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Motion sensor: a learning tool for reading and understanding function graphs

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ABSTRACT
The aim of the lead experimental research was to underline improvements about the reading and the understanding of function graphs, through didactic activities in the laboratory, using a sensor motion. Particularly, there were improvements in the reading of maxima of functions and of interval sizes on 10 year-old students. While the activities developed with students of the 4th year of high school (17 years) conducted satisfying results about the reading of relative maxima and growth of function and about forming hypothesis on the base of experimental data.
Such research, born from a historical-epistemological analysis of the function concept, finds its basis in the *Embodiment Theory and Metaphorical Thought* of Lakoff e Nuñez and in the numerous works about the use of Microcomputers-Based Laboratory (MBL).
The experimental methodology references *Brousseau's Theory of Didactic Situations*; it is based on the qualitative analysis of lessons and on the use of the software *Chic* of implicative statistics (Gras, 2008), for the interpretation of the pre-test and the post-test.

Key Words: function, graph, motion sensor, learning.

1. INTRODUCTION

The research work consists of the analysis of didactical activities, conducted in a 5th class of Primary School (10 years) and in a 4th class of High School (17 years). These activities were based on the study of space-time graphs and velocity-time, representative motions of bodies, produced in real time through the use of a motion sensor interfaced with a computer. This instrument is utilized in Physics laboratories to study rectilinear motion and its graphical representations. It reveals, instant by instant, through the emission and the reception of ultrasounds, the distance of bodies put in front of it and it transmits the measurements to the computer that, making use of opportune software (called *Data Logger*), visualizes the data in tabular and graphic way.
By achieved results it was observed that not only the students improved reading and understanding of motion graphs (Thornton & Sokoloff, 1990) but they improved these competences with function graphs in other context.
This research is part of a doctoral working thesis, regarding *The Acquisition of the Concept of the Function and of its Representations.* The idea is that one of the approaches to introduce the concept of function could be the graphical one, using graph representing of real physics phenomena. This proposal comes from some considerations of a historical-epistemological analysis of the concept of function, which finds its historical and contextual origins in the ambit of kinematics. Considering this analysis and taking into account the fact that history, «as a source of problems, from the teacher's point of view can become an instrument of projection and for the student an occasion to give meaning to mathematical concepts» (Spagnolo, 1998, p.76), we propose introducing the concept of function through a physical approach and graphical representation.
Nevertheless, in this paper the obtained results are relative to the reading and comprehension of Cartesian graphs. These competences are part of the prerequisites necessary for the introduction of the concept of function through a graphical representation.

1.1 SOME DIDACTICAL CONSIDERATIONS

To clarify the connection between the competences with regard to the reading and the understanding of motion graphs and of mathematical functional graphs, we made a comparison between them. The table below shows this analysis:

#	Mathematical competences	Physics competences
1	Reading the coordinates of points of a function graph	Reading the values of spatial variable in relation to the values of temporal variable
2	Reading extremes and size of intervals	Reading space and time of departure and arrival, the length of space and the time spent
3	Reading the correspondence between intervals of the independent and dependent variables	Reading the correspondence between spatial intervals and temporal intervals
4	Distinguishing among increase, decrease and constancy of a function	Distinguishing between motion of approach, motion of leaving and situation of still bodies
5	Individuating absolute maximum and minimum of a function	Reading absolute maximum and minimum distance reached with respect to the system of reference
6	Individuating relative maxima and minima of a function	Reading points of inversion of the motion
7	Confronting the slope of a curve in different tracts	Confronting the velocity of a body in different tracts of motion
8	Forming hypothesis and conjecture	Forming hypothesis and predictions based on experimental data

2. THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

The motion sensor is one of the MBL tools (Microcomputer Based Laboratory), introduced into physics teaching in the 90's to improve the students' learning and comprehension of physics concepts (Thornton & Sokoloff, 1990).

The advantages of the use of the MBL tools are multiple as they allow to visualize in tables and graphs the experimental results in real time and to analyze of such data. This facilitates the comprehension of abstract representations as the data are revealed and represented in real time and students can make observations on the physical phenomenon and interpret, discuss and analyse the data as they see fit (Tinker 1996, Thornton 1997).

