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Two Grade 5 teachers' enactment of mathematical problem solving and their classroom talk: contrasting approaches

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ABSTRACT. *The classroom practices of two Grade 5 teachers were analyzed to address the extent they reflect the intents of a mathematical problem solving curriculum. Observed lessons were video recorded, transcribed and coded into four main categories of heuristic-instruction, teaching of concepts and skills, going over assigned work and student activities. Classroom talk, particularly in the heuristic-instruction and student activities, revealed two contrasting approaches best described as traditional and more reform-minded, highlighting different understandings of what it means to enact a mathematical problem solving curriculum.*

Keywords: Primary mathematics teacher classroom talk, word problems, problem solving.

1. INTRODUCTION

Research on classroom practices is crucial to understanding the teaching/learning process within schools (Good and Brophy 2003; Goodchild and English 2002). There are various approaches. One particular approach is to start from a given curriculum mandated to be implemented in schools. To what extent would classroom practices *reflect* the mandates and bring about the learning as *intended* in the curriculum? The question is a broad one. This study focuses on the classroom practices of two Grade 5 teachers as they implement the mandated curriculum.

The Singapore mathematics curriculum has a pentagonal framework (Ministry of Education 2006, p. 6), conceived to show "the underlying principles of an effective mathematics programme." It places mathematical problem solving (MPS) as central to mathematics learning involving "the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems… dependent on five interrelated components, namely, *Concepts*, *Skills*, *Processes*, *Attitudes* and *Metacognition*."

There are many conceptions of what having MPS as a central focus means. For example, teaching *about*, teaching *for*, and teaching *via* problem solving (Schroeder and Lester 1989), and *problematizing* mathematics as a way to think about problem solving (Hiebert et al. 1996). Just these various conceptions of what it means to have MPS as a central focus would lead teachers to a wide variety of ways of *enacting* the curriculum. Stacey (2005), in her review of contemporary mathematics curriculum documents, raised the important question about whether and how such a framework might "influence teachers' understanding of the goals of teaching mathematics, and whether these different understandings make a real difference in the attention that teachers give to mathematical problem solving beyond the routine" (p. 345). Crucially, the question is how a curriculum framework influences teachers' classroom practices.

One particular broad perspective to explore this issue is the use of Cazden's (2001) notion of *traditional* and *nontraditional* lessons. Does the three-part sequence of teacher Initiate, student Response, and teacher Evaluate or Feedback (IRE/F) (Sinclair and Coulthard 1975; Mehan 1979) predominate in classrooms? Or has the nontraditional approaches advocating "inquiry", leaning more on reformed curricula made its way through the classroom? This paper focuses on two teachers involved in a project undertaken to address the above questions. Based on preliminary observations and review, the two teachers were selected to highlight contrasting styles of practice. Their classroom practices were studied with the purpose of understanding the process better through identifying predominating pedagogical features in classrooms particularly those that relate closely with MPS. Specifically, the research questions were: (1) How often do teachers *do* word problems in class and (2) What classroom discourse do teachers/students engaged in when they are *doing* word problem solving?

2. METHOD

The focus was on two teachers from two different schools. The two classrooms were observed over a two-week period. The researcher video recorded the lessons. The videos were streamed into computers and rendered into video compact disc format. They were transcribed, and the transcripts served as the data source for coding as well as investigations of classroom talk.

The coding scheme

The scheme was developed using the Grounded Theory approach (Glaser and Strauss, 1967) where ideas of pedagogical phases *emerged* as the videos were reviewed together with the transcripts. After several iterations, a coding scheme that segmented each lesson to five categories of action comprising *heuristic-instruction*, *teaching of skills and concepts*, *going over assigned work*, *student activities* and *others* (Ho and Hedberg, 2005), was developed. The heuristic-instruction category of action occurs when the teacher introduces a new word problem to be solved. It is divided into four subcategories based on Pólya's (1988) four stages, namely, understanding, planning, executing and reflecting. In the going over assigned work category, there are three types: reworking or going over the word problems thoroughly, procedural with the main focus on steps to solve word problems and quick answer checking. For student activities category, there are student presentation, group work and individual seat work. The last category is for events not coded in the previous four and it is mainly for more accurate account of time. It is divided into 'on' and 'off', with 'on' referring to events directly related to the lesson and 'off', not related. Altogether there are 14 subcategories within the five main categories.

