' argumentation schemes. The findings show that the students perceive: a) area conservation when they refer both the parallelism relation and the elements of the formula to calculate the area of triangles or simulate movements on figures; b) area comparison when they realize the relationship between the areas of the three polygons (triangles), and; c) area measurement when they manage to use the formula to calculate the area of the triangles. The teacher's role becomes complex when he confronts students' arguments. The study of the interactions in the mathematics classroom provides evidence of the importance of communication. Finally, the examination of the arguments show how students build their justifications, give reasons, argue, think about knowledge, share knowledge and the methods they use to argue.

Key words: Arguments, perception, teacher's role, classroom interactions, conservation, comparison and measurement of the area.

1. Introduction

The study of arguments given by mathematics students has been a central subject of research in our discipline, Mathematics Education (Inglis, Mejía-Ramos and Simpson, 2007). Its analysis must be kept in mind to understand the value that the students give to the forms of validation in mathematics classroom (Crespo-Crespo, Farfán & Lezama, 2009). The analysis of students´ arguments in this contribution is a central aspect to explore the perception of mathematics students on notion of area. The results obtained become in a long term research that investigates the role of the conservation of the area in the study both the concept of area and definite integral in the mathematics classroom (Cabañas and Cantoral, 2006, 2008). Particularly this paper examines students´ arguments while working on a learning situation in the context of geometrical transformations on a plane region –convex polygons–. Our main interest is in understanding through classroom interactions, university students' perceptions of how the area on plane regions is conserved, compared and measured and also the role of the teacher in this process. We used Toulmin´s argumentation model to reconstruct students' argumentation schemes and to analyze their arguments. Generally these types of analysis are of two kinds: those that concentrate on the argument's content and those that concentrate on the argument's structure (Inglis et al, 2007). This paper fits into the first category to interpret students arguments given in the observed lesson. Then, the written, verbalized and gestured arguments became fundamental sources to this exploration.

2. Student-teacher interactions

In the mathematics classroom is the teacher who introduces the mathematical knowledge as an activity or task, knows the purpose of the situation and organizes the classroom interactions, etc. Following to Laborde and Perrin-Glorian (2005), we maintain that the teacher's role is necessarily fundamental when the classroom situation becomes an object of study. In the classroom the construction of mathematical knowledge relies on discursive practices. The discursive practices happen, in a narrow relationship with the argumentative forms and its use. Thus the language is a fundamental means of mathematical communication (Pirie, 1998, Voigt, 1995; Yackel & Cobb, 1998). Language in its broadest sense is the mechanism by which teachers and pupils alike attempt express their mathematical understanding to each other (Pirie, 1998).

In mathematics classroom not necessarily all the arguments become verbalized or written. They sometimes become evident through gestures or by using visuals resources (Crespo-Crespo et al, 2009). Therefore it is important to consider all the resources the students are using in an argumentation process.

3. Perception of the conservation, comparison and measurement of the area.

To examine how students perceive that the area is conserved, compared and measured, we based in a view from the knowledge that considers to the perception as the fundamental capacity that it maintains us in contact with the world (González, 2006). This researcher describes three fundamental aspects of the perception: functional one, phenomenology and symbolic one. To perceive the world from the functional aspect is equal to operate appropriately in it. From the phenomenology aspect to perceive the world is equal to experience it (i.e. auditory, smell and visual form). And finally from the symbolic aspect or conceptual linguistics it allows perceiving the world depending on concepts and communicable descriptions that, on having had them available and to apply them give an identity to what we perceive. It is from the second and third aspect from which we analyze to the perception of the notions of comparison, conservation and measurement of the area. From the second aspect we took into account both visual elements and gestures, which contribute in identifying properties as: the *change of shape* or *change of position* of the figures or both properties. From the third aspect we considered the mathematical relationships that are established between the figures (for example the relation of parallelism), the movements, algorithms, etc.

