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The explanations of teachers. A research experiment on the notion of similarity in high school.

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Abstract

This report focuses on the explanations given in high school geometry classes, when trying to teach mathematical concepts and processes linked to the notion of similarity. One of the obstacles in the evolution of this concept has been the relationship between figurative and numerical aspects. Here we will present sections of analysis that illustrate these aspects in the development of one class in particular. We will consider a qualitative model of investigation, based on an ethnographic method.

Key words: similarity, ethnography, interaction patterns, explanation.

Introduction

In recent years, there has been a notable rise in the number of investigations that have dealt with understanding the teaching practice of mathematics. Some are oriented towards identifying the influence of the teacher's different areas of knowledge in relation to their practice (Aubrey, 1996; Escudero, 1999). Other works adopted a more socio-cultural nature, based on a perspective of teaching that "implies understanding and negotiating meaning through communication" (Herbst, 2006, 2002; Martín, et al. 2005). These investigations have tried to describe and interpret teachers' activities, searching for regularities in the interactions that teacher and students develop during daily practice.

Some investigators (Hersant and Perrin-Glorian, 2005; Laborde and Perrin-Glorian, 2005) analyze the practices of teachers in ordinary classes using the theory of didactic situations. They show that this theoretical model is relevant for the study and understanding of ordinary teaching in a way which takes into account the progress in students' learning of the knowledge at stake. Other perspectives are based on the empirical analysis of classroom interaction and the relationships that are generated between teacher-student-content, placing emphasis on the teacher-content relationship (Bromme and Steinbring, 1994; Steinbring, 2005).

Recently, in the area of didactics of mathematics, based on different theoretical approaches, new theoretical tools have been suggested in order to analyze teacher-student interactions by way of a mathematical task (Cobb and Bauersfeld, 1995; Voigt 1994; Brousseau, 1986; Escudero, 1999). Constructs such as <<interaction pattern>>, <<negotiation of meanings>>, <<sociomathematical norms>>, <<didactical contract>>, are useful tools that permit analysis of teacher practice.

The investigation that is now being reported does not relate to investigations aimed at describing the thoughts of teachers, but instead, studies an aspect of the teacher's practice when a certain mathematical notion is used among their students, subject to the restrictions of the didactic system’s operation. Some studies along this line have shown evidence of the difficulties of acquiring the notion of similarity. One of the obstacles in the evolution of this concept has been the relationship between the figurative and numerical aspects (Escudero, 1999). The organization of both registers and the weight that each of them has in the management of the subject (Lemonidis, 1991) are important components to bear in mind when the time comes to consider similarity as an object of teaching-learning**.**

In this paper, we want to understand: what is the role that explanations play in the development of the notion of similarity in a particular teaching situation? We will observe and analyze interactions between teachers and students, the role of the teacher and the effect of explanations, the role of students and their commitment to knowledge. In our investigation, explanation refers to those actions aimed at the establishment of relationships between data, phenomenon, processes or events, or those actions aimed at the construction of conceptual networks by students. Therefore, the intention is not only to offer new information, but also to ensure that this information can be blended with previously incorporated information as an outline for thought. Thus, explanation constitutes one of the means utilized by teachers to make students understand or to “give sense”, and it turns into the object of communication or argumentation.

In our work, we follow the interpretive framework of Cobb and Yackel (1996), which contributed to our interpretations of classroom events, because this model acknowledges the reciprocal influences of both individual and collective learning in the social context of the classroom; in particular, social norms and socio-mathematical norms (Cobb, 1999). Social norms are defined as normative aspects of the classroom that may apply to any subject area, such as the expectation that all solutions must be justified. Sociomathematical norms refer to evolving classroom norms that specifically refer to mathematics, such as what counts as an acceptable argument.

Our analysis of classroom episodes as a means of investigating the interplay between teacher actions and student actions requires a close look at the maturity of the classroom discourse and its role in the development of student learning with regards similarity. In this study, discourse is broadly interpreted to mean both the rules of communication among members of the community, the content of verbal and non-verbal exchanges, as well as the mathematical level of the discourse.

Methodology

The investigation is marked by the qualitative paradigm, based on the ethnographical method (Erickson,1986). The ethnographic focus permits us to obtain information about the context of the class that is relevant for its interpretation. The ethnographic perspective consists of discovering and analytically rebuilding the scenes and groups that they are involved in, and which participate in educational practices, putting them in a linguistic register that allows their readers to represent them, as seen by the investigator.