The analytical framework

Classroom practices are multifaceted and too complex to be described completely. The approach adopted was to use the coding scheme to segment the lesson with a particular focus on the teaching/learning of word problem solving. Whilst there are diverse views of what the teaching/learning of problem solving might entail (Schroeder and Lester 1989; Hiebert et al, 1996; De Corte et al. 1999), here the scope is limited to segments of the lessons when the teacher presents a new word problem to the class. The conception of what explication of problem solving involves is defined by Pólya's (1988) four stages. Thus what is meant by teachers *doing* problem solving in class is defined as teachers presenting a new word problem and explicating it through Pólya's four stages. Through the coding scheme, the observed lessons were segmented and the relative amount of time spent on each category gleaned. On this basis the first research question was addressed.

To address the question of what classroom discourse do teachers/students engaged in when they are *doing* problem solving, it is noted that there are diverse methodological perspectives and procedures in investigating classroom talk (cf. Edwards and Westgate 1994). For this paper, a micro-ethnographic approach was adopted and classroom conversation related to problem solving was considered the unit of analysis. The structure and length of the utterances, the order in the flow of talk, and how teachers and students respond to each other as they talk and go about solving word problems. By looking at these conversational practices, the kinds of problem solving instructions in the different classrooms could be investigated, particularly the question of whether the lessons are *traditional* or *nontraditional* (Cazden 2001).

Based on initial observations, talk related to problem solving occurred mainly in the heuristic-instruction category and the student activities (presentation) category of action. The type of talk that began with the teacher introducing a new word problem and the following exchanges (if any) with students were coded in the heuristic-instruction category as Understanding, Devising a plan, Carrying out the plan and Looking back (Pólya 1988). In the student activities (presentation) category, the form usually involved students presenting their solutions to assigned word problems. The focus was on the details of these exchanges. By examining the ways participation was structured around these exchanges, the contrasting styles of *engaging* in solving problems of the two teachers could be investigated.

3. Results and analyses

The observations and results based on the coding scheme

At the time of observation, both teachers were teaching the topic on fractions. A total of nine lessons of Betty and eight of Chan were observed and video recorded. The average length of the Betty's lesson was 51 minutes while Chan's was 54. The lessons were coded based on the scheme. The following Table 1 shows the details:

Table 1. *Amount of time each teacher spend on each category of action in h:mm:ss (percentage of total)*

Teacher	Heuristics-instruction	Teaching concepts & skills	Going over assigned work	Student activities	Other events	Total
Betty	3:04:13 (40%)	0:37:35 (8%)	0:53:20 (12%)	2:45:20 (36%)	0:17:25 (4%)	7:37:53 (100%)
Chan	0:12:15 (3%)	1:02:15 (14%)	0:48:45 (11%)	3:42:50 (51%)	1:29:10 (20%)	7:15:15 (100%)

On the heuristics-instruction category of action

Betty spent about 40% of her class time in the heuristics-instruction category, in other words, presenting new word problems, and explicating the solutions. Chan was coded as spending only 3% of her overall class time in the heuristic-instruction category. The relative amount of time each spent on Pólya's four stages is shown in Figure 2. While it is difficult to compare the two because of the big difference in amount of time each spent in this category, two pertinent points can be gleaned. First, both spent proportionately similar amounts of time in 'Understanding' and 'Planning', and second, the differences in emphasis on 'Executing' and 'Reflecting' – Betty's emphasis was more on 'Executing' while Chan's relatively small amount of time spent on 'Executing' and large amount spent on 'Reflecting' suggests that her emphasis leaned less towards showing the steps to execute the problem and more towards reflection and thinking about the problem.