4. Toulmin's Argumentation Model

The reconstruction of the argumentation schemes is based in the argumentation model proposed by Toulmin (1958). Particularly we used Toulmin´s model described in Inglis et al (2007). This model is made up of six basic types of statement, each of which plays a different role in an argument. The *conclusion* (C) is the statement of which the arguer wishes to convince their audience. The *data* (D) is the foundations on which the argument is based, the relevant evidence for the conclusion. The *warrant* (W) justifies the connection between data and conclusion by, for example, appealing to a rule, a definition or by making an analogy. The warrant is supported by the *backing* (B) of new evidence. The *modal qualifier* (Q) specifies the degree of certainty, the strength of the conclusion, expressing the degree of confidence in the thesis; and the *rebuttal* (R) presents the exceptions to the conclusion. The six categories of the model are connected in the structure shown in the Figure 1.

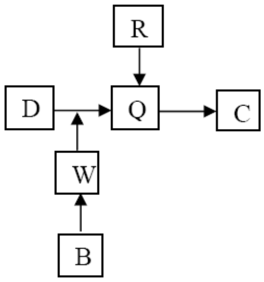


Figure 1: Toulmin's Model of general argument

As Inglis et al (2007) say, it should be noted that, in any given argument, not all of these roles will necessarily be explicitly verbalized.

In this study Toulmin´s argumentation scheme was used to interpret the mathematical arguments or the logical reasons that students gave to confirm their proposed conclusions on task.

5. Methods and participants

The data reported in this paper come from the item a, of one learning situation –in the context of the notion of area–, as well as from an interview. The participants in this study were 13 students (21-24 years old) who were studying their Bachelor studies in Mathematics Education. The activity was performed in two sessions. In the first one, the students were invited to solve it individually and by team. They were asked to write all the procedures and justifications. In the second one, they were applied a semi-structured interview in a private office. Both classroom interactions and students interviews were videotaped.

5.1. Teacher activity

The professor who agreed to collaborate in this activity was asked to develop his activities as follows:

- To organize to the students in small groups (one team was made up by four members and the others by three);
- To give the research activities to the students by series (each students had the research activities);
- To ask the students to work on the activities individually at the beginning, to write their procedures, reflections, reasoning, etc. Working by team they had to explain their procedures to the team members;
- To ask to the team to write the final procedures and the reasoning they used in the solution process.
- To provide additional information during the lessons;
- To interact with the students during the work in groups (by team)

5.2. Students´ activity:

- Worked on the research activities individually and wrote their procedures;

- Shared the procedures with their classmates and discussed the methods used in the solution process of the activities.

In the individual work stage the students were allowed 15 to 20 minutes to analyze the activities and work on the solution process. For team work (Team 1= T1, Team 2= T, Team 3= T3) the time depended on the discussion by the members. In the item of the activity analyzed here the students had around 20 minutes. The students were interviewed two days after they solved the activity. Before this, both the teacher and researcher analyzed the students' productions and the videotapes of the individual and team work in order to plan the interview questions.

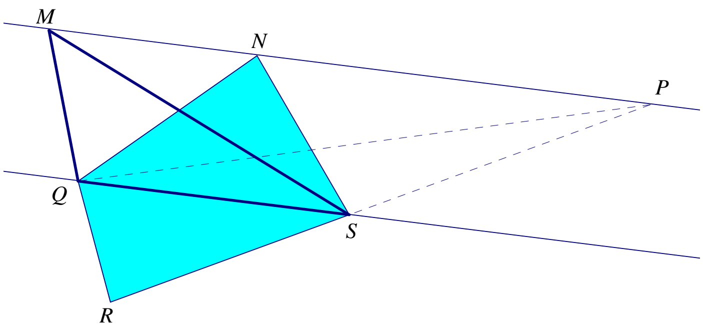
6. Interactions and argumentation schemes in the classroom

The concepts, properties, relations and processes associated with the tasks became a fundamental unit of analysis in the study of the arguments given by the students during the sessions and the interview. They were, therefore, asked to write down their reasoning as they worked both individually and by team. It was interesting, then, to analyze the activity solution process from their verbalized and written arguments, as well as the gestures and other forms of arguments they exhibited.

