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A Theoretical Characterisation of Service Mathematics

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Abstract

A qualitative study of Service mathematics in Irish universities was carried out by the author (OG) in order to examine the context, the practice and experience of Service mathematics teaching in Irish universities on recognition of the fact that Service mathematics was under-theorised (Gill, 2006). The findings were used to serve as a basis for developing a meaningful characterisation of Service mathematics in Irish Universities today. Also an effort was made to define more clearly the teaching/learning contract that exists between the actors in this sphere of activity. In addition the authors built on this preliminary profile to develop a theoretical model of Service mathematics teaching using Bernstein's curriculum theory and aspects of Realistic Mathematics Education. In this paper the authors utilise this characterisation to define more clearly the classroom practices of university service mathematics classes in Ireland.

Keywords: mathematics problem; service mathematics; didactical contract; third level

Introduction

Over the past 10 to 15 years, there has been widespread concern among third level (i.e. university) academics about the poor level of mathematical preparedness of first year undergraduates in mathematics intensive courses (Smith, 2004; IMA, 1996; LMS, 1995). Research shows also, that the problem is not just that some students are “under-prepared” but that even students with good Leaving Certificate/ A-Level grades have difficulty with even the most fundamental aspects of mathematics (NCCA, 2005; LMS, 1995). This ‘Mathematics problem’ as it is styled in the UK research on mathematics education includes issues in the transition from school mathematics to university Service mathematics. Service mathematics refers to degree courses where mathematics constitutes a significant part, but is not the main focus of the students' studies.

The existence of the ‘Mathematics problem’ in Irish third level institutions was the inspiration for the author's (OG) doctoral research. At the outset of the study there was only anecdotal evidence to suggest that the problem existed in Ireland. The author established the existence of the ‘Mathematics Problem’ and the scale of the problem in the University of Limerick (UL) through analysis of the UL database.

The author carried out a qualitative investigation into Service mathematics in Irish universities. The enquiry involved a close inspection of how Service mathematics is perceived, planned, delivered, evaluated, assessed and experienced by both lecturers and students. Murphy (2002) used Brousseau's concept of didactical contract to uncover the implicit contract present in Irish

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second level classrooms. The author emulated this work in her study to discover the hidden learning contract in third level lectures. A major outcome of this phase of the study was the development of a characterisation of Service mathematics in Irish universities and insight into the nature of the didactical contract at work in the Service mathematics courses surveyed.

Although this characterisation provided a clear picture of Service mathematics in Irish universities, it was less than adequate in a more global sense. It became apparent that the nature of Service mathematics was not fully understood. Even in the literature, it has never been clearly characterised until now being identified variously as Engineering mathematics, Mathematics for Business etc. Prior to this, the ‘Mathematics Problem’ had been viewed as just that: a mathematics problem. Following the work of O’Donoghue (1999), the authors decided to view the problem through a mathematics education problem lens instead and treat it as a mathematics education problem. Mathematics education could extend the problem definition and solution spaces beyond mathematics per se by providing the curriculum theories needed to characterise Service mathematics. Bernstein’s theories of curriculum and pedagogy provided the means necessary to characterise Service mathematics in a theoretical way. This was accomplished through a concurrent research effort directed at putting Service mathematics on a more theoretical basis and finally culminated in a theoretical model of Service mathematics. This paper utilises the aforementioned model to define more clearly the practices of Irish third level service mathematics classrooms. In this way, a better understanding of the practices is achieved and helps one to identify where issues lie so that they can then be addressed.

Methodology

The aim of the qualitative study on Service mathematics in Irish universities was to examine the context, the classroom practice and experience of Service mathematics teaching in Irish universities so that the findings might serve as a basis for developing a characterisation of Service mathematics in Irish universities. There are 7 universities in Ireland. The majority of the research, therefore, was carried out within the seven Irish universities. Non-probability/purposive sampling methods are utilised to gather the data required often followed by observational modes of inquiry to get an holistic insight into the world of the group of individuals to be studied (Cohen et al, 2000). For the authors to gain an holistic insight into the world of Irish third level Service mathematics classes and programmes, a multi-method approach was required in order to attain validity and reliability in this study. The research data was obtained via: interviews with 12 students and 9 lecturers to see both viewpoints; observations of lectures and tutorials; analysis of course documentation; questionnaires.