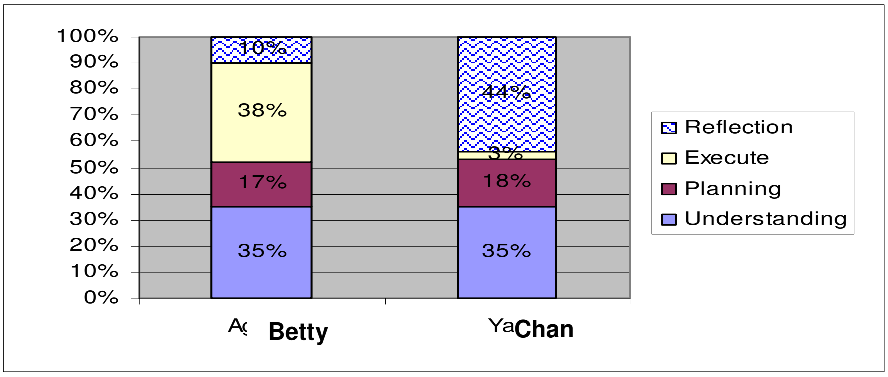

Figure 1. Proportion of time the teachers spent on each stage of heuristics-instruction

On student activities

Overall Betty spent about 36% (or about 2 hr 45 min) of the total observed class time on student activities while Chan spent slightly more than half her class time (3 hr 43 min). Interestingly, all of Betty's student activities involved only individual seat work. She did not task the students to do group work nor presentation. For Chan, she spent about 73% (or about 2 hr 43 min) on seat work, 10% group work (or about 22 min) and 17% student presentation (or about 38 min).

So, in terms of talk related to word problem solving, Betty's students did not have the opportunity to talk about problems they solved, while Chan's students were tasked to solve problems in groups and present their solutions in front of the class.

How Betty and her class talked about solving word problems

The dominant pattern for classroom talk about problem solving that was observed in Betty's class was teacher-centered. It occurred only in the heuristic-instruction category. The three-part IRE/F format prevailed. Typically she stood in front of the class, her students seated in double columns, all facing her. She began by reading or having a student read a new word problem, and proceeded to explicate the problem. Often she asked questions and two or three students would respond without her calling upon them, and sometimes she called upon volunteers to respond one at a time. All exchanges were teacher-initiated.

The following Table 2 shows the number of turns, words and average number of words per turn in the heuristic-instruction category in the series of nine lessons. She spent time in the heuristic-instruction category in all except Lesson 8 where she did mainly 'Going over assigned work'. So doing new word problems were very much part of her regular lessons. In terms of number of turns, Betty took about 50% more turns than students. Her average length of words per turn is almost 12 words compared with her students' 2.7. Most of her students' responses were one word or one answer.

Table 2. *The number of turns, words and average number of words per turn in the heuristic-instruction category of Betty*

Heuristic-instruction category		**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**L9**	**Over-all**
Student	**# of turns**	92	50	120	164	100	51	55	0	97	**729**
	# of words	174	140	259	534	349	132	123	0	287	**1998**
	ave. # words	1.9	2.8	2.2	3.3	3.5	2.6	2.2	0.0	3.0	**2.7**

	per turn										
Teacher	**# of turns**	112	69	170	239	187	60	82	0	174	**1093**
	# of words	1572	674	1725	2769	2030	1030	1010	0	2155	**12965**
	ave. # words per turn	14.0	9.8	10.1	11.6	10.9	17.2	12.3	0.0	12.4	**11.9**

A typical exchange between Betty and her class in a heuristic-instruction category is illustrated by the following transcripts extract. About two minutes into Lesson 4, Betty introduced a new word problem and began her exchange with her students.

Transcript extract 1:

(S indicates one student responding; Ss, more than one; names of students are used if it is known.)

Turn	Who	Exchange	IRE	Codes
1	Teacher:	We have, Mrs Lim bought 120 eggs. She used 2/3 of them for baking cakes and used 1/4 of the remainder for baking cookies. How many eggs has she left? Can you find the answer?	I	Understanding
2	S:	20	R	Understanding
3	Teacher:	Mrs Lim bought, Mrs Lim bought 120...	I	Understanding
4	Ss:	Cakes	R	Understanding
5	Teacher:	Cakes. Okay, look at this. 120 eggs. Okay, so you know she bought 120 eggs. She used 2/3 of them. For what?	F I	Understanding
6	Ss:	Baking cakes	R	Understanding
7	Teacher:	For baking cakes. Okay…	E	Understanding
8		Okay, so if I want to draw the model, how many parts do I have to divide? ((starts to draw a rectangle for a model))	I	Planning
9	Ss:	3	R	Planning
10	Teacher:	We have to divide it into how many parts?	I	Planning
11	Ss:	3	R	Planning
12	Teacher:	3, okay, where do we get the 3 from?	E; I	Planning
13	S:	From 2/3	R	Planning
14	Teacher:	From?	I	Planning
15	S:	2/3	R	Planning
16	Teacher:	From this right? ((teacher is underlining the 2/3 on the board)) from 2/3 right? Okay. Then I divide it into 3 parts. And what do I do next?	I	Planning
17	S:	Shade.	R	Planning
18	S:	Shade 2 parts.	R	Planning
19	Teacher:	Shade 2 parts, ok. ((teacher shades)) Then?	E; I	Planning
20	S:	Desmond know	R	Planning
21	Teacher:	Desmond.	I	Planning
22	Desmond:	Label	R	Planning