In particular, the possibility for the students to visualise and analyse the graphs of the bodies in movement or of objects physically perceptible at a sensory level, finds strong support in the cognitive theories *of the Embodiment of the mind,* for which «the detailed nature of our bodies, of our brains, and of our daily functioning in the world structures human concepts and reasoning», and from *Metaphorical Thought*, according to which «for the most part, human being conceptualize abstract concepts in concrete terms, utilising ideas and models of reasoning founded on a sensor-motor system» (Lakoff & Núñez, 2005, p.27). Moreover, «the functions on the Cartesian plane are often conceptualized in terms of motion on a route» (Lakoff & Núñez, 2005, p.70) and motion sensor induces this type of conceptualisation so that the student sees the graph constructed under his own eyes as "motion of a point that leaves a wake". This process facilitates the acquisition of the ability to read graphs of function and the learning of such concept so that «revealing the cognitive structure of mathematics renders it decisively more accessible and understandable» (Lakoff & Núñez, 2005, p.30).

As above, under the hypothesis that *the motion sensor is a good learning tool to study kinematics graphs*, we put forth the following research question: *Using the motion sensor to learn to study kinematics graphs, will the student acquire competences relative to reading, comprehension and making conjectures about graphs of mathematical function?*

3. EXPERIMENTAL WORK AND RESEARCH METHOD

The experimentation consisted of a laboratorial lesson[1] of two hours, where the students compared reading and prediction of the graphs realised with the motion sensor and the software *Logger Lite*[2].

The research methodology adopted is *Theory of Didactic Situations* by Brousseau (Brousseau, 1997). The laboratorial lesson was preceded and followed by the administration of a test, with the aim of evaluating the a-priori and a-posteriori state of the students. The lessons were conducted positing question-stimulus to the students so as to solicit observations and discussions between pairs and with the teacher, allowing for the active construction of knowledge (Kilpatrick, 1987).

The experimental work was leaded in two classes:

1. May 2007, 5th class of Primary School (10 year), *School: II° Circolo of Villabate (PA) "G. Rodari", Italy;*
2. December 2007, 4th class of High School (17 years) of type classical lyceum, *School: Liceo Classico "Scaduto", Bagheria (PA), Italy.*

It was decided to perform the research activity on students of different grades to confront the obtained results, which were thought to be different cause of the age of students and their courses of study. In fact, before the experimental work the students had got different competences, showed in succession:

Competences possessed by students before the experimental work:	
5th class of Primary School	**4th class of High School** (classical lyceum)
1.Knowing, representing on a straight line and operating with positive decimal numbers 2.Knowing and operating with the Cartesian plane (it was utilised, for instance, to study and to represent translations and similarities of images) 3.Knowing, from experiences lived in physical education and in every day life, motion of his own body and of other bodies	1. Knowing, representing on a straight line and operating with real numbers 2. Knowing and operating with the Cartesian plane (studied and visualized various times during the scholastic curriculum) 3. Knowing, by experiences lived in physical education and in the life of every day, motion of his own body and of other bodies 4. Knowing analytical and graphical (space-time) representations of the rectilinear uniform motion and of the rectilinear uniformly accelerated motion

3.1 PHASES OF THE DIDACTICAL ACTIVITY

The phases of the didactical activity for both the classes were the following:

1. Discovery of how the motion sensor works, by the observation of graphs and tables representing a student's motion, and reflections on the variable studied by the sensor
2. Prediction and reading of the graphs of rectilinear student's motion of three types:
    a. Leaving motion from the sensor
    b. Approach motion to the sensor
    c. Situation of still body with respect to the sensor
3. Prediction and reading of the graphs of various rectilinear motions of students, combining the three motion typology of phase 2.

During phase **2** the students observed and calculated space and time of departure and arrival, the length of space and the time spent. In phase **3** these values were observed and calculated for every tract of leaving, approach or stilling of the curve of the motion. Also the maximum

[1] Lessons was conducted by the teacher-researcher M. L. Lo Cicero

[2] In the research we used instruments and *Data Logger* from the Venier Software & Technologies. In the same way we could have used instruments and software from other equally valid companies.