In order to derive a theoretical characterisation of Service mathematics the authors drew on mathematics education tools throughout the research implicitly and explicitly using:

- Realistic Mathematics Education (Freudenthal)
- Didactical contract (Brousseau)
- Curriculum theory (Bernstein).

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Didactical Contract

Brousseau (1997) looked for reasons other than children’s maturity or mathematical ability to explain aberrations in their mathematical education. He contended that a child’s learning is as affected by its relationship with knowledge as it is by the characteristics of the child itself. Brousseau proposed that an implicit contract exists within every mathematics classroom between all actors in the sphere. Students are presented with mathematical tasks/problems by their teacher/lecturer. The students are required to work on the tasks whilst adhering to various constraints imposed by the teacher/lecturer. The expected behaviours of the students from their teacher/lecturer and vice versa determine the didactical contract present in the classroom. Brousseau believed that this contract has a significant impact on the teaching and learning that occurs in the class and has a key role in the analysis and the development of effective conditions for the teaching and learning of mathematics.

Preliminary Characterisation of Service Mathematics

A better understanding of the nature and practice of Service mathematics in Irish universities is a significant outcome of the qualitative study carried out by the author (OG) in her doctoral studies in 2006. It was the authors’ objective to investigate the current practices and attitudes of lecturers and students within third level Service mathematics courses to ascertain exactly what ‘Service mathematics’ means, in theory and in practice, in Ireland. No such study has ever been undertaken in Ireland and consequently the findings constitute a significant source of new data on Service mathematics teaching in Ireland.

As a result the authors were able to develop a preliminary profile of Service mathematics based on this work. This profile is summarised as follows:

- Service mathematics is distributed across many disciplines and faculties and is identified in various ways such as engineering mathematics, mathematics for engineers and scientists, mathematics for computing, mathematics for business, technology mathematics etc.
- Service mathematics is directed at client groups composed of non-specialist users of mathematics.
- In service mathematics classes, there is more emphasis on techniques, followed by applications than theory.
- Mathematics content is negotiated between mathematics and client departments with no external input from industry or employers.
- Courses are usually offered in the traditional large lecture/tutorial format.
- There is a large diversity in mathematical background and attainment of learners.
- Lecture style is usually ‘chalk and talk’ supported with limited resources e.g. white/blackboard, overhead projector.
- There is very little interaction or questioning in lectures.
- The use of relevant real-life mathematical examples is acknowledged as being important but is invariably absent.

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- Assessment varies between end of term examination and end of term examination combined with some form of coursework or continuous assessment.
- Additional learning support may be available and comes in a variety of formats including drop-in centres, support tutorials and other learning centre activities.
- It is common for class notes to be supplied in book form supported by appropriate textbooks for non-specialists.
- The didactical contract evident in classes displays mismatches in expectations between students and lecturers, for example, lecturers expect students to take more responsibility for their own learning i.e. to become more self-directed in their approach whereas the students expect everything explained in detail which the lecturers do not do.

The authors' mathematics education perspective directs one to take a close look at curriculum issues such as aims, objectives, planning, implementation, assessment and evaluation which is what the author has done in this study in the Irish context. In addition, the nature of Service mathematics needs to be examined. There have been various interpretations/working definitions of 'Service mathematics' e.g. engineering mathematics, mathematics for engineers and scientists, mathematics for computing, mathematics for business, technology mathematics etc. One could describe these approaches as 'naïve' because they are not supported by any theoretical perspective. One of the aims of this research was to explore the possibilities of moving from ad hoc working definitions to finding a characterisation that can be supported theoretically. This, the authors proposed to do, by drawing on existing curriculum models.

Bernstein's classification and framing

Bernstein developed the concepts of classification and framing as a means of systematically analysing pedagogic discourses and practices. Classification refers to the degree of insulation between categories. Strong classification implies strong insulation between categories. Each category is distinct from the next. Weak classification implies weak insulation. In a strongly classified curriculum, there is restricted interaction between lecturers and students and vice versa.