Turns 1 to 7 was coded as ‘Understanding’, i.e. understanding the given word problem. It was straightforward; Betty read the problem and highlighted its salient aspects. In Turn 8, she started to devise a plan of drawing a model to solve the problem. The crucial *thinking* about *how* to proceed with the problem was decided at this point by the teacher (sic). Betty continued to lead the students through ‘Planning’, mainly in the IRE/F format, and at Turn 74 when the class was ready to carry out the plan, she initiated the ‘Executing’.

Transcript extract 2:

Turns	Who	Exchange	IRE	Codes
74	Teacher:	Okay? How would write the steps out?	I	Executing
75	S:	3 units equal to 120 eggs.	R	Executing
76	Teacher:	Okay, you say, 3 units, equals to 120, …	E/F	Executing
77	S:	1 unit equal to 120 divided by 3 equal to 40.	R	Executing
78	Teacher:	Alright, what else do you need to do?	I	Executing
79	Teacher:	Is that all? From here, where can we go? ((boy raises hand))	E; I	Executing
80	Teacher:	Guo Wei		Executing
81	S:	4 units, 4 units equals to 40	R	Executing
82	Teacher:	Okay, so we talk about 40, erm, okay, from here, do we write the statement?	E; I	Executing
83	S:	Yes	R	Executing

Betty proceeded in a similar way for another 40 turns before finishing the problem in Turn 126 with the remark: "Okay, we will (have) work(ed) it out step by step okay." This remark reflected Betty's approach to problem solving and the way to go about teaching and guiding students. She continued with another three more word problems immediately thereafter, explicating it in a more or less similar fashion. After which the students copied the solutions written on the board, and continued on with seat work solving two or three more similar problems on their own.

Betty's way of explicating problem solving and the structure of her classroom talk fit a *traditional* lesson format. The scope for student 'maneuvering' in the problem presented was limited to responding to her questions, leaving little or no room for "inquiry". Interestingly, her students' talk was all directed towards her and *rarely* at each other, reinforcing the notion of her classroom as a collection of individual learners.

How Chan and her class talked about solving word problems

The pattern for classroom talk about problem solving in Chan's class was a little more complex. In the observed series of lessons, she spent about 12% of her class time in the heuristic-instruction category (compares with Betty's 40%). Her Lessons 1 and 2 focused more on concepts and skills related to fractions. She started word problems only from Lesson 3. Her talk in giving instructions on problem solving was relatively short (in terms of the number of words).

Table 3. *The number of turns, words and average number of words per turn in the heuristic-instruction category of Chan*

Heuristic-instruction category		**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**Overall**
Student	# of turns	0	0	12	4	11	0	0	0	27
	# of words	0	0	50	32	46	0	0	0	128
	ave. # words per turn	0.0	0.0	4.2	8.0	4.2	0.0	0.0	0.0	4.7
Teacher	# of turns	0	0	45	17	12	0	0	0	74
	# of words	0	0	721	208	250	0	0	0	1179
	ave # words per turn	0.0	0.0	16.0	12.2	20.8	0.0	0.0	0.0	15.9

Table 4. *The number of turns, words and average number of words per turn in the student activities (presentation) category*