and minimum distance reached with respect to sensor was read. The students noted that the slope of every tract of the curve depended on the corresponding velocity of the student. Then the students were asked to make a relationship between spatial intervals and temporal intervals about tracts of a curve and to make comparisons.
Moreover the Secondary school students calculated the mean velocities and compared them and the observations about the slopes of the tracts of the curve with the graphs velocity-time.
This was followed by a phase **4**, in which the students observed graphs of rectilinear uniform motions and uniformly accelerated motions of a train on tracks, developing the activity described above.
The Primary school students met difficulties in calculating the length of space and the time spent. To get over these difficulties, they had to put up a metric scale on the pavement to compare the space physically run with the graphic representation. The overcoming of this obstacle can be explained with the cognitive theories *of the Embodiment of the mind.* In fact, students lived through their body the measurement of the space.
The motion sensor induced curiosity and desire to learn in the students. They were encouraged to experiment with typologies of motion from these same suggestions and to compare the graphics produced with their own predictions. It be noted that the process of prediction is important to acquire the competence of *forming hypothesis on the base of experimental data.*
A qualitative analysis of the laboratory lessons was developed but it is not decrypted in this paper. Only the results of the quantitative analysis of the tests are shown here.

3.2 TEST

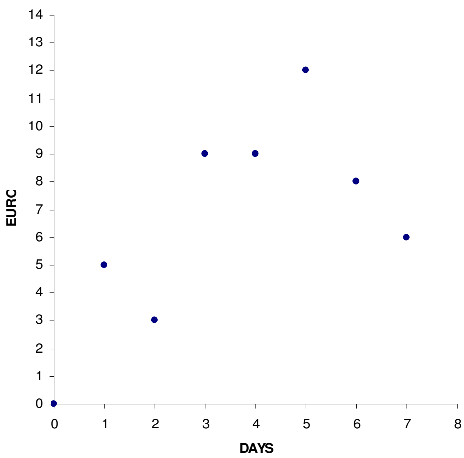
A test was administered before and after the laboratorial lesson, to investigate the a-priori and a-posteriori students' competences. The pre-test was equal to the post-test. The students worked individually, they were not allowed consulting books or notes. They were given sixty minutes.
In the tests of both classes there were exercises concerning reading of space-time graphs but the typology of graphs proposed and of questions were different in the two classes. In fact, in the kinematics part, the questions were open in the Primary school test and closed in the other one. It was made this chose because the Primary school students did not study kinematics in systematic way before, so it necessary to make the cognitive resources emerge liberally. This process was stimulated by the comparison among graphs. Instead, in the case of students of Secondary school the acquisition of specific competences was investigated, through questions expressed in scientific language, familiar to the students jet. In the following table the items of the test are summarized:

Test for students of Primary school	Test for students of Secondary school
• Two exercise of reading and two of writing of points on a Cartesian plane • Two questions about understanding of a *euro-days* graph • Request of commenting and comparing on the three graphs representing a rectilinear uniform motion of leaving, a rectilinear uniform motion of approach and a still body. • Request of commenting on two graphs of rectilinear uniform motions, in which the time intervals were the same but the covered distances were different • Request of giving a definition of velocity	• Five questions about understanding of a *euro-days* graph • Questions about four types of motion: constant motion, rectilinear uniform motion, accelerated rectilinear uniform motion and various motion. They were requested to explain the following characteristics: the space covered, the time spent, the space of departure and arrival, the maximum and minimal distance reached with respect to the system of reference, the type of motion, the mean velocity and the Law of the Time.

In both cases, the obtained results have confirmed the effective validity of the utilization of motion sensor as learning tool of kinematics competences. So, in this paper we would like to focus the reader's attention on our research results about the understanding of a graph representing a phenomenon of real life. So we will show the results about the test's item: *euro-days*. This item is formulated in natural language and it consists of the interpreting of a Cartesian graph representing a situation of every-day life.
It the following the item *euro-days* is showed:

Sara's dad decided to reward his daughter every time she got a good grade at school by giving her five euros, which she could decide to spend or save as she pleased; but this would be her only source of income. The adjacent graph shows the money Sara possessed on each day of a week. Observe it and answer the following questions:

a. How many euros did Sara have on the 4th day? *(C:1)*
b. On which day/days did Sara surely get a good grade? *(C:6)*
c. Knowing that on the second day Sara didn't get a good grade, how much money did she spend that day? *(C:2)*
d. Could she have gotten a good grade on the 6th day? (Justify your answer.) *(C:8)*
e. On which day/days did Sara posses the most money? *(C:5)*
f. On which day/days did Sara accumulate more money's quantity in comparison to the previous day? *(C:7)*