Whereas classification is concerned with the selection and organisation of knowledge to be taught, framing is related to the transmission of knowledge from teacher/lecturer to student. The concept of framing was determined to analyse different types of communication in a pedagogic practice. Framing refers to how meanings are to be put together. In the context of Service mathematics it is about the relations between lecturers and students. It is about who controls what. It refers to the levels of control over:

- Selection of the communication (how the mathematics are to be taught)
- The sequencing of the subject matter (in what order topics are to be taught)
- The pacing (how quickly one moves from one topic to the next)
- The criteria
- The control over the social base that makes this possible (Bernstein, 1999:12-13)

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Bernstein (1999:9) distinguished between discourses as singulars and discourses as regions. Singular discourses produce their own knowledge for themselves. In contrast, discourses as regions are defined as the interface between the field of production of knowledge and the area of its application. Service mathematics falls into the latter category as the mathematics to be taught/learned is essentially to be utilised in other fields outside the mathematics domain such as Science, Business etc.

Realistic mathematics education (RME)

RME endorses Freudenthal’s notion that real life is the source of mathematics, not exclusively the place where mathematics is applied. Horizontal and vertical mathematising are key elements in RME. Freudenthal (199: 133) illustrated 4 types of curricular standpoints: mechanistic, empiricist, structuralist and realistic each characterised by vertical and/or horizontal mathematising as Table 1 shows.

Table 1 Treffer’s 4 types of mathematical education

	Horizontal	Vertical
Mechanistic	-	-
Empiricist	+	-
Structuralist	-	+
Realistic	+	+

Source: Freudenthal (1993:33)

Clearly, a realistic mathematics curriculum, which encompasses both vertical and horizontal mathematising should be available to all students studying mathematics whether they are mathematics specialists or Service mathematics. The move from real-life to the abstract (horizontal and vertical mathematising), as emphasised in RME, is critical for students to progress in the discourse of mathematics. Benn (1997) states that we must provide ‘pathways’ for students to do so.

From the research conducted in this study within Irish universities, it seems that Service mathematics is based on a vertical continuum. First students learn mathematics in an abstract, mechanical way, real life examples and applications are not taught until after they have learned the required skills. They are not taught in tandem. O’Donoghue (1999) states that while this may work for a number of students, it will not work and does not work for many others. In Service mathematics, context and real-world situations really need to be exploited more. ‘Deferred reward’ in terms of applications needs to be avoided in Service mathematics teaching. If common sense knowledge (local knowledge) was exploited more often, student real-life experiences could be integrated more into their mathematics education.

Within the discourse of mathematics, Benn differentiates between ‘separate’ and ‘connected’ reasoning. Separate reasoning refers to the solving of a mathematical problem in an abstract me-

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thodical manner, detached from context. Connected reasoning solves problems within a real-world/relevant context.

Separated reasoning appears to be the dominate within Irish Service mathematics classes and this is consistent with the preparation of these students in their mathematical preparation at second level (Murphy, 2002). Examples and exercises test mechanical skills. They are often stripped of context and students fail to see the relevance of their mathematical studies. Service mathematics students learn mathematical skills first and applications come later. A well selected real life example used to introduce a mathematical topic serves many purposes, one of the most important being showing students the importance of mathematics in the real world and its relevance to their other studies. As Service mathematics is mathematics embedded in other disciplines e.g. Engineering, Business Studies etc. it would appear to make sense to apply aspects of RME to Service mathematics curricula. Relevance was one of the key issues to arise from this research into Service mathematics. Many students are unaware of why/how the mathematics they are studying are relevant to them (Gill, 2006). This is problematic as it affects student motivation as a result. RME offers a model which has the potential to alleviate this problem.

A new classification for Service mathematics

The authors conclude that Service mathematics may be characterised as a pedagogic discourse within the discipline of mathematics. Service mathematics is essentially a pedagogical task demanded by client departments (e.g. Engineering, Business etc.). The characterisation of Service mathematics as a discourse is viable because we can interpret its defining characteristics. There is a community of practitioners comprising mathematicians in mathematics departments mainly but some in other faculties, there is a reasonably well defined agreement as to the mathematics content of Service mathematics courses at various levels together with a well articulated debate on its teaching e.g. the literature contains an ongoing concern for issues such as tools versus rigour/theory, modelling etc. There is general agreement as to the purposes of Service mathematics teaching e.g. to service client departments by providing the toolkit of skills needed for other areas of study. It is a strongly classified discourse in Bernsteins’s sense. Further, following Bernstein, this discourse is viewed as a region because it is distributed across areas of mathematics and other disciplines where it is applied.