The episode presented in this report forms part of a group of classroom observations and interviews, as part of interpretative work. Audio and video tapings were used in its design when the teacher undertook the notion of similarity. Those recordings and interviews were completely transcribed.

The analysis of the obtained transcripts was developed in two stages. In the first part, recordings of the videos and notes taken during observations of the class were used. The second part was based on the transcriptions, and the inspections of the videos identified segments of interaction which are characterized by: the explanations given by the teacher in relation to the notion of similarities and the concrete actions that are put into play at the moment of undertaking this concept.

Results and discussion

Teacher Alfonso, a participant in the episode that will be explained next, has six years of experience teaching in High Schools, which in Mexico is the formal level of education after secondary school (16 to 18 years old). His willingness to participate in all types of initiatives which could help him to improve in his profession was the principal reason for which he agreed to participate in this study.

Teacher Alonso sees the concept of similarity as a way to connect the numerical vision with graphic images. For him, similarity is students being able to distinguish from graphic images that something is proportional. This form of perceiving similarity by emphasizing that which is numeric-algebraic is manifested in the selection of exercises that he presents to the students.

During the previous class, the teacher had given the students some sheets which he had called "worksheets". These sheets consisted of a series of definitions relating to similarity, similar triangles, fundamental theorem and the similarity of triangles. The teacher began to draw a right-angled triangle and, upon doing so, the group was silent. He spoke to the group and began talking in a slightly louder voice, generally used for public speaking, very different from the voice used in the personal conversation held with some of the students before beginning class.

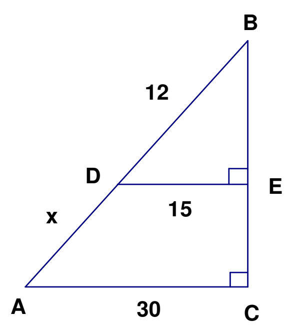

Figure 1

In this episode, the teacher wanted the students to find the value of "x", which raised the question, "How can we obtain the value of 'x' from this point to this other point?" He mentioned that he had given them the necessary elements to solve the problem. He was referring to the information provided to them on the worksheets he had handed out during the previous class.

<u>Episode 1</u>

Teacher: How do we obtain the value of "x" from this point to this other point? (he points to segment *AD* of the triangle *ABC*). We need to know this segment. I gave you the necessary elements in the last class. You have the information on the worksheets in order to solve it. If you can't, we will do it as a class. You have three minutes to solve the problem individually. If you cannot solve it, you may consult with a classmate. I will sign the sheets of the first ten students who find the value of "x". The triangle has measurements; do not make any mistakes (he comments to a student who already finished the problem).

Student: Yes.

Teacher: Copy what you did on the board (the student does what he is asked). Jonathan, not yet?, and the group in the corner?

This form of procedure on the part of the teacher is representative of many of the activities which are carried out in math classes. The teacher starts with a question, which is then followed by a series of instructions to execute the activity: *How can we obtain the value of "x" from this point to this other point?* We call this type of question a *question of continuity;* they are generally short interrogative statements which perform the function of assuring the continuity of discourse and engage the attention of listeners.

In this episode, the teacher Alfonso approaches the concept of similarity by means of the *intrafigural* relationship since the idea of transforming one figure into another is absent, considering the *aspect of projection* (see figure 1). However, he does not explicitly say that they are dealing with a *Thales Configuration*, directly presenting by way of graphic representation the identification of *numerical/algebraic* data with the corresponding segments of the configuration and asking for the calculation of the numerical value.

<u>Episode 2</u>

Teacher: We have an ordinary triangle and we pass a straight line parallel to either of its sides, let's call it "*m*" or "*l*", any letter will work to distinguish this straight line. In this case, if the line crosses the segments *AB* and *BC* at the points *D* and *E,* such that *BE : EC and BD : DA*. In which, $\angle D$ will be equal to $\angle A$, and $\angle E$ will be equal to $\angle C$.