The symbols into round parenthesis near the questions, that are "*(C:i)*" with *i=1,…,8,* were inserted for the readers of this paper to focus the mathematical competences that questions wanted to investigate. In fact, the writing *(C:i)* suggests that question wanted to investigate the mathematical competence number *i.* (We refer to the competence listed in paragraph 1.1.) For example, the question *a* wanted to investigate the acquisition of the competence number 1.
The description and the graphical representation of the phenomenon, showed above, are the same in the tests of Primary and Secondary school. But, the number of questions is different in the two tests. In fact, in the Primary school test there were only questions ***b*** and ***c***, while the Secondary school test was complete like above. The reason of this difference is due to the fact that we chose to focus the attention on some competences in Primary school. However, in Primary school test, question ***a*** was profusely substituted by two specific items about reading of Cartesian coordinates of 10 points. The outcomes were that, in the pre-test, only 1 student wrongly read one point and another student wrote wrong coordinates of two points. So, these items were kept off from the post test.

3.3 A-PRIORI ANALYSIS OF STUDENTS' BEHAVIOUR

As it is indicated by *Theory of Didactic Situations*, we made an a-priori analysis of students' behaviour in working out the test. In succession, for reasons of space, only the students' behaviours extrapolated from protocols are reported.

Behaviours of Primary and Secondary school students:

B1 Correct reading of the value of the ordinate in correspondence with abscissa [a: 9][3];
B2 Correct identification of the days corresponding to the relative maxima [b: 1,3,5]

[3] In square parenthesis we indicate the letter of the question and the answer of the students to that question.

B3 Confusion between the concept of relative maxima and of absolute maximum [b: 5]
B4 Writing, beside of the days which correspond to the relative maxima, also, of the 4th day, in which the euro remained constant [b: 1,3,4,5]
B5 Correct identification of the size of the interval [c: 2]
B6 Confusion between the concept of interval and of value of the coordinate [c: 3]

Other behaviours of Primary school students:

B7 Writing all the days except the absolute minimum [b: *1,3,4,5,6,7*]
B8 Writing the highest values [b: *3,4,5*, or, *5,3,4,6*]
B9 Writing of the days without apparent criterion [b: *4, 6, 7,* or, *1, 3, 4, 6, 8*]

Other behaviours of Secondary school students:

B10 Correct identification of the absolute maximum [e: *5*]
B11 Correct identification of the day corresponding to the relative maximum with major degree of growth on left neighbourhood of size "a day" [f: *3*]
B12 Confusion between the absolute maximum and the day corresponding to the relative maximum with major degree of growth on left neighbourhood of size "a day" [f: *5*]
B13 Correct identification of the day corresponding to the relative maximum with greater degree of growth on left neighbourhood of size "a day" but writing of the next day too, in which the money remains constant [f: *3,4*]
B14 Affirmative answered to the question *d*, justifying with the affirmation "she could have spent the earned money": forming correct hypotheses on the base of experimental data [d: *yes, because* ….]
B15 Negative answer to the question *d*, justifying with the affirmation "she spent 4 euro" or "her budget would have become 13 euro: not forming correct hypotheses on the base of experimental data [d: *not, because…*]
B16 Affirmative answer to the question *d*, justifying with the affirmation "she earns 8 euro": not forming correct hypotheses on the base of experimental data and wrong interpretation of the graph as earned money [d: *yes, because* ….]

3.4 STATISTICAL METHODOLOGY AND ANALYSES OF THE TESTS

Through the questions of the euro-days test, we would have had to investigate the acquisition of the following competences:

	Primary school	Secondary school
Initial investigated competences	C1, C2, C6	C1, C2, C5, C6, C7, C8

Really the results of the pre-test showed that students of Primary school just possessed C1 and students of Secondary school possessed C1, C2, C5. So our analysis focused only on the acquisition of following competences:

	Primary school	Secondary school
Actual investigated competences	C2, C6	C6, C7, C8