In the early undergraduate years the discourse is best described as horizontal but in more advanced applications, vertical aspects are implicated. Horizontal discourses are segmentally organised. By definition, students are trained to achieve a list of competences, in this case, mathematical skills. When one learns the skills in one segment, they move on to the next segment and next list of mathematical skills, for example, in one of the service mathematics classes observed students learn how to differentiate various functions and apply rules such as the product and quotient rules. When this section is finished, they move on to linear equations and learn a list of techniques such as how to solve systems of equations by Gaussian elimination etc., this list seemingly having no relation or bearing on the list of skills learned in the last segment. Vertical discourses integrate fundamental skills and concepts learned and build on these to work at

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increasingly theoretical levels. This rarely happens in the first year of the Service mathematics courses observed. Lecturers implied it happened more so in later years.

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Conclusion

The authors' work and the theoretical model offer the opportunity to reformulate the 'Mathematics Problem' in a different way. Service mathematics is a pedagogic discourse within mathematics that deserves attention in its own right thus the issue becomes one of appropriate teaching of Service mathematics. This is a permanent rather than transient concern and should not focus on falling mathematical standards or students' mathematical deficits but rather on the educational task of Service mathematics teaching in its own right. The authors' theoretical model offers insights into aspects of the educational tasks and how they might be approached e.g. the skills/rigour/concepts debate, aspects of pedagogy such as the use of applied contexts and integration of topics. In the contexts of mass higher education and the ever-widening range of mathematical competencies of entering students a concern for effective Service mathematics teaching is more likely to benefit students, institutions and the economy than a constant replication of coping strategies as is dominating the current situation and thinking.

References

- Balachev, M., Cooper, M. & Sutherland, R. (1997). *Theory of Didactical Situations in Mathematics 1970-1990 (Didactique des Mathematiques – Guy Brousseau)*. Volume 19, Dordrecht: Kluwer Academic Publishers.
- Bernstein, B. (1996). *Pedagogy, symbolic control and identity: theory, research, critique*. London: Taylor and Francis.
- Benn, R. (1997). *Adults count too. Mathematics for empowerment*. Leicester: National Institute of Adult Continuing Education.
- Cohen, L., Manion, L. and Morrison, K. (2000). *Research Methods in Education*. London: Routledge/ Falmer Academic Publishers.
- Freudenthal, H. (1991). *Revisiting Mathematics Education. China Lectures*. Dordrecht: Kluwer Academic Publishers.
- Gill, O. (2006). *What Counts as Service Mathematics? An Investigation into the ‘Mathematics Problem’ in Ireland*. Unpublished thesis (PhD), Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland.
- I.M.A. (1995) *Institute of Mathematics and its Applications, Mathematics Matters in Engineering*, Southend-on-Sea:
- LMS, IMA and RSS (1995). *Tackling the Mathematics Problem*, London Mathematical Society, London, UK
- Murphy, M. (2002). *An Investigation into the Mathematical Under-Preparedness Present Among Third Level Entrants: the Possible Contribution of the Second Level Mathematics Experience*. Unpublished thesis (M.A.), Department of Mathematics and Statistics: University of Limerick.
- NCCA , (2005). (National Council for Curriculum and Assessment) *Review of Mathematics in Post-Primary Education – a Discussion Paper*, Dublin. Available at <http://www.ncca.ie/uploadedfiles/MathsReview/MathsDiscusPaperEng.pdf> .
- O’Donoghue, J. (1999). *An intervention to assist at risk students in service mathematics courses at the University of Limerick: University of Limerick teaching fellowship scheme*. University of Limerick, Limerick.
- Smith, A. (2004). *Making Mathematics Count: The Report of Professor Adrian Smith’s Enquiry into Post-14 Mathematics Education*, London: HMSO
- Treffers, A. (1993). *Wiskobas and Freudenthal: Realistic Mathematics Education*. In Streefland, L., ed., *The Legacy of Hans Freudenthal*. Dordrecht: Kluwer Academic Publishers, 89-108.