Chan’s Student presentation		**L1**	**L2**	**L3**	**L4**	**L5**	**L6**	**L7**	**L8**	**Overall**
Student	# of turns	0	0	0	0	12	50	42	0	104
	# of words	0	0	0	0	201	1065	838	0	2104
	ave words per turn	0.0	0.0	0.0	0.0	16.8	21.3	20.0	0.0	20.2
Teacher	# of turns	0	0	0	0	16	51	25	0	92
	# of words	0	0	0	0	158	854	305	0	1317
	ave words per turn	0.0	0.0	0.0	0.0	9.9	16.7	12.2	0.0	104

Reading

Table 3 together with Table 4, the talk on heuristic-instruction in Lessons 3 and 4 was lead by the teacher (see Transcript Extract 3). In Lesson 5, Chan went through some assigned word problems, did one more word problem, and then assigned problems as group work for the students to present their solutions, all within 52 minutes. Only one group presented their solution because of time constraints. Lesson 5 seemed to serve a transition phase – Chan talked less and made provisions for her students to talk more about problem solving. By Lessons 6 and 7, students (as a group) talked more than the teacher both in terms of the number of turns and the average number of words. She also stopped solving problems as she had in Lessons 3 to 5, and switched the focus on students doing, solving and talking about problems. Chan’s role changed from one of the ‘sage at the centre stage’ to one of mediator and facilitator of the whole class participation in problem solving talk. Students’ talk was not just directed at the teacher, but the whole class. There were also provisions for students to initiate and ask questions of their classmates (see Transcript Extract 4). She wrapped up the topic on fractions in Lesson 8.

The following excerpt taken from Lesson 5 shows how Chan talked about a problem taken from the textbook. She used the IRE/F format sparingly. The crucial point in the exchange was at the end of Turn 191 where she asked “… what would they do?” The lead up to this point was done in such a way that it seemed *obvious* that the students would respond with: “Cut the pie.”

Transcript Extract 3:

Turn	Who	Exchange	IRE	Codes
191	Teacher:	4 boys shared 2/3 of the pie equally. You can see the pie drawn there, okay mine is not a proper circle, 2/3 of the pie ((draws a circle on the whiteboard, drawing lines in it to divide into 3 equal parts))		Understanding
		… that means actually this part is missing ((erases 1/3 of the circle she drew on the board away))		Understanding
		They only have 2/3 of the pie. By drawing this dotted line, you actually can see the whole pie ((draws a curved dotted line in replacement of the line erased)) So out of this 2/3 of the pie, 2 boys are supposed to share ((shades 2/3 of the pie))		
		If, if they have the pie in front of them, and they want to share immediately, what would they do?	I	Planning
192	Ss:	Cut the pie…	R	Planning
193	Teacher:	Yah, into equal parts, alright. Since there are two parts, there are 4 of them, 2 parts make into 4 equal parts, so this is what they will do ((draws 2 lines on the shaded area to separate the area into 4 equal parts)).	E/F	
		So it becomes 1,2,3,4 equal parts…		Executing

Chan's explication of a problem did not have as many exchanges of turns as Betty's. She would explain, clarify the problem and devise a plan. Her execution of the plan was usually done quickly as she did not focus too much on the calculational steps. Between 'Executing' and 'Looking back', she appeared to emphasize more on looking back at the solution, rather than carrying out the 'mechanics' of calculating an answer.

In the following excerpt from Lesson 6, students have been doing some prior group work solving word problems and preparing for their presentation. The extract is from the second group of students about 7 minutes into the lesson, to highlight the transited phase where the classroom talk centers around students.

Transcript Extract 4:

(Note: S# refers to a particular student identified, * in IRE/F to indicate students')