The analysis of the tests was made by the implicative statistical software *Chic*. In particular, the *supplementary variables* method (Fazio & Spagnolo, 2008) was utilized. A table with double input was filled in which the 1st column was constituted by the symbols B_j (j= 1, …,16) of students' behaviours, showed in the previous paragraph. Instead, in the 1st row there were the symbols S_i that represent the students. S_i are vector variables, called "student variables", having components s_{ij}, putted in column under S_i, with binary values "0" or "1":

$$s_{ij} = \begin{cases} 1 & \text{if the student } Si \text{ follows the behaviour } Bj \\ 0 & \text{if the student } Si \text{ doesn't follow the behaviour } Bj \end{cases}$$

Moreover, on the right side of the "student variables" were inserted "supplementary variables", which we used as correct models of students' behaviour. By the software *Chic* the "student variables" and the "supplementary variables" are analyzed together in *trees of* *similarity, using* Lermann's similarity index. This index classifies variables and groups them according to hierarchical levels (similarities). It follows the Poisson law and it is defined as follows: $s(a,b)=\frac{n_{a\wedge b}-\frac{n_a n_b}{n}}{\sqrt{\frac{n_a n_b}{n}}}$. Where n_a and n_b are, respectively, the occurrences of A and B. The Lermann's similarity index is related to the implication index, $q(a,\bar{b})$, by the following formula: $\frac{q(a,\bar{b})}{s(a,b)}=-\sqrt{\frac{nb}{n\bar{b}}}$

The purpose of this type of analysis is to visualize how the "student variables" are similarly grouped respect to the "supplementary variables". If the behaviour of a student is similar to a certain behaviour model it will belong to its similarity group. Of course, the choice of supplementary variables is fundamental to investigate about the research questions. They depend by the competences that we want to analyze.

So we chose supplementary variable to investigate these competences, adjusted for school level. In the case of Primary school we have 3 supplementary variables, one for C2 (that we call *Int*), one for C6 (that we call *MaxR*) and one that combines together C2 and C6 (that we call *MaxR-Int*). The name of the supplementary variables remind the concerning competences. They are showed in the following tables:

Primary school Students						
	Student variables			**Supplementary variables**		
	…	**S_i**	**…**	**MaxR**	**Int**	**MaxR-Int**
B2	0 or 1	0 or 1	0 or 1	1	0	1
B3	0 or 1	0 or 1	0 or 1	0	0	0
B4	0 or 1	0 or 1	0 or 1	0	0	0
B7	0 or 1	0 or 1	0 or 1	0	0	0
B8	0 or 1	0 or 1	0 or 1	0	0	0
B9	0 or 1	0 or 1	0 or 1	0	0	0
B5	0 or 1	0 or 1	0 or 1	0	1	1
B6	0 or 1	0 or 1	0 or 1	0	0	0

For reason of space, we do not show all the student variables. Supplementary variables are chosen to cover every type of correct student's behaviours. So, for instance, **MaxR** represents students who correctly answer to the questions about the reading of relative maximum and answer wrongly or do not answer to the question about the size of interval. **Int** is complementary to **MaxR**. **MaxR-Int** represents students that posses both competences, about relative maxima and size of intervals.

The supplementary variables for the Secondary school are the following seven:

Secondary school Students										
	Student variables			**Supplementary variables**						
	…	**S_i**	**…**	**MaxR**	**Grow**	**Hp**	**MaxR-Grow**	**MaxR-Hp**	**Grow-Hp**	**MaxR-Grow-Hp**
B2	0 or 1	0 or 1	0 or 1	1	0	0	1	1	0	1
B3	0 or 1	0 or 1	0 or 1	0	0	0	0	0	0	0
B4	0 or 1	0 or 1	0 or 1	0	0	0	0	0	0	0
B10	0 or 1	0 or 1	0 or 1	1	1	1	1	1	1	1
B11	0 or 1	0 or 1	0 or 1	0	1	0	1	0	1	1
B12	0 or 1	0 or 1	0 or 1	0	0	0	0	0	0	0
B13	0 or 1	0 or 1	0 or 1	0	0	0	0	0	0	0
B14	0 or 1	0 or 1	0 or 1	0	0	1	0	1	1	1
B15	0 or 1	0 or 1	0 or 1	0	0	0	0	0	0	0