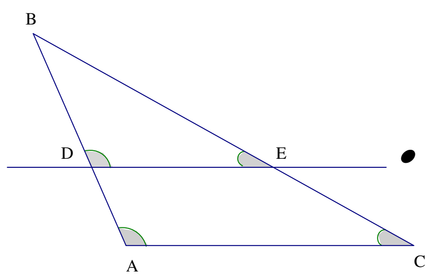

Figure 2

We can see that teacher Alonso does not refer to the Thales Theorem. Consider, as a starting point, that the students formalize the theorem through the division of proportional segments. To do so, it is necessary to identify those distances as sides of similar triangles, thus, being in approximation to the concept within the *intrafigural* relationship (Lemonides, 1991) in which the properties concerning two configurations are studied, in such a way that, although the correspondence between elements of one figure and the correspondences of the other are emphasized, any and all ideas about transforming one figure into another is absent.

Considering this problem and the others that teacher Alonso had prepared, aspects arose which were necessary in order to establish the connection between numerical and geometric aspects of proportionality. In this problem, yet another aspect of the perception of similarity as an object of teaching-learning is incorporated: the idea of the similarity of figures in homothetic dispositions.

<u>Episode 3</u>

Teacher: Ok, now we are going to solve the next exercise, number two. The sides of a triangle measure 7cm, 8cm and 10cm respectively. Find the sides of a similar triangle whose perimeter measures 75cm. I’m going to draw the triangle right now while you reflect on, analyze and understand the problem.

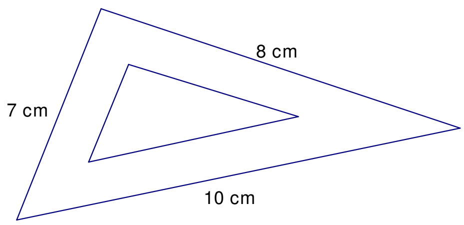

Figure 3

The teacher seeks to facilitate the exercise by means of explanations and justifications which are linked to the content regarding proportions, with the aim of producing reflection among the students in relation to the problem to be solved. Potential conflicts during the negotiation are minimized through routines and obligations, as can be seen in the following dialogue:

<u>Episode 4</u>

Teacher: There’s the figure right? What is the value of “x” and what is the value of “y” (he asks the group while he erases the board). What is the ratio? Remember that the ratio is the quotient of both quantities. What's up? You’ve already solved it (he says to a student who is approaching him and showing him his notebook, to whom he asks), What did you get as the ratio?

Student: I don’t understand (outloud)

Teacher: ¡Shhh!, Do you have your results yet?
Students: No
Teacher: What? Don’t you understand it?
Student: No, I don’t understand.
Teacher: It’s right here, (pointing to the sheets that he gave out)
Student: I don’t get it.
Teacher: Wait, (the teacher notices that the students haven’t understood the problem) OK, let’s see students! OK students, I think that the explanation that I am going to give you about proportions of a triangle will help you a lot. Let’s see, look, this is going to solve everything…

The teacher, upon discovering the conflicts, tried to quiet the student who had spoken out loud. Faced with his inability to give him an affirmative answer, he showed the student the sheets of notes that he had given him beforehand. Then, he gave a supposedly easier explanation of the problem. This interactive pattern was named the “*funnel pattern”* by Voigt (1985) and Bauersfeld (1995), that is to say, the type of interaction that is generated between the teacher and students. The actions that characterize said pattern are shown below and are those which coincide with the actions preformed by the teacher:

- The teacher gives the students a problem.
- The students are incapable of solving it.
- The teacher proposes easier questions which relate to the problem and whose solution leads to its resolution, but without the students risking minimum significant intellectual activity.

The teacher, as well as the students, tend to maintain their classroom behavior within predictable boundaries. When students break a rule, they do so only in a limited number of specific forms: they respond out loud without having been called upon, they talk amongst themselves, they look out the window, etc. One can suppose that the <<official>> rules of classroom behavior at this educational level are so well known by students that they are never explicitly mentioned during the course of the class.

In episode 5, reprimand is weaved into an extensive explanation given by the teacher. He senses that the students are having difficulty understanding the problem, which sets into motion an explanation of the proportions of a triangle. He feels that if they pay attention, they will be able to solve all of the problems. However, during the course of the explanation, he notices that a student is talking, and so he interrupts in order to reprimand the student *“what you are doing is using you hands, but your brain, your mind is somewhere else”*. It is worth pointing out that this incident did not seriously distract or interrupt the group and only a few students noticed what had happened.