Teacher's interventions are studied from his: open questions; questions about the conclusion at which students arrived; questions about the conclusion or method or procedure; intervention to confront an argument or procedure, or perhaps, as persuasion. Students´ interventions are studied from their: arguments to convince about their conclusion; intervention to reject or confront a procedure or argument; or perhaps, to assign meaning to a mathematical object.

The activity under discussion in this work is the following:

Activity. In the following figures, straight lines MP and QS are parallel.



a) Determine the relationship that exists between the areas of triangles MQS, NQS and PQS. Justify your answer.

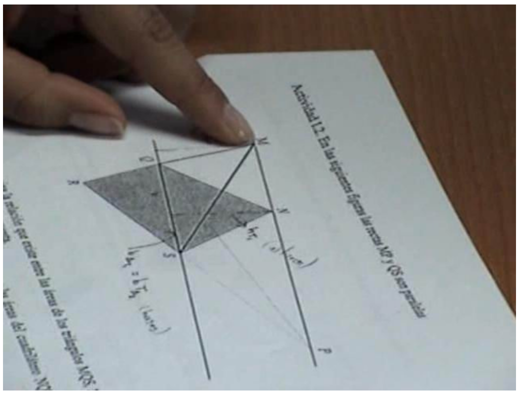
6.1. Analysis of students arguments on item a

The result of the analyses is shown in this section through Toulmin´s argumentation model. The analysis indicates that:

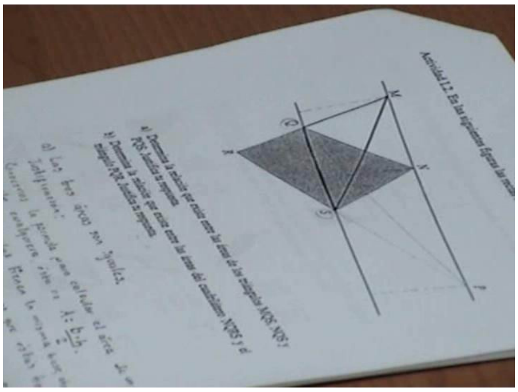
The thesis (*Conclusion*) being justified by the teams is that the areas of the MQS, NQS and PQS triangles are equal. The members of three teams justified their conclusions (*warrant*) in the fact that the triangles are placed between parallel lines and constructed on the same base. The foundations come from the *data*, the parallel lines and the base of the triangles. The members of

T1 and T3 supported their conclusion in the formula to calculate the area of the triangles (*backing*). T2 conclusion was supported both the formula to calculate the areas on triangles (see figure 2.b.) and the translation movement on MP parallel line. The movement was justified in the fact that it is possible to make the vertices M, N and P coincide in any position of the straight MP or to change the position of these vertices on the straight MP in any position –students used their hands to simulate the movement of the vertices–. T2 members also declared that the measure of *the given area is conserved* when the vertices M, N and P change position on the straight line MP. Nevertheless, they stated that both the *shape* and the *position* of the triangles change, when the vertices M, N and do not coincide. The *modal qualifier* was explicitly verbalized only by the members of the T1 when introducing the formula to calculate area of the triangle in their arguments. In the others appeared implicitly in their arguments.

We chose two episodes to show the interactions between the teacher (T) and the members of team 1 as well as one argumentation scheme which interpretation come from students' arguments.



a. Student shows the heights of the triangles they took into account in their justification



b. Written arguments shows how student supported their conclusion in the formula to calculate the area of the triangles

Figure 2: Students´ activity

Episode 1. This episode shows how students justify their conclusion through the parallelism relation

[15] Teacher: Did you find any relationship between the triangles?
[11] Adriana: Yes, we found that the areas of the three triangles are equal.
[12] Teacher: Why?
[13] Adriana: Because the *triangles are constructed between parallels lines*
[14] Teacher: Is it the reason that the areas of the triangles are equal?
[15] Enrique: Well... no ... we also saw in the figure that they were constructed on the same base... then... the triangles have the same height
[16] Teacher: Could you explain me why do you say that the triangles have the same height? Any height?
[...]