B16	0 or 1	0 or 1	0 or 1	0	0	0	0	0	0	0

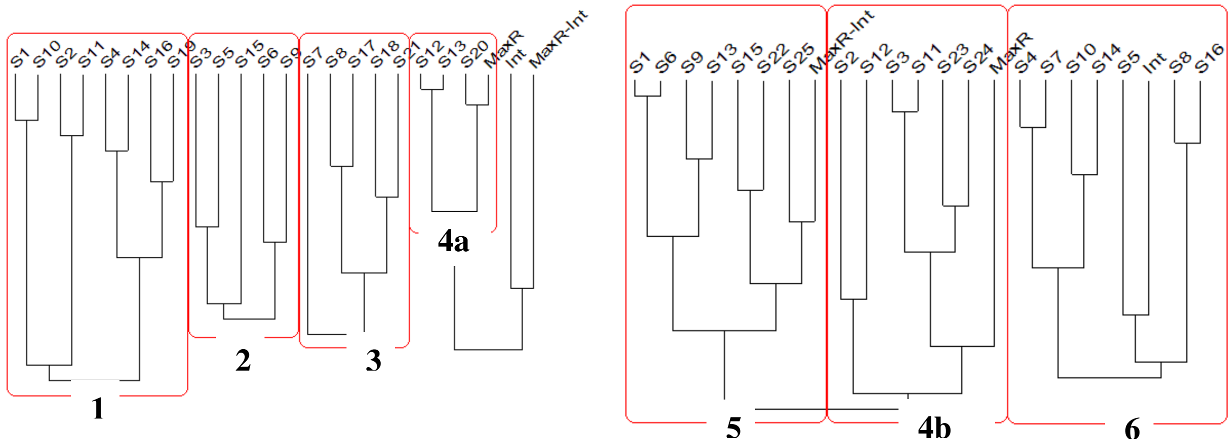
They were constructed in similar way respect to supplementary variables of Primary school. In the next paragraph we will show the graphs of the statistical implicative analysis. It will be visualized how "student variables" similarly grouped respect to "supplementary variables". So it is understood for each student if his behaviour is similar to a certain behavioural model rather than to another. Of course, since we chose "supplementary variables" as correct model of student behaviours, the research question would have had a positive answer if there had been an improvement from the pre-test to the post-test in the similarity of the "student variables" respect to the "supplementary variables".
Moreover the tree of similarity can be read also in individual terms, observing the improvements of the single student in base to the group of which he takes part in the graphs of the pre-test and of the post test.

4. EXPERIMENTAL RESULTS

4.1 EXPERIMENTAL RESULTS FOR PRIMARY SCHOOL

Analysing the tests of the 5th class of Primary school with *Chic* the following graphs were obtained:

PRE-TEST PRIMARY SCHOOL **POST-TEST PRIMARY SCHOOL**

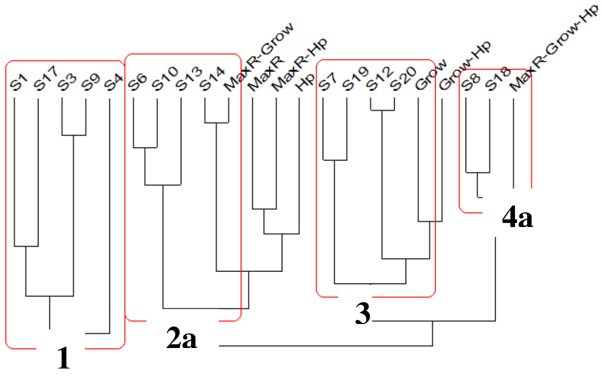


In the tree of similarity of the pre-test for Primary school it is observed that the variables are divided in four groups: 1, 2, 3, 4a. The supplementary variable *MaxR* characterizes the group **4a**. So the three students that belong to this group are students that correctly read the relative maxima. The other two supplementary variables belong to no similarity group. It means that the remaining students did not possess the competences of reading of relative maxima and size of the interval.
In the graph of the post-test there are three groups of similar variables: 4b, 5, 6. Each group is characterized by a supplementary variable. The group **4b,** as **4a** of the pre-test, is characterized by *MaxR*, but it is increased since it consists of 6 students. In the group **5** there is *MaxR-Int*, so the 7 students correctly read the relative maxima and the size of the intervals. The group **6,** with variable *Int,* is the group of the students who correctly identify the size of the interval. So we can affirm that all the students have shown improvement, some in one competence, and others in both.
Also, the improvement of single students can be observed comparing their membership to similarity groups in pre-test and post-test. For example, S1, after the didactical activity, acquired both the competences.