Episode 5
Teacher: OK, hold on (the teacher notices that the students haven’t understood the problem). OK students! I feel that the explanation I am going to give you about triangular proportions will be of great use to you. Look, you are going to be able to solve all of them, just pay attention.
Students: No!

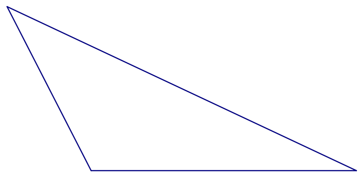

Figure 4

Teacher: We have an ordinary triangle (he draws one on the board), it can be right-angled like this one (he points to the other side) or it can be obtuse, like this one here because this one has an obtuse angle right? (He labels it with the letters *A, B, and C*) Or, you can use any letters you want. No, well that won't work…(a student is talking in class).

Student: Teacher, I am paying attention.

Teacher: What you're doing is using your hands, but your brain, your mind is somewhere else (annoyed, he speaks to the student who interrupted him).

Lemke (1997) has recorded observations of similar classes, and he found that talking in class has three main functions: Firstly, it provides a channel of communication between the students who are maintaining the dynamic of their interpersonal relationships, whether they are friendly, joking or hostile. Secondly, talking during class satisfies the need that students have to talk to someone else other than the teacher about what is going on in class in that moment of 'what page are we on? What does it say on the board? Is this the answer?' etc. The third essential function of talking amongst themselves is to distract themselves from the main activity of the class and talk to their neighbor about something completely different.

The medium term repercussion that I find in a classroom in which students do not talk amongst themselves is that there is little possibility that a group dynamic will exist, causing all interaction to be carried out through the teacher. Without the possibility to clear up doubts or receive support from their classmates or teacher, or to practice first what they want to say, it is very unlikely that they will say something in public. They will only hear the words of the teacher, not their own.

On the other hand, classroom negotiation can be concealed by an asymmetric power in the relationship between the teacher and the students. Certainly, the adaptations of the teacher and the students are motivated by different intentions and take diverse shapes. The students try to respond correctly to the teacher's questions, adapting their answers to the teacher's intentions. They try to indentify the teacher's expectations which differ from their previous knowledge in order to adjust them to the conditions of the math class. The teacher, although consciously trying to influence the mathematical content in play, is interested in the process of developing an adaptation of the student's answers.

In this process of teaching-learning, the teacher and the students have, from the beginning, very different experiences and knowledge (Ponte, 1994). For the teacher, mathematical concepts have a rich meaning, full of relationships with other mathematical concepts and processes. For the students, mathematical concepts have no meaning at the beginning. The negotiation of mathematical meanings in the classroom implies that each one of the participants, teacher and students, form their own meanings which are visible in the process. Through explanations, everyone can come to better understand the reference points of the other and their relationships with mathematical knowledge.

Conclusions

This investigation affirms how the teacher explains and justifies different forms of representation in order to achieve the intended articulation between that which is numerical and that which is graphical, which allows for the visualization of the proportionality and the symmetry of triangles in a geometric context. We can see that the teacher tries to relate the activity generated in the recording of graphic representation, to that which is numerical-algebraical, and in his explanations he appreciates the attempt to foster both registers. However, apart from the aspects related to the different dominions and the way to integrate them, these characteristics allow us to establish inferences about the teacher’s performance that bring us closer to, in some way, influencing the immediate future of his practice.

The episodes described show that some modifications are produced in the explanations based on the interaction brought about by a search for complementarity between the students’ and the teacher’s versions. Studies by Reséndiz (2006) found that the teacher’s interventions have a dual function: to solicit explanations and to try to guide them by regulating the course of the class.

In this respect, explanations by teachers can be classified in two moments of conversation which have very clear purposes: in the first moment, the initial departure context is created and in the second, certain questions and tasks are considered which the student needs to resolve. Once the initial starting point has been guaranteed, that which characterizes the professor in the development of the explanation is the formulation of a series of questions, which regularly obtain little or no participation from students. In this case, interaction between the teacher and student in order to encourage communication and the negotiation of meanings is not stimulated.

We would like to finish by highlighting the fact that ethnographic investigations are impossible to accomplish without the willingness and help of the teacher. They can help us to deepen our understanding, providing information which allows us to contribute to the practice all the way to professional development.

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