[18] Nicolas: Not really. . . we are talking about the height is placed from the base of the triangles . . . to the opposite vertex. Then ... triangles conserve the same base and the same height because they were constructed on parallel lines.

Episode 2. The following episode shows how students support their conclusion in the formula to calculate the area of the triangles

[34] Teacher: I realized that you alluded to the formula to calculate the area of the triangle. . . Why did you use this formula?
[35] Enrique: We know that the area of any triangle is definite by the base and by the height… then, we used the formula to prove that the area is the same.
[36] Teacher: The area is the same. . . is it in any case?
[37] Enrique: We had to take into account the data of the problem . . . then. . . it doesn't matter what is the *form* or *position* of the triangles . . . if you have information about the bases and the height. . . you can know the area of the triangles.
[38] Teacher: It is possible to know the measurement of the area of these triangles?
[...]
[39] Adriana: Not exactly ... but ... we realized that the measurement of the area *always* is maintained in the three triangles . . . through parallelism relation and the base they share. . . but to prove it we use the formula... we were assigning measures to the base and height of the triangles.
[40] Teacher: Is it necessary to do that?
[41] Adriana: No . . . but is another way to prove it

Argumentation scheme reconstructed from Team's 1.

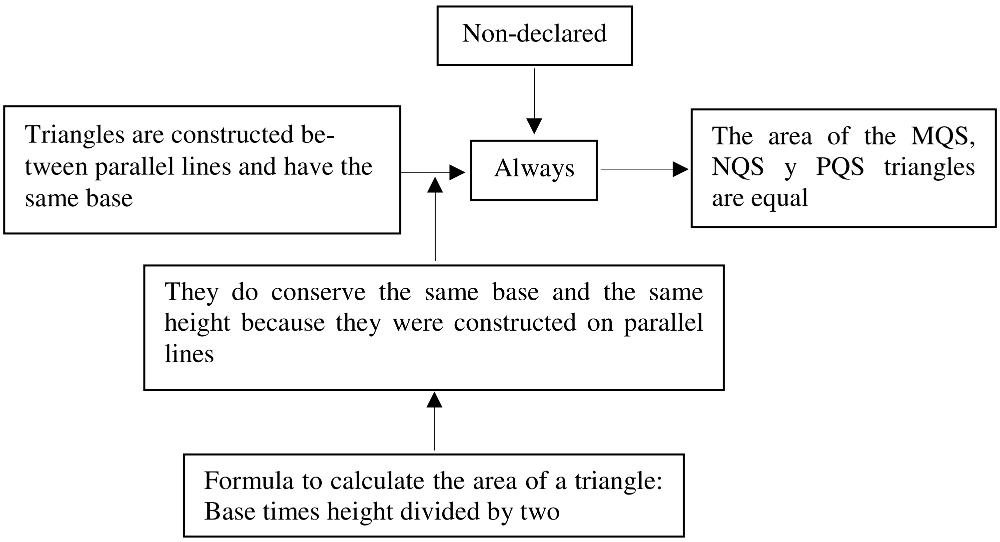


Figure 3: Team 1´s argumentation scheme.

This scheme displays one interpretation on mathematical arguments or the logical reasons that students gave to confirm the proposed conclusions on task and the relationship between them, according Toulmin´s model.

7. Discussion

The discussion presented here is based both teacher and students arguments, contextualized in a learning situation during interactions in the mathematics classrooms. The study provides some useful insights on students' perceptions of how the area on plane regions is conserved, compared and measured and also the role of the teacher in this process.

The complete Toulmin´s argumentation model was used in the analysis of the arguments given by the students. Particularly to interpreter the mathematical arguments or the logical reasons that they gave during the lesson and the interview to argue their conclusions. Means that it was not under discussion in the class.

The findings show that the students perceive: a) area conservation when they refer both the parallelism relation and the elements of the formula to calculate the area of triangles or simulate movements on figures; b) area comparison when they realize the relationship between the areas of the three polygons (triangles), and; c) area measurement when they manage to use the formula to calculate the area of the triangles. Students made gestures to show both teacher and fellows how the area of the triangles is conserved; they simulated to move the vertices of the polygons through the parallel lines. Conservation of the area is verbalized by the students in two ways: maintained the area or conserved the area.