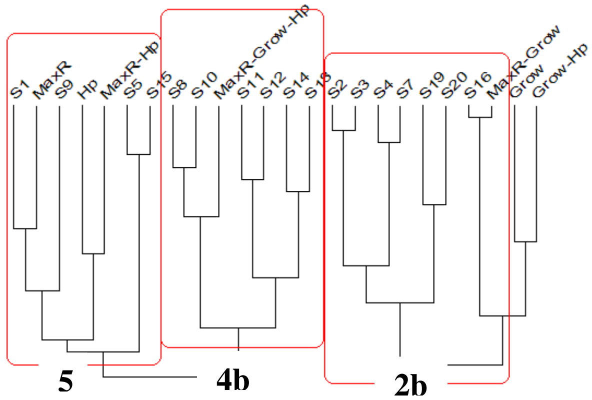
4.2 EXPERIMENTAL RESULTS FOR SECONDARY SCHOOL

Analysing the tests of the 4th class of Secondary school with *Chic* the following graphs were obtained:

PRE-TEST SECONDARY SCHOOL



POST-TEST SECONDARY SCHOOL:



In the graph of the pre-test, variables are divided into 4 groups: 1, 2a, 3, 4a. Group **1** has not got supplementary variables, so it is constituted by students who do not possess any of the investigated competences. Group **2a** is characterized by *MaxR-Grow*, so it is formed by students who correctly read the relative maxima and confront the slope of a curve in different tracts. The students in group **3**, similar to *Grow*, correctly confront the slopes of a curve. In group 4a the students possess the three mentioned competences, similarly to *MaxR-Grow-Hp*.
In the graph concerning to the post-test, student variables are divided into three groups: 5, 4b, 2b. Groups **2b** and **4b** are analogous to groups 2a and 4a of the pre-test, but they are increased. The group **5** is constituted by students who correctly read the relative maximum, like *MaxR*.
The improvement of the single students can be noted from these graphs. For instance, *S*12 before the didactical activity, correctly confront the slopes of a curve, after he learnt to read the relative maximum and to form hypotheses as well.
Moreover it is observed by both the graphs that competence *forming hypotheses on the experimental data base* is subordinate to the competences of reading of the relative maxima and confronting the slope of a curve.

5. CONCLUSIONS

In the historical development of the concept of function, strong intersections of this concept with kinematics can be found. Besides, Cartesian representation of function preceded the analytical one. So, this paper shows a didactic proposal to introduce function concept with a kinematics-graphical approach, using a motion sensor. Moreover, the authors made an analysis of competences concerning to reading and understanding of a function graph. Since the didactical activities scheduled the study of Cartesian graph with kinematics variables, a comparison between mathematical and physical competences was elaborated. This comparison conducted to chose the phases of the didactical activities, to optimize the acquisition of the competences. The activities, conducted in a 5th class of Primary school and in a 4th class of Secondary school, were adjusted to the school level and to the competences that were wanted to reach.
The effectiveness of teaching/learning process was valued through a statistical analysis implicativa of pre-test and post-test. In particular, the method of the supplementary variables was used. The supplementary variables were correct models of students' behaviours. The comparison of similarity between student variables and supplementary variables, in the pre-test and in the post-test, allows analyzing the improvement of the students subsequent to the didactic activity. The results of such analysis showed that all students of Primary school

improved in one or both the investigated competences: individuating relative maxima and size of intervals. Likewise, students of Secondary acquired one, two or three of the investigated competences: individuating relative maxima, confronting the slope of a curve in different tracts and forming hypothesis and conjecture

Another result of the analysis consists in the fact that, the students of Primary school showed an improvement more significant respect to the other students. In fact, as it can be noticed by the similarity tree, the improvement was decisive in the case of Primary school. This result can be justified in terms of *didactical contract* (Brousseau, 1997). The didactical contract becomes stronger with advancing of school levels. Equally, mathematical language used at school becomes more formal in the highest classes and, often, less contextualized in real life. In both cases, these research outcomes showed in this paper give a positive answer to the research question regarding the usefulness of the motion sensor for reading and understanding function graphs.

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