Turn	Who	Exchange	IRE	Codes
30	S3:	Page 60 question 3. Mr Wang had 400 dollars. He spent 2/5 of it on a vacuum cleaner and 1/4 of the remainder on a fan. How much money had he left?		Student presenta-tion
31	S4:	1 whole stands for (the total sum of money).( ). 1 whole stands for (the total 400) ( ). 2/5( ). 1 whole stands for ( ). 2/5 stands for the money..fraction ( ). 1 whole stands for the total … two fifths stands for the amount of money spent on the vacuum cleaner...( ).		
32	Teacher:	Is 2/5 the amount of money?	?	
33	S4:	Fraction ((pointing to the transparency)) ( ) the remainder ...( )…spends 1/4 of ( ) so one fourth times the remainder equals to 60. 60 times 3 equals to 180. He had $180 left. (….)		
34	Teacher:	Why look at me? ((responding to the whole class who didn't seem to understand S4's explanation)) You all should look at (the thing) and see if you understand. Whatever you don't understand, raise your hand and ask.(.) Benedict?		
35	Benedict:	Why do you put 60 times 3? ((directed at S4))	I*	
36	S4:	60 times 3 ( ) one fourth the remainder…( ) So if ( )	R*	
37	Benedict:	That means you are saying you are (calculating) [three fourths?	F*	
38	Teacher:	[Okay, it's "quarter", don't say fourth, fourth, fourth. It's "quarter"	E	
39	Teacher:	Okay, you understand already? You know why? Even though Benedict knows, understands, if you still don't understand, you still can raise it up ah. ( )? Harvis, any question?	E I	
40	Harvis:	Same thing.	R	
41	Teacher:	Same thing. So now you understand. Sure? Zhongxi, do you understand?	E I	
42	Zhongxi:	Yes.	R	
43	Teacher:	Okay now you explain to me. The last step. [His last step. Not yours.	I	
44	Zhongxi:	[huh? He is telling us that 60 equals to 1/4. Then he times 3 to get 3/4 lah.	R	
45	Teacher:	Why times 3?	F/I	
46	Zhongxi:	(….) He needs to find out how much money Mr Wang left.	R	

In the preceding excerpt, Turns 30 to 37 did not quite fit an IRE/F structure. Chan still asked questions (e.g. in Turns 32 and 45), but more to probe further what students said rather than to ask "display" questions to which she already knew the answer. In Turn 31, S4's explanation of their group's solution was somewhat unclear, and many students appeared puzzled and looked to the teacher to explain (briefly before Turn 34). Notably Chan did *not* overlay her students' explanation with her own. She redirected the attention back to her students, that the questions of seeking clarification *should* be directed at the student presenters. Her students' uptake on this was notable. Student Benedict directed his question at S4 who responded at Turn 36 (albeit not audible to the transcriber). Student Harvis who had raised his hand earlier was called upon to ask questions of S4. He responded that his question was similar to Benedict's. And Zhongxi did not get away with a 'Yes I understood'. Instead Chan asked him to explain S4's last step. He did for the next few turns. These exchanges between teacher-student and student-student were typical features during the student activities (presentation) phase. Such *nontraditional* student discourse can occur (Cazden 2001), and this excerpt clearly is an example. S4's not-too-clear exposition of his group's solution was *accepted*, and students were *redirected* to ask the group instead of the teacher for clarification. And that Zhongxi had to explain not his own but S4's last step in the solution brings to bear the mathematical community in the making.

4. SUMMARY AND CONCLUSIONS

The first purpose of this study was to explore classroom practices. In particular, it sought to address the question of how often teachers engaged in solving word problems in class. By focusing on the practices of two teachers, some attention is drawn to the ways in which they differed in their approaches. It is important to note that both differences and similarities are expected. Based on the coding scheme, the series of lessons observed were segmented. The different proportions of time spent on the different phases suggest different emphasis. Through the series of observations, investigations showed the contrasting ways each teacher approached mathematical problem solving.

The details of their differences were examined through their discourse, particularly when they talked about solving word problems. For Betty, she introduced new word problems as part of her regular lessons. The prevailing format she used was the IRE/F. Her "display" questions occurred almost every other utterance and her students' responses and talk were limited two or three words. All her student activities were individual seat work where students copied solutions on the board, and finished problem exercises, reinforcing the idea of her classroom as a collection of individuals.

Chan's approach contrasts markedly from Betty. The sequence of her lessons suggests that she started each topic focusing on the related concepts and skills. She introduced word problems only in her third lesson. Her time spent in the heuristic-instruction category was little compared to Betty. Instead, by the fifth lesson, she shifted the main part of the classroom talk to the students for them to present their solutions as well as to question each other's work. She continued in a similar fashion in Lessons 6 and 7, allowing for her and her students to build their classroom as a mathematical community rather than a collection of individual learners (Lampert et al. 1996, p.739).