From visual aspects, the students realized that the polygons change both its shape and position, but not the measurement of the area. The notion of area comparison is explored through students´ analyses on the triangles: a) how the triangles share the same base and the measurement of one of its heights is the same as well (students showed the height they are talking about, see figure 2.a), and b) the relationship among the area of the three triangles. The area measurement is perceived when students used the formula to calculate the area of the triangles to support their conclusion.

The professor played the role of organizing student's mathematical activities and the communication in the mathematics classroom. During classroom interactions the teacher confronted students' arguments both to support them and to argue their conclusions and the interactions between them. Following to Crespo-Crespo et al. (2009), to favor the construction of the knowledge, one of teacher's challenges is to understand the forms of argumentation that students use and construct in mathematics classroom.

The results of this study indicate that almost all students´ arguments are referred to concepts, properties, relations and processes associated with the tasks. To identify how they refer to properties as: the *change of shape* or *change of position* of the figures or both properties, we analyzed their verbalized and written arguments, as well as the gestures and other forms of arguments they exhibited.

8. Recommendations

The study reported in this paper is part of a research project. Thus, the analyzed learning situation is a research activity. Nevertheless future investigations must study the daily routine of the mathematics classroom. However Toulmin´s model is not adequate to describe the complexities on classrooms interactions.

References

Cabañas, G. & Cantoral, R. (2006). La integral definida: un enfoque socioepistemológico. En Dolores, C., Martínez, G., Farfán, R.M., Carrillo, C. López, I. y Navarro, C. (Eds.). *Matemática Educativa: Algunos aspectos de la socioepistemología y al visualización en el aula* (pp. 9-32). México: Diaz de Santos- Fondo Mixto Guerrero-Conacyt.

Cabañas, G. & Cantoral, R. (2008). La conservación en el estudio de Área. En R. Cantoral, O. Covián, R. Farfán, J. Lezama & A. Romo (Eds.), *Investigaciones sobre enseñanza y aprendizaje de las matemáticas: Un reporte Iberoamericano* (pp. 199-226). México DF, México: Diaz de Santos-Comité Latinoamericano de Matemática Educativa A.C.

Crespo-Crespo, C., Farfán, R.M. & Lezama, J. (2009). Algunas características de las argumentaciones y la matemática en escenarios sin influencia aristotélica. *Revista Latinoamericana de Investigación en Matemática Educativa 12* (1), 287-317.

González, J.C. (2006). Presentación: La cognición como objeto de estudio filosófico y científico. En J. C. González (Ed.), *Perspectivas contemporáneas sobre la cognición: categorización, percepción y conceptualización* (pp. 11-36). México DF, México: Siglo XXI Editores.

Inglis, M., Mejia-Ramos, J.P. & Simpson, A. (2007). Modelling mathematical argumentation: the importance of qualification. *Educational Studies in Mathematics 66* (1), 3 – 21.

Inglis, M. & Mejia-Ramos, J.P. (2005). La fuerza de la aserción y el poder persuasivo en la argumentación matemática. *Revista Ema 10* (2), 327-352.

Toulmin, S. (1958). *The uses of argument*. UK: Cambridge University Press.

Pirie, S. (1998). Crossing the Gulf between Thought and Symbol: Language as (Slippery) Stepping- Stones in H. Steinbring, M:G: Bartolini Bussi & A. Sierpinska (Eds), *Language and communication in the mathematics classroom* (pp. 7-28). E.U.A.: National Council of teachers of mathematics INC.

Laborde, C. & Perrin-Glorian, M.J. (2005). Introduction Teaching situations as object of research: empirical studies within theoretical perspectives. *Educational Studies in Mathematics, 59* (1), 1-12.

Voigt, J. (1995). Thematic patterns of interaction and sociomathematical norms. In Cobb (Ed), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 163-201). Hove, UK: Lawrence Erlbaum Associates, publishers.

Yackel, E., & Cobb, P. (1998). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458-477.