The exploratory study of these two teachers is not to highlight one approach is necessarily better than the other. The coding scheme and classroom talk analysis allowed for some exploratory investigation in the classroom practices of the teachers. The salient features of Betty and Chan's classroom discourse and practices do suggest what Cazden (2001) would call traditional and non-traditional lessons. Betty's predominant use of IRE/F would mark her approach as traditional. In terms of the first research question of how often she did problem solving in class, the answer would be very regularly or about 40% of her class time. In contrast, Chan's frequency of giving heuristics-instruction appeared to be proportionately much less at 3%. However her approach seemed to promote classroom discourse in which her students presented solutions, listened and responded to one another. They also initiated questions, and tried to convince themselves and one another of the validity of their solutions, fulfilling the partial list of National Council of Teachers of Mathematics’ set of guidelines (1989 and 1991), as well as part of the overall aims of mathematics education spelt out by MOE (2000, p.4). This was in marked contrast with Betty’s more traditional discourse. To this extend Chan’s classroom practices *was* closer to the mandates of the curriculum of ‘inquiry’, ‘communication’; closer to the *spirit* of problem solving. Notwithstanding the above, both teachers have in their own way focused on problem solving in their classes. Their differences no doubt have to do with their particular conceptions of what problem solving entails (Schroeder & Lester, 1989).

Hence to address the question of how the *intended* curriculum of mathematical problem solving gets enacted in the classroom, looking at the frequency of teacher lead problem solving alone would suggest only an incomplete picture. The *nature* of classroom talk surrounding problem solving would provide some pertinent details, offering perhaps a more refined picture of how teachers enact the curriculum focusing on problem solving.

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REFERENCES

Cazden, C. (2001). *Classroom Discourse* (2nd Ed.). Heinemann, Portsmouth, N. H.

De Corte, E., Verschaffel, L., Lasure, S., Borghart, I. & Yoshida, H. (1999). Real-world knowledge and mathematical problem solving in upper primary school children. In J. Bliss, R. Säljö & P. Light (Eds.), *Learning sites: social and technological resources for learning* (pp. 61-79). Amsterdam: Pergamon.

Edwards, A. D. & Westgate, D. P. G. (1994). *Investigating classroom talk* (2nd Ed.). London: Falmer Press.

Glaser, B. & Strauss, A. (1967). *The discovery of grounded theory*. Chicago: Aldine.

Good, T. L. & Brophy, J. E. (2003). *Looking in classrooms* (9th Ed.). Boston: Allyn & Bacon.

Goodchild, S. & English, L. (Eds.) (2002). *Researching mathematics classrooms*. Greenwich, Connecticut: Information Age Publishing.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A. & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, 25(4), 12-21.

Ho, K. F. & Hedberg, J. (2005). Teachers' pedagogies and their impact on students' mathematical problem solving. *Journal of Mathematical Behavior*, 24 (3&4), 238-252.

Lampert, M., Rittenhouse, P. & Crumbaugh, C. (1996). Agreeing to disagree: Developing sociable mathematical discourse. In D.R. Olson & N. Torrance (Eds.), *Handbook of education and human development: New models of learning, teaching, and schooling* (pp. 731-764). Cambridge, MA: Blackwell Publishers.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom.* Cambridge, MA: Harvard University Press.

Ministry of Education, Singapore (MOE) (2006). *Primary Mathematics Syllabus*. Singapore: Curriculum Planning and Development Division, Ministry of Education.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: author.

National Council of Teachers of Mathematics (1991). *Professional Standards for Teaching Mathematics*. Reston, VA: author.

Pólya, G. (1988). *How to solve it – a new aspect of mathematical model* (2nd Ed). Princeton, NJ: Princeton University Press.

Schroeder, T. L. & Lester, F. K. (1989). Developing understanding in mathematics via problem solving. In P.R. Trafton & A.P. Shulte (Eds.), *New directions for elementary school mathematics* (pp. 31-42). Reston, VA: National Council of Teachers of Mathematics.

Sinclair, J.M. & Coulthard, R.M. (1975). *Towards an analysis of discourse: The English used by teachers and pupils*. London: Oxford University Press.

Stacey, K. (2005). The place of problem solving in contemporary mathematics curriculum documents. *Journal of Mathematical Behavior*, 24, 